International Journal of Mathematical Archive-8(7), 2017, 19-24
Available online through www.ijma.info ISSN 2229 – 5046

CHAOS SYNCRONIZATION OF THE PHOTOGRAVITATIONAL MAGNETIC BINARIES PROBLEM VIA NONLINEAR CONTROL

MOHD. ARIF*

Dept. of Mathematics,
Zakir Husain Delhi College (Delhi University), New Delhi 2, India.

(Received On: 16-06-17; Revised & Accepted On: 05-07-17)

ABSTRACT

In this article we have discussed the chaos synchronization of the photogravitational magnetic-binaries problem when the bigger primary is a source of radiation. We have designed a nonlinear controller based on the Lyapunov stability theory. Simulation studies are conducted to show the effectiveness of the proposed method.

Key words: *Space dynamics, photogravitational magnetic-binaries problem, synchronization, Lyapunov stability theory.*

1 INTRODUCTION

In the last several decades, much effort has been devoted to the study of nonlinear chaotic systems. Chaos control and synchronization are especially noteworthy and important research fields leveling to affect dynamics of chaotic systems in order to apply them for different kinds of applications that can be examined within many different scientific research. At present, there are different kinds of control methods and techniques that have been proposed for carrying out chaos control and synchronization of chaotic dynamical systems. Chaotic synchronization did not attract much attention until Pec-ora and Carroll introduced a method to synchronize two identical chaotic systems with deferent initial conditions in 1990 and they demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device. Various techniques have been proposed and implemented successfully for achieving stable synchronization between identical and non-identical systems notable among these methods, the active control scheme proposed by E. W. Bai & K. E. Lonngren 1997 has been received and successfully implemented in almost all the field of nonlinear sciences for synchronization for different systems with various techniques.

Nonlinear control is an effective method for making two different chaotic systems by synchronized. However, this method usually assumes that the Lyapunov function of error dynamic of synchronization is formed as $V = \frac{1}{2}\, e^{t} e$.

The synchronization problem via nonlinear control scheme is studied by Chen and Han in 2003, Chen 2005, Ju H. Park 2005, Amir Abbas Emadzadeh, and Mohammad Haeri 2005 and M. Mossa Al-sawalha, M.S.M. Noorani in 2009.

The different cases of the magnetic binaries problem have been studied by A. Mavragnais (1978, …… 1988) .

In 2015 Mohd Arif. have studied the equilibrium points of the photogravitational magnetic binaries problem.

In this article we have discussed the chaos synchronization of the photogravitational magnetic-binaries problem when the bigger primary is a source of radiation, here we have designed a nonlinear controller based on the Lyapunov stability. The system under consideration is chaotic for some values of parameter involved in the system. Here two systems (master and slave) are synchronized and start with deferent initial conditions. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time.

*Corresponding Author: Mohd. Arif**
Dept. of Mathematics, Zakir Husain Delhi College (Delhi University), New Delhi 2, India.

2 EQUATION OF MOTION

In formulating the problem we shall assume that the two primaries (dipoles) in which the bigger primary is a source of radiation with magnetic fields move under the influence of gravitational force and a charged particle P (small body) of charge q and mass m moves in the vicinity of these primaries. The equation of motion and the integral of relative energy in the rotating coordinate system including the effect of the gravitational forces of the primaries on the small body written as:

$$\ddot{x} - \dot{y}\, f = U_x \tag{1}$$
$$\ddot{y} + \dot{x}\, f = U_y \tag{2}$$
$$\dot{x}^2 + \dot{y}^2 = 2\mathrm{U} - \mathrm{C} \tag{3}$$

Where

$$f = 2 - \left(\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{r_2^3}\right),\ U_x = \frac{\partial U}{\partial x} \text{ and } U_y = \frac{\partial U}{\partial y}$$
$$U = (x^2 + y^2)\left\{\frac{1}{2} + \frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{r_2^3}\right\} + x\left\{\frac{q_1\mu}{\mathrm{r}_1^3} - \frac{\lambda(1-\mu)}{r_2^3}\right\} + \frac{q_1(1-\mu)}{\mathrm{r}_1} + \frac{\mu}{\mathrm{r}_2} \tag{4}$$

$r_1^2 = (x-\mu)^2 + y^2$, $r_2^2 = (x+1-\mu)^2 + y^2$, $\lambda = \frac{M_2}{M_1}$ (M_1, M_2 are the magnetic moments of the primaries which lies perpendicular to the plane of the motion). q_1 is the source of radiation of bigger primary.

Here we have assumed that the distance between the primaries as the unit of distance and the coordinate of one primary is (μ, 0, 0) then the other is ($\mu-1$, 0, 0). We also assumed that the sum of their masses as the unit of mass. If mass of the one primaries μ then the mass of the other is ($1-\mu$). We choose the unit of time in such a way that the gravitational constant G has the value unity and $q = mc$ where c is the velocity of light.

3. CHAOS SYNCHRONIZATION VIA NONLINEAR CONTROL

Let

$$x = x_1,\ \ \dot{x} = x_2,\ y = x_3,\ \dot{y} = x_4$$

Then the equation (1) and (2) can be written as:

$$\dot{x_1} = x_2 \tag{5}$$
$$\dot{x_2} = 2\, x_4 + x_1 + A_1 \tag{6}$$
$$\dot{x_3} = x_4 \tag{7}$$
$$\dot{x_4} = -2\, x_2 + x_3 + A_2 \tag{8}$$

Where

$$A_1 = -3\, x_1 \left\{\frac{q_1(x_1-\mu)^2}{\mathrm{r}_1^5} + \frac{\lambda\,(x_1+1-\mu)^2}{\mathrm{r}_2^5}\right\} - 3{x_3}^2\left\{\frac{q_1(x_1-\mu)}{\mathrm{r}_1^5} + \frac{\lambda\,(x_1+1-\mu)}{\mathrm{r}_2^5}\right\} + \left\{\frac{q_1(x_1-\mu)}{\mathrm{r}_1^3} + \frac{\lambda\,(x_1+1-\mu)}{\mathrm{r}_2^3}\right\}$$
$$+x_1\left\{\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{\mathrm{r}_2^3}\right\} - \left\{\frac{q_1(1-\mu)(x_1-\mu)}{\mathrm{r}_1^3} - \frac{\mu(x_1+1-\mu))}{\mathrm{r}_2^3}\right\} - x_4\left(\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{r_2^3}\right),$$
$$A_2 = -3\, x_1 x_3 \left\{\frac{q_1(x_1-\mu)}{\mathrm{r}_1^5} + \frac{\lambda\,(x_1+1-\mu)}{\mathrm{r}_2^5}\right\} - 3{x_3}^3\left\{\frac{q_1}{\mathrm{r}_1^5} + \frac{\lambda}{\mathrm{r}_2^5}\right\} + x_4\left(\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{r_2^3}\right) + 2\, x_3\left\{\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{\mathrm{r}_2^3}\right\} - x_3\left\{\frac{q_1(1-\mu)}{\mathrm{r}_1^3} - \frac{\mu}{\mathrm{r}_2^3}\right\},$$
$$r_1^2 = (x_1-\mu)^2 + {x_3}^2,\ \ r_2^2 = (x_1+1-\mu)^2 + {x_3}^2,$$

Corresponding to master system (5, 6, 7 and 8), the identical slave system are:

$$\dot{y_1} = y_2 + u_1(t) \tag{9}$$
$$\dot{y_2} = 2\, y_4 + y_1 + A_3 + u_2(t) \tag{10}$$
$$\dot{y_3} = y_4 + u_3(t) \tag{11}$$
$$\dot{y_4} = -2\, y_2 + y_3 + A_4 + u_4(t) \tag{12}$$

Where

$$A_3 = -3\, y_1 \left\{\frac{q_1(y_1-\mu)^2}{\mathrm{r}_1^5} + \frac{\lambda\,(y_1+1-\mu)^2}{\mathrm{r}_2^5}\right\} - 3{y_3}^2\left\{\frac{q_1(y_1-\mu)}{\mathrm{r}_1^5} + \frac{\lambda\,(y_1+1-\mu)}{\mathrm{r}_2^5}\right\} + \left\{\frac{q_1(y_1-\mu)}{\mathrm{r}_1^3} + \frac{\lambda\,(y_1+1-\mu)}{\mathrm{r}_2^3}\right\}$$
$$+y_1\left\{\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{\mathrm{r}_2^3}\right\} - \left\{\frac{q_1(1-\mu)(y_1-\mu)}{\mathrm{r}_1^3} - \frac{\mu(y_1+1-\mu))}{\mathrm{r}_2^3}\right\} - y_4\left(\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{r_2^3}\right),$$
$$A_2 = -3\, y_1 y_3 \left\{\frac{q_1(y_1-\mu)}{\mathrm{r}_1^5} + \frac{\lambda\,(y_1+1-\mu)}{\mathrm{r}_2^5}\right\} - 3{y_3}^3\left\{\frac{q_1}{\mathrm{r}_1^5} + \frac{\lambda}{\mathrm{r}_2^5}\right\} + y_4\left(\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{r_2^3}\right) + 2\, y_3\left\{\frac{q_1}{\mathrm{r}_1^3} + \frac{\lambda}{\mathrm{r}_2^3}\right\} - y_3\left\{\frac{q_1(1-\mu)}{\mathrm{r}_1^3} - \frac{\mu}{\mathrm{r}_2^3}\right\},$$
$$r_1^2 = (y_1-\mu)^2 + {y_3}^2,\ \ r_2^2 = (y_1+1-\mu)^2 + {y_3}^2,$$

where $u_i(t)$; i =1 ,2,3,4 are control functions to be determined. Let $e_i = y_i - x_i$; i = 1, 2, 3, 4 be the synchronization errors. From (5) to (12), we obtain the error dynamics as follows:

$$\dot{e_1} = e_2 + u_1(t) \tag{13}$$
$$\dot{e_2} = 2e_4 + e_1 + A_3 - A_1 + u_2(t) \tag{14}$$
$$\dot{e_3} = e_4 + u_3(t) \tag{15}$$
$$\dot{e_4} = -2e_2 + e_3 + A_4 - A_2 + u_4(t) \tag{16}$$

According to the Lyapunov stability theory, when controller satisfies the assumption with $V(e) = \frac{1}{2}\, e^t e$ a positive definite function and the first derivative of this function $\dot{V} < 0$ the chaos synchronization of two identical systems (master and slave) for different initial conditions is achived.

Let a positive definite Lyapunov function

$$V(e) = \frac{1}{2}\, e^t e$$

Then we have the first derivative of $V(e)$:

$$\dot{V} = e_1[e_2 + u_1(t)] + e_2[2e_4 + e_1 + A_3 - A_1 + u_2(t)] + e_3[e_4 + u_3(t)] + e_4[-2e_2 + e_3 + A_4 - A_2 + u_4(t)] \tag{17}$$

Therefore, if we choose the controller u as follows,

$$u_1 = -2e_2 - e_1 \tag{18}$$

$$u_2 = -e_2 - A_3 + A_1 \tag{19}$$

$$u_3 = -e_3 - 2e_4 \tag{20}$$

$$u_4 = -A_4 + A_2 - e_4 \tag{21}$$

Then

$$\dot{V} = -{e_1}^2 - {e_2}^2 - {e_3}^2 - {e_4}^2 < 0 \tag{22}$$

Hence the error state

$$\lim_{t\to\infty} \|e(t)\| = 0$$

which gives asymptotic stability of the system. This means that the controlled chaotic systems (5, 6, 7, 8) and (9, 10, 11, 12) is synchronized for any initial conditions.

4. NUMERICAL SIMULATION

We select the parameters $\mu = .0001$ $q_1 = 1.5$ and $\lambda = 1$, the state orbits of the chaotic system are shown in Figure 1, with the different initial conditions $[x_1(0) = 0.0, x_2(0) = 12.0, x_3(0) = 1.0, x_4(0) = 4.0]$ for master systems and $[y_1(0) = 10.0, y_2(0) = 2.0, y_3(0) = 4.0, y_4(0) = 0.0]$ for slave systems Simulation results for uncoupled system are presented in figures.2,4,6,8 and that of controlled system are shown in figures.3,5,7 and 9 respectively. Figures (10, 11, 12, 13) display the chaos-synchronization errors of systems.

It can be seen from the figures that the chaos-synchronization errors converge to zero rapidly.

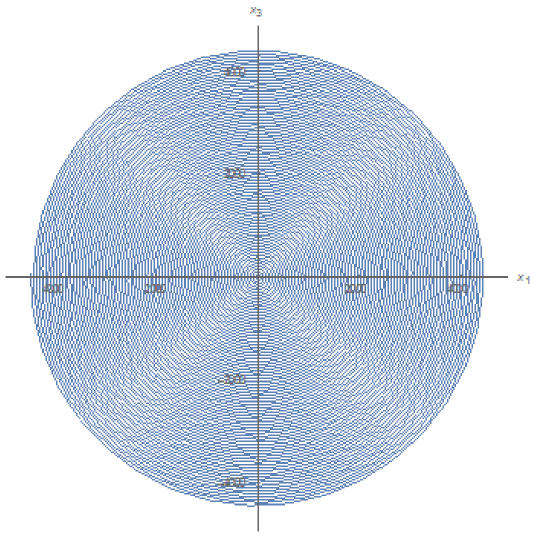

Figure-1

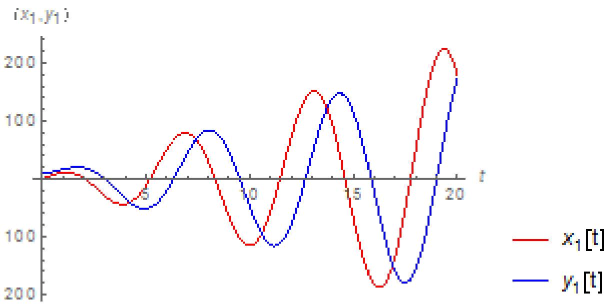

Figure-2

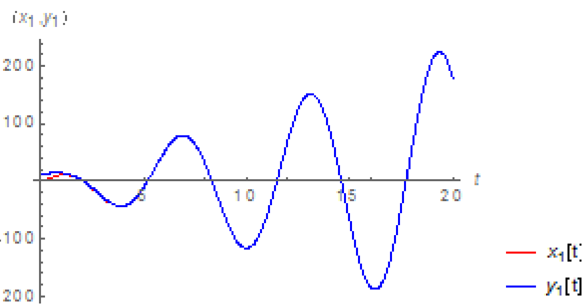

Figure-3

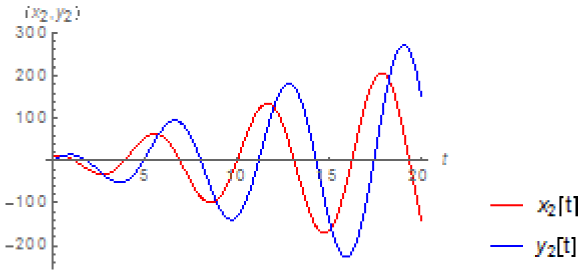

Figure-4

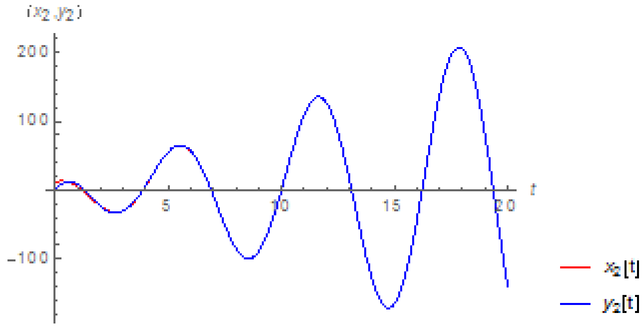

Figure-5

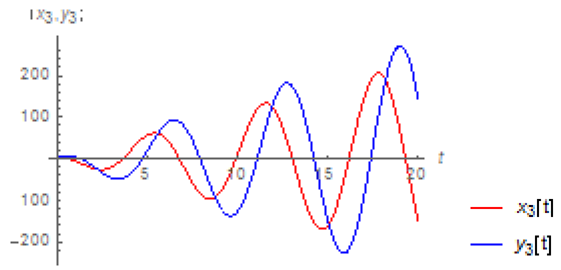

Figure-6

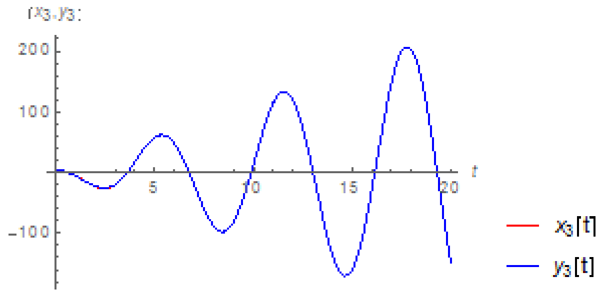

Figure-7

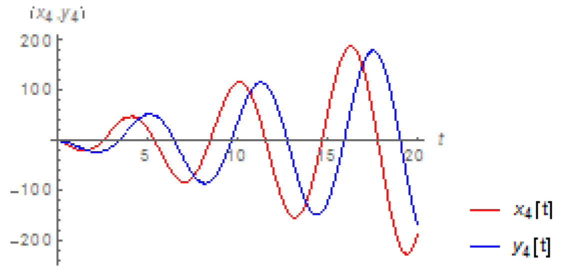

Figure-8

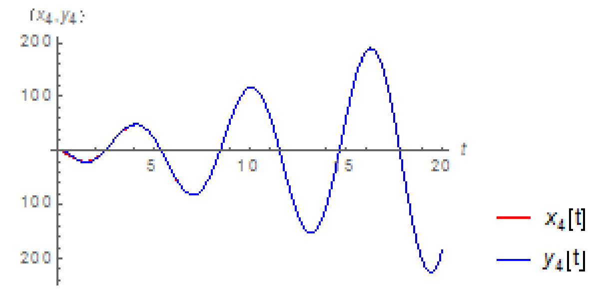

Figure-9

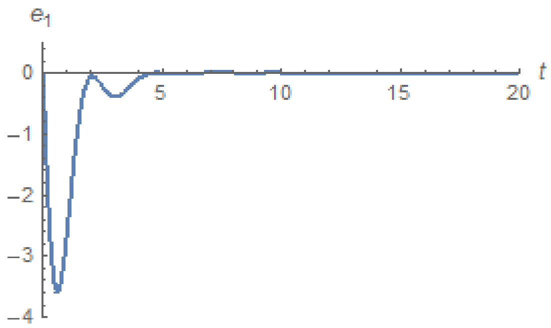

Figure-10

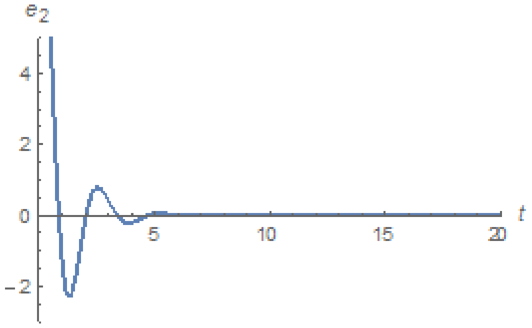

Figure-11

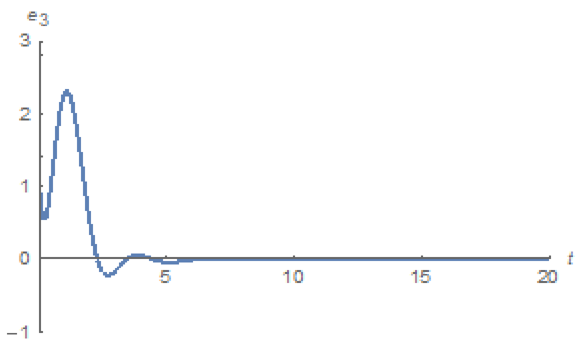

Figure-12

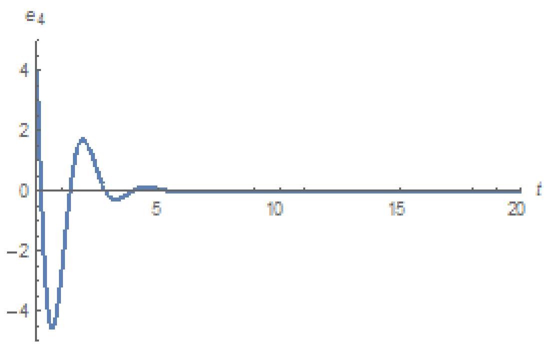

Figure-13

5. CONCLUSION

An investigation on chaos synchronization in the photogravitational magnetic-binaries problem when the bigger primary is a source of radiation including the effect of the gravitational forces of the primaries on the small body, via nonlinear controller based on the Lyapunov stability theory have been made. Here two systems (master and slave) are synchronized and start with deferent initial conditions. This problem may be treated as the design of control laws for chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with master system. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time. The results were validated by numerical simulations using Mathematica.

REFRENCES

1. Amir Abbas Emadzadeh, and Mohammad Haeri, (2005) Global Synchronization of Two Different Chaotic Systems via Nonlinear Control KINTEX, Gyeonggi-Do, Korea ICCAS Jun 2-5.
2. N. NJAH, K. S. OJO. (2010) Backstepping control and synchronization of parametrically and externally excited Φ6 van der pol oscillators with application to secure communications International Journal of Modern Physics B Vol. 24, No. 23 4581–4593.
3. Ahmad Taher Azar. (2012) Modeling and Control of Dialysis Systems: Volume 2: Biofeedback Systems and Soft Computing Techniques of Dialysis, Springer-Verlag GmbH, Heidelberg Germany.
4. Ayub Khan, Priyamyada Tripathi. (2013) Synchronization, Anti-Synchronization and Hybrid-Synchronization Of A Double Pendulum Under The Effect Of External Forces International Journal Of Computational Engineering Research (ijceronline.com) Vol. 3 Issue. 1 pp.166-176.
5. Ayub Khan, Rimpipal, (2013) Complete synchronization, anti-synchronization and hybrid synchronization of two identical parabolic restricted three body problem Asian Journal of Current Engineering and Maths 2: 2 118 - 126.
6. Khan, M. Shahzad.(2013) Synchronization of a circular restricted three body problem with Lorenz hyper chaotic system using robust adaptive sliding mode controller. *Complexity*, 18(6), 58-64.
7. Bai E. W., Lonngren K. E. (1997). Synchronization of two Lorenz systems using active control, Chaos, Solitons and Fractals, Vol. 8.
8. C. Liu, T. Liu, L. Liu and K. Liu, (2004) A new chaotic attractor, Chaos, Solitons and Fractals, Vol. 22, pp. 1031-1038pp. 51-58.
9. Chen M, Han Z. (2003) Controlling and synchronizing chaotic Genesio system via nonlinear feedback control. Chaos, Solitons & Fractals; 17:709–16.
10. Chen HK. (2005) Global chaos synchronization of new chaotic systems via nonlinear control. Chaos, Solitons & Fractals; 23:1245–51.
11. G. Leonov, H. Nijmeijer, A. Pogromsky and A. Fradkov,(2010). Dynamics and Control of Hybrid Mechanical Systems, World Scientific, Singapore.
12. Henon, M., (1965), Ann. Astrophys. 28, 499.
13. H. K. Chen. (2005). Synchronization of Two Different Chaotic System: A New System and Each of the Dynamical Systems Lorenz, Chen and Lu, Chaos Solitons & Fractals 25, 1049– 1056.
14. Hill, G. W. (1878). Reasearches in the lunar theory, Am.J.Math. 1, 5-26, pp. 129-147, 245-261.
15. Israr Ahmad, Azizan Bin Saaban, Adyda Binti Ibrahim, Said Al-Hadhrami, Mohammad Shahzad, Sharifa Hilal Al- Mahrouqi, (2015) A research on adaptive control to stabilize and synchronize a hyperchaotic system with uncertain parameters. An International Journal of Optimization and Control: Theories & Applications Vol.5, No.2, pp.51-62.
16. J.H. Park, (2006).Chaos synchronization of nonlinear Bloch equations," Chaos, Solitons and Fractals, vol. 27, pp. 357-361.
17. *L. Lu*, *C. Zhang* and Z.A. Guo, (*2007).* Synchronization between two different chaotic systems with nonlinear feedback control, Chinese Physics, 16, pp.1603-1607.
18. Lei Y., Xu W., Xie W.(2007) Synchronization of two chaotic four-dimensional systems using active control, Chaos, Solitons and Fractals, Vol. 32, pp. 1823- 1829.
19. M. C. Ho and Y.C. Hung. (2002). Synchronization two Different Systems by using Generalized Active Control, Phys. Lett. A 301, 424–428.
20. M. Haeri and A. Emadzadeh, (2007). Synchronizing diferent chaotic systems using active sliding mode control, Chaos, Solitons and Fractals. 31 pp.119-129.
21. M. Mossa Al-sawalha and M.S.M. Noorani. (2009). Anti-synchronization Between Two Different Hyperchaotic Systems. Journal of Uncertain Systems Vol.3, No.3, pp.192-200.
22. Mavraganis A (1978). Motion of a charged particle in the region of a magnetic-binary system. Astroph. and spa. Sci. 54, pp. 305-313.
23. Mavraganis A (1978). The areas of the motion in the planar magnetic-binary problem. Astroph. and spa. Sci. 57, pp. 3-7.
24. Mavraganis A (1979). Stationary solutions and their stability in the magnetic-binary problem when the primaries are oblate spheroids. Astron Astrophys. 80, pp. 130-133.

25. Mavraganis A and Pangalos. C. A. (1983) Parametric variations of the magnetic -binary problem Indian J. pure appl, Math. 14(3), pp. 297- 306.
26. Moulton, F. R., (1920), An Introduction to Celestial Mechanics., Macmillan, New York.
27. Mohd. Arif. (2013). Motion around the equilibrium points in the planar magnetic binaries problem international journal of applied math and Mechanics. 9(20), pp.98-107
28. Mohd Arif. (2015). A study of the equilibrium points in the photogravitational magnetic binaries. Problem International journal of modern sciences and, eengineering technology Volume 2, Issue 11, pp.65-70.
29. Mohd Arif. (2016). Complete syncronization of a planner magnetic binaries problem when bigger primary is Oblate Spheriod and smaller primary is ellipsoid. International journal of mathematical Archive (IJMA) 7(8), 45-52.
30. M. Shahzad, I. Ahmed.(2013) Experimental study of synchronization & Anti-synchronization for spin orbit problem of Enceladus. *International Journal of control science & Engineering*, 2(3), 41-47.
31. Pecora L. M.,Carroll T.L.,(1990) Synchronization in chaotic systems Rev Lett., Vol. 64, pp. 821-824.
32. Papadakis. K.E. (2005), Motion Around The Triangular Equilibrium Points Of The Restricted Three-Body Problem Under Angular Velocity Variation, Astroph. and spa. Sci. 2 pp. 129-148.
33. S. H. Chen and J. Lu. (2002). Synchronization of an Uncertain Unified System via Adaptive Control, Chaos Solitons & Fractals 14, 643–647.
34. Stormer, CF (1907). Arch. Sci. Phys.et Nat.Geneva, pp. 24-350.
35. Szebehely. V. (1967) Theory of orbit Academic press New York.
36. Tang F.,Wang L. (2006) An adaptive active control for the modified Chua's circuit, Phys. Lett. A, Vol. 346, pp. 342-346.
37. U. E. Vincent.(2008). Synchronization of Identical and Non-identical 4-D Chaotic Systems Using Active Control, Chaos Solitons Fractals 37, no. 4, 1065–1075.
38. Ucar A., Lonngren K. E., Bai E. W. (2007) Chaos synchronization in RCL-shunted Josephson junction via active control, Chaos, Solitons and Fractals, Vol. 31, pp. 105-111.
39. W. Hongwu and M. Junhai, , (2012).Chaos Controland Synchronization of a Fractional-order Autonomous System, WSEAS Trans. on Mathematics. 11, pp. 700-711.
40. Y. Wang, Z.H. Guan and H.O. Wang, (2003).Feedback an adaptive control for the synchronization of Chen system via a single variable, Phys. Lett A. 312 pp.34-40.
41. Z. M. Ge and C. C. Chen. (2004) Phase Synchronization of Coupled Chaotic Multiple Time Scale Systems, Chaos Solitons & Fractals 20, 639–647.

Source of support: Nil, Conflict of interest: None Declared.