

**EFFECTS OF MAGNETIC FIELD WITH VARIABLE VISCOSITY AND AN ENDOSCOPE ON PERISTALTIC TRANSPORT OF COUPLE STRESS FLUIDS THROUGH A POROUS MEDIUM**

**Dr. N. G. SRIDHAR\***

**Govt. First Grade College & P. G. Centre, Sedam-585 222, Karnataka, India.**

*(Received On: 14-06-17; Revised & Accepted On: 04-07-17)*

---

**ABSTRACT**

*The peristaltic transport of couple stress fluid in a variable viscosity through the gap between coaxial tubes through a porous medium with magnetic field has been studied. Where outer tube is non-uniform sinusoidal wave traveling down in wall and inner tube is rigid. The relation between viscosity, pressure gradient and friction force on inner and outer tubes have been obtained in terms of couple stress parameter through a porous medium. The numerical solution of pressure gradient, outer friction, inner friction and flow rate are shown graphically.*

**Keywords:** *Peristaltic transport, Couple Stress fluid, Porous Media, Magnetic field, Viscosity parameter.*

---

**1. INTRODUCTION**

The aim of this paper is to study a physiological situation with the presence of an endoscope placed concentrically. Alsaedi *et al.* [1] studied peristaltic flow of couple stress fluid through uniform porous medium. Ayman and Sobh [2] investigated peristaltic transport of a magneto-newtonian fluid through a porous medium. Jayarami Reddy *et al.*, [3] have studied the peristaltic flow of a Williamson fluid in an inclined planar channel under the effect of a magnetic field. Gupta and Sheshadri [4] studied peristaltic transport of a Newtonian fluid in non-uniform geometries. El-dabe and El-Mohandis [5] have studied magneto hydrodynamic flow of second order fluid through a porous medium on an inclined porous plane. Habtu and Radhakrishnamacharya [6] studied dispersion of a solute in peristaltic motion of a couple stress fluid through a porous medium with slip condition. Mekhemier [7] studied non linear peristaltic transport a porous medium in an inclined planar channel. Mekhemier and Abd elmaboud [8] studied peristaltic flow of a couple stress fluid in an annulus: application of an endoscope.

Srivastava and Srivastava [9] have investigated peristaltic transport of a non-newtonian fluid: applications to the vas deferens and small intestine. Srivastava *et al.* [10] investigated peristaltic transport of a physiological fluid: part I flow in non- uniform geometry. Sridhar [11] studied effect of thickness of the porous material using porous media on the peristaltic pumping of couple stress fluid through non - erodible porous lining tube. Cotton and Williams [12] investigated practical gastrointestinal endoscopy. Ramchandra and Usha [13] studied the influence of an eccentrically inserted catheter on the peristaltic pumping in a tube under long wavelength and low Reynolds numbers approximations. Raptis and Peridiki [14] are investigated flow of a viscous fluid through a porous medium bounded by a vertical surface. Latham [15] investigated the fluid mechanics of peristaltic pump and science. Rathod and Sridhar [16] investigated Peristaltic transport of couple stress fluid in uniform and non-uniform annulus through porous medium. Rathod and Asha [17] studied Effect of couple stress fluid and an endoscope in peristaltic motion. Rathod and Asha [18] studied effects of magnetic field and an endoscope on peristaltic motion. Rathod *et.al.*, [19] studied peristaltic pumping of couple stress fluid through non - erodible porous lining tube wall with thickness of porous material Rathod and Laxmi [20] studied effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in a channel. Rathod and Laxmi [21] investigated effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in an inclined channel. Rathod and Laxmi [22] studied peristaltic transport of a conducting fluid in an asymmetric vertical channel with heat and mass transfer. Rathod and Laxmi [23] investigated slip effect on peristaltic transport of a conducting fluid through a porous medium in an asymmetric vertical channel by Adomian decomposition method. Rathod and Mahadev [24] studied of ureteral peristalsis with Jeffrey fluid flow. Rathod and Mahadev [25] Studied of ureteral peristalsis in cylindrical tube through porous medium. Rathod and Mahadev [26] studied Slip effects and heat transfer on MHD peristaltic flow of Jeffrey fluid in an inclined channel. Rathod and Mahadev [27] studied effect of magnetic field on ureteral peristalsis in cylindrical tube. Rathod and Asha [28] have investigated peristaltic transport of a couple stress fluid In a uniform and non-uniform annulus. Rathod and

---

**Corresponding Author: Dr. N. G. Sridhar\***

**Govt. First Grade College & P. G. Centre, Sedam-585 222, Karnataka, India.**

Mahadev [29] investigated effect of thickness of the porous material on the peristaltic pumping of a Jeffry fluid with non - erodible porous lining wall. Rathod *et al.*, [30] studied effect of magnetic field on peristaltic transport of a couple stress fluid in a channel. Rathod and Sridhar [31] studied peristaltic flow of a couple stress fluid in an inclined channel. Rathod and Pallavi [32] studied the effect of slip condition and heat transfer on MHD peristaltic transport through a porous medium with complaint wall. Rathod and Pallavi [33] studed the influence of wall properties on MHD Peristaltic transport of dusty fluid. Rathod and Pallavi [34] investigated the influence of wall properties on Peristaltic transport of dusty fluid through porous medium. Rathod *et al.*, [35] studied peristaltic flow of a couple stress fluid in an inclined channel under the effect of magnetic field. Rathod *et al.*, [36] studied peristaltic transport of a conducting couple stress fluid through a porous medium in a channel. Abd elmaboud and Mekheimer [37] study non-linear peristaltic transport of a second-order fluid through a porous medium.

We propose to study peristaltic transport of a viscous incompressible fluid (creeping flow) through gap between coaxial tubes, where outer tube is non-uniform and has a sinusoidal wave traveling down its wall and inner one is rigid, uniform tube and moving with a constant velocity. This investigation may have clinical applications such as endoscopes problem. In this paper, peristaltic transport of a couple stress fluid, variable viscosity in an endoscope through a porous medium with magnetic field is investigated.

## 2. FORMULATION OF THE PROBLEM

Consider the peristaltic flow of an couple stress fluid through coaxial tubes such that the outer tube is non-uniform and has a sinusoidal wave traveling down its wall and the inner one is rigid, uniform and moving with a constant velocity. The geometry of the two wall surfaces are:

$$r_1^* = a_1 \quad (1)$$

$$r_2^* = a_{20} + b \sin\left(\frac{2\pi}{\lambda}(z^* - ct^*)\right) \quad (2)$$

Where  $a_1$  is the radius of endoscope tube,  $a_{20}$  is radius of the small intestine at inlet,  $b$  is the wave amplitude,  $\lambda$  is the wavelength,  $t$  is time and  $c$  is the wave speed.

We choose cylindrical coordinate system  $(r^*, z^*)$ , the flow in the gap between inner and outer tube is unsteady but if we choose moving coordinates  $(r^*, z^*)$  which travel in the  $z$ -axis lies along centerline of inner and outer tubes and  $r^*$  is distance measured radially. The coordinate frames are related through

$$z^* = Z^* - ct^*, \quad r^* = R^*, \quad (3)$$

$$w^* = W^* - c, \quad u^* = U^*$$

Where,  $U^*$ ,  $W^*$  and  $u^*$ ,  $w^*$  are the velocity components in the radial and axial direction in the fixed and moving coordinates, respectively.

The Navier - Stokes equations are:

$$\frac{1}{r^*} \frac{\partial(r^*, u^*)}{\partial r^*} + \frac{\partial(w^*)}{\partial z^*} = 0 \quad (4)$$

$$\rho \left\{ u^* \frac{\partial u^*}{\partial r^*} + w^* \frac{\partial u^*}{\partial z^*} \right\} = -\frac{\partial p^*}{\partial r^*} + \frac{\partial}{\partial r^*} \left[ 2\mu^*(r^*) \frac{\partial u^*}{\partial r^*} \right] + 2 \frac{\partial \mu^*(r^*)}{r^*} \left( \frac{\partial u^*}{\partial r^*} - \frac{u^*}{r^*} \right) + \frac{\partial}{\partial z^*} \left[ \mu^*(r^*) \left( \frac{\partial u^*}{\partial z^*} + \frac{\partial w^*}{\partial r^*} \right) \right] - \eta \nabla^2 (\nabla^2 (u^*)) - \frac{\mu}{K} u^* - \sigma B_z^2 (u^*) \quad (5)$$

$$\rho \left\{ u^* \frac{\partial w^*}{\partial r^*} + w^* \frac{\partial w^*}{\partial z^*} \right\} = -\frac{\partial p^*}{\partial z^*} + \frac{\partial}{\partial z^*} \left[ 2\mu^*(r^*) \frac{\partial w^*}{\partial z^*} \right] + \frac{1}{r^*} \frac{\partial}{\partial r^*} \left[ \mu^*(r^*) r^* \left( \frac{\partial u^*}{\partial z^*} + \frac{\partial w^*}{\partial r^*} \right) \right] - \eta \nabla^2 (\nabla^2 (w^*)) - \frac{\mu}{K} w^* - \sigma B_z^2 (w^*) \quad (6)$$

Where,  $\nabla^2 = \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \frac{\partial}{\partial r^*} \right)$

Where  $p^*$  is pressure,  $\mu^*(r^*)$  is the viscosity function,  $\rho$  is density,  $\mu$  is viscosity,  $\eta$  is couple stress parameter,  $K$  is porous media,  $\sigma$  is electric conductivity and  $B_0$  is applied magnetic field.

The boundary conditions are:

$$\begin{aligned} w^* &= -c, \quad \nabla^2(w^*) \text{ finite at } r^* = r_1^* \\ u^* &= 0, \quad \nabla^2(w^*) = 0 \text{ at } r^* = r_2^* \end{aligned} \quad (7)$$

We introduce the non-dimensional variable and Reynolds number (Re) and wave number ( $\delta$ ) introduced:

$$\begin{aligned} z &= \frac{z^*}{\lambda^*}, \quad Z = \frac{Z^*}{\lambda^*}, \quad R = \frac{R^*}{a_{20}}, \quad \mu(r) = \frac{\mu^*(r^*)}{\mu}, \quad u = \frac{\lambda u^*}{a_{20}c}, \quad U = \frac{\lambda U^*}{a_{20}c}, \quad p = \frac{a_{20}^2}{\lambda \mu c} p^*(z^*), \quad t = \frac{t^* c}{\lambda}, \\ \text{Re} &= \frac{\rho c a_{20}}{\mu}, \quad w = \frac{w^*}{c}, \quad W = \frac{W^*}{c}, \quad K = \frac{K^*}{\lambda}, \quad \eta = l^2 \rho \gamma, \quad \delta = \frac{a_{20}}{\lambda} \ll 1, \quad M = B_0 \sqrt{\frac{\sigma}{\mu a_{20}^2}}, \quad r_1 = \frac{r_1^*}{a_{20}} = \varepsilon < 1, \\ r_2 &= \frac{r_2^*}{a_{20}} = 1 + \phi \sin 2\pi z, \quad \phi = \frac{b}{a_{20}} < 1 \end{aligned} \quad (8)$$

Where,  $\varepsilon$  is the radius ratio,  $\phi$  is the amplitude ratio,  $\mu$  is the viscosity on the endoscope.

Equation of motion and boundary conditions in dimensionless form becomes

$$\frac{1}{r} \frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \quad (9)$$

$$\begin{aligned} \text{Re} \delta^3 \left\{ u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right\} &= -\frac{\partial p}{\partial r} + \delta^2 \frac{\partial}{\partial r} \left( 2\mu(r) \frac{\partial u}{\partial r} \right) + 2\delta^2 \frac{\mu(r)}{r} \left( \frac{\partial u}{\partial r} - \frac{u}{r} \right) \\ + \delta^2 \frac{\partial}{\partial z} \left[ \mu(r) \left( \delta^2 \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) \right] &- \frac{\delta^2}{\gamma^2} \nabla^2 (\nabla^2 (u)) - \delta^2 h^2 (u) - \delta^2 M^2 (u) \end{aligned} \quad (10)$$

$$\begin{aligned} \text{Re} \delta \left\{ u \frac{\partial u}{\partial r} + w \frac{\partial w}{\partial z} \right\} &= -\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left[ \mu(r).r. \left( \delta^2 \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) \right] + \delta^2 \frac{\partial}{\partial z} \left( 2\mu(r) \frac{\partial w}{\partial z} \right) \\ - \frac{1}{\gamma^2} \nabla^2 (\nabla^2 (w)) &- h^2 (w) - M^2 (w) \end{aligned} \quad (11)$$

Where,  $\gamma = \sqrt{\frac{\eta}{\mu a_{20}^2}}$  couple-stress parameter,  $h = \sqrt{\frac{a_{20}}{K}}$  porous media &  $M = B_0 \sqrt{\frac{\sigma}{\mu a_{20}^2}}$  Hartmann number.

The dimensionless boundary conditions are:

$$\begin{aligned} w &= -1, \quad \nabla^2(u, w) \text{ finite at } r = r_1 \\ u &= 0, \quad \nabla^2(u, w) = 0 \text{ finite at } r = r_2 \end{aligned} \quad (12)$$

Using long wavelength approximation and neglecting the wave number  $\delta$ , Navier Stokes equations reduces to:

$$\frac{\partial p}{\partial r} = 0, \quad (13)$$

$$\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left[ \mu(r).r. \frac{\partial w}{\partial r} \right] - \frac{1}{\gamma^2} \nabla^2 (\nabla^2 (w)) - h^2 (w) - M^2 (w) \quad (14)$$

The instantaneous volume flow rate in the fixed coordinate system is given by

$$Q^* = \int_{r_1^*}^{r_2^*} 2\pi R^* W^* dR^* \quad (15)$$

Where,  $r_1^*$  is a constant and  $r_2^*$  is a function of  $z^*$  and  $t^*$ . On substituting (3) into (15) and on integrating, we obtain

$$Q^* = q^* + \pi c(r_2^{*2} - r_1^{*2}) \quad (16)$$

Where,

$$q^* = \int_{r_1^*}^{r_2^*} 2\pi r^* w^* dr^* \quad (17)$$

is the volume flow rate in the moving coordinate system and is it independent of time? Here,  $r_2^*$  is a function of  $z^*$  alone and is defined through (2). Using the dimensionless variable, we find that (17) becomes

$$F = \frac{q^*}{2\pi a_{20}^2 c} = \int_{r_1}^{r_2} r w dr \quad (18)$$

The time-mean flow over a period  $T = \lambda/c$  at a fixed Z position is defined as

$$Q = \frac{1}{T} = \int_0^T Q^* dt^* \quad (19)$$

Using (16) and (17) in (19) and integrating, we get

$$Q^* = q^* + \pi c \left( a_2^2 - a_1^2 + \frac{b^2}{2} \right) \quad (20)$$

which may be written as

$$\frac{Q^*}{2\pi a_{20}^2 c} = \frac{q^*}{2\pi a_{20}^2 c} + \frac{1}{2} \left( 1 - \varepsilon^2 + \frac{\phi^2}{2} \right) \quad (21)$$

On defining the dimensionless time-mean flow as

$$\Theta = \frac{Q^*}{2\pi a_{20}^2 c} \quad (22)$$

Writing (21) as

$$\Theta = F + \frac{1}{2} \left( 1 - \varepsilon^2 + \frac{\phi^2}{2} \right) \quad (23)$$

and using boundary conditions (12) to eqns.(13) & (14), we obtain

$$w = \frac{1}{2} \frac{\partial p}{\partial z} \left[ X_1 - \frac{1}{8\gamma^2} X_2 \cdot X_3 - \frac{2}{(h^2 - M^2)} \right] + \frac{1}{4\gamma^2} X_2 \quad (24)$$

Where,  $X_1 = I_1(r) - I_1(r_1) + \frac{I_1(r_1) - I_1(r_2)}{I_2(r_2) - I_2(r_1)} \{I_2(r) - I_2(r_1)\}$ ,

$$X_2 = r^2 - r_1^2 + (r_1^2 - r_2^2) \left( \frac{\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)} \right), \quad X_3 = r^2 - r_1^2 + 2(r_1^2 - r_2^2) \left( \frac{\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)} \right),$$

$$I_1(r) = \int \frac{r}{\mu(r)} dr, \quad I_2(r) = \int \frac{dr}{r \cdot \mu(r)} \quad (25)$$

Using (18) we obtain the relationship between  $dp/dz$  and F as follows:

$$F = \frac{1}{2} \frac{dp}{dz} \left[ \begin{array}{l} I_3 - I_1(r_1) \cdot \frac{(r_2^2 - r_1^2)}{2} + \frac{I_1(\eta) - I_1(r_2)}{I_2(r_2) - I_2(\eta)} \left\{ I_4 - I_2(r_1) \frac{(r_2^2 - r_1^2)}{2} \right\} \\ - \frac{1}{8\gamma^2} \left\{ \frac{\left( \frac{r_2^6 - r_1^6}{6} + \frac{r_1^2 \cdot r_2^2 \cdot (r_1^2 - r_2^2)}{2} \right)}{3 \cdot \frac{(r_1^2 - r_2^2)}{2} \{Z_1\} - \frac{(r_1^2 - r_2^2)}{2} \left\{ \frac{r_1^2}{\ln\left(\frac{r_2}{\eta}\right)} - \frac{4 \cdot (r_1^2 - r_2^2)}{\ln\left(\frac{r_2}{\eta}\right)} \right\} \cdot \{Z_2\} - \frac{(r_2^2 - r_1^2)}{(h^2 - M^2)}} \right\} \end{array} \right] + \frac{1}{4\gamma^2} \left[ \frac{(r_2^4 - r_1^4)}{4} - \frac{r_1^2 \cdot (r_2^2 - r_1^2)}{2} + \frac{(r_1^2 - r_2^2)}{\ln\left(\frac{r_2}{\eta}\right)} \{Z_2\} \right] \quad (26)$$

Where,  $Z_1 = \frac{(-1+4 \cdot \ln(\eta)) \cdot r_1^4 + (1-4 \cdot \ln(r_2)) \cdot r_2^4}{16(\eta - r_2)} - \frac{\ln(\eta) \cdot (r_2^4 - r_1^4)}{4}$ ,  $I_3 = \int_{r_1}^{r_2} \frac{r^2}{\mu(r)} dr$ ,

$Z_2 = \frac{(-1+2 \cdot \ln(\eta)) \cdot r_1^2 + (1-2 \cdot \ln(r_2)) \cdot r_2^2}{4 \cdot (\eta - r_2)} - \frac{\ln(\eta) \cdot (r_2^2 - r_1^2)}{2}$ ,  $I_4 = \int_{r_1}^{r_2} \frac{dr}{\mu(r)}$  (27)

Solving (26) for  $dp/dz$ , we obtain

$$\frac{dp}{dz} = \frac{2F - \frac{1}{2\gamma^2} \left[ \frac{(r_2^4 - r_1^4)}{4} - \frac{r_1^2 \cdot (r_2^2 - r_1^2)}{2} + \frac{(r_2^2 - r_1^2)}{\ln\left(\frac{r_2}{\eta}\right)} \{Z_2\} \right]}{\left[ \begin{array}{l} \frac{\left( \frac{r_2^6 - r_1^6}{6} + \frac{r_1^2 \cdot r_2^2 \cdot (r_1^2 - r_2^2)}{2} \right)}{3 \cdot \frac{(r_1^2 - r_2^2)}{2} \{Z_1\} - \frac{(r_1^2 - r_2^2)}{2} \left\{ \frac{r_1^2}{\ln\left(\frac{r_2}{\eta}\right)} - \frac{4 \cdot (r_1^2 - r_2^2)}{\ln\left(\frac{r_2}{\eta}\right)} \right\} \cdot \{Z_2\} - \frac{(r_2^2 - r_1^2)}{(h^2 - M^2)}} \right]} \quad (28)$$

The pressure rise  $\Delta p_\lambda$  and friction force (at the wall) on outer and inner tubes  $F_\lambda^{(o)}$  and  $F_\lambda^{(i)}$  respectively, in their non-dimensional forms, are given by

$$\Delta P_\lambda = \int_0^1 \left( \frac{dp}{dz} \right) dz, F_\lambda^{(o)} = \int_0^1 r_2^2 \left( -\frac{dp}{dz} \right) dz, F_\lambda^{(i)} = \int_0^1 r_1^2 \left( -\frac{dp}{dz} \right) dz \quad (29)$$

The effect of viscosity variation on peristaltic transport can be investigated through (29) for any given viscosity function  $\mu(r)$ .

For the present investigation, we assume viscosity variation in the dimensionless form following Srivastava *et al.* (1) as follows:

$$\mu(r) = e^{-\alpha r} \quad (30)$$

Or

$$\mu(r) = 1 - \alpha r \quad \text{for } \alpha \ll 1 \quad (31)$$

where,  $\alpha$  is viscosity parameter. The assumption is reasonable for the following physiological reason. Since a normal person of animal or similar size takes 1 to 2 L of fluid every day, another 6 to 7 L of fluid are received by the small intestine daily as secretion from salivary glands, stomach, pancreas, liver, and the small intestine itself. This implies that concentration of fluid is dependent on the radial distance. Therefore, the above choice of  $\mu(r) = e^{-\alpha r}$  is justified.

Substituting (31) into (25) & (27), and using (28), we obtain

$$\frac{dp}{dz} = \left[ \begin{aligned} & \left\{ 2\theta - \left( 1 - \varepsilon^2 + \frac{\phi^2}{2} \right) \right. \\ & \left. - \frac{1}{2\gamma^2} \left( \frac{r_2^4 - r_1^4}{4} + \frac{r_1^2 (r_2^2 - r_1^2)}{2} + \frac{(r_2^2 - r_1^2)}{\ln\left(\frac{r_2}{r_1}\right)} \left( \frac{(-1+2\ln(r_1))r_1^2 + (1-2\ln(r_2))r_2^2}{4(\eta-r_2)} - \frac{\ln(r_1)(r_2^2 - r_1^2)}{2} \right) \right) \right\} \\ & \left( \frac{2\alpha + \frac{2(\ln(-1+\alpha\eta) - \ln(-1+\alpha r_2))}{(\eta-r_2)} + \alpha^2(\eta+r_2)}{2\alpha^3} \right) - \left( -\frac{\eta}{\alpha} - \frac{\ln(-1+\alpha r_1)}{\alpha^2} \right) \left( \frac{r_2^2 - r_1^2}{2} \right) \\ & + \left( \left( -\frac{\eta}{\alpha} - \frac{\ln(-1+\alpha r)}{\alpha^2} \right) - \left( \frac{-r_2}{\alpha} - \frac{\ln(-1+\alpha r_2)}{\alpha^2} \right) \left( \frac{-\ln(1-\alpha r_1) + \ln(1-\alpha r_2)}{\alpha(r_1 - r_2)} \right) \left( \frac{r_2^2 - r_1^2}{2} \right) \right) \\ & \left. \left( \frac{\left( \frac{r_2^6 - r_1^6}{6} + \frac{r_1^2 r_2^2 (r_2^2 - r_1^2)}{2} + \frac{3(r_2^2 - r_1^2)}{\ln\left(\frac{r_2}{r_1}\right)} \left( \frac{(-1+4\ln(r_1))r_1^4 + (1-4\ln(r_2))r_2^4}{16(\eta-r_2)} - \frac{\ln(r_1)(r_2^4 - r_1^4)}{4} \right) \right)}{8\gamma^2} \right) \right\} \right. \\ & \left. \left( \frac{(-\ln(-1+\alpha r_2) + \ln(r_2))}{(-(-\ln(-1+\alpha r_1) + \ln(r_1)))} \right) - \frac{1}{8\gamma^2} \left( \frac{(r_2^2 - r_1^2)}{\ln\left(\frac{r_2}{r_1}\right)} \left( r_1^2 + \frac{4(r_2^2 - r_1^2)}{\ln\left(\frac{r_2}{r_1}\right)} \right) + \frac{(-1+2\ln(r_1))r_1^2}{4(\eta-r_2)} + \frac{(1-2\ln(r_2))r_2^2}{2} - \frac{\ln(r_1)(r_2^2 - r_1^2)}{2} \right) \right) \right. \\ & \left. - \frac{(r_2^2 - r_1^2)}{(h^2 - M^2)} \right) \end{aligned} \right] \quad (32)$$

Substituting (32) in (29) yield:

$$\Delta P_\lambda = \int_0^1 \left( \frac{dp}{dz} \right) dz \quad (33)$$

$$F_\lambda^{(o)} = \int_0^1 r_2^2 \left( -\frac{dp}{dz} \right) dz \quad (34)$$

$$F_\lambda^{(i)} = \int_0^1 r_1^2 \left( -\frac{dp}{dz} \right) dz \quad (35)$$

### 3. RESULT AND DISCUSSIONS

The dimensionless pressure rise ( $P_\lambda$ ) and the friction forces on the inner and outer tube for different given values of the dimensionless flow rate  $\Theta$ , amplitude ratio  $\phi$ , radius ratio  $\varepsilon$ ,  $\gamma$  is couple stress parameter, K is porous media M magnetic field and viscosity parameter  $\alpha$  are computed using the (33) to (35). As the integrals in (33) to (35) are not integrable in the closed form, so they are evaluated using

$$a_{20} = 1.25\text{cm}, \quad \frac{a}{\lambda} = 0.156 \quad (36)$$

The values of viscosity parameter  $\alpha$  as reported by Srivastava et al. (1) are  $\alpha = 0.0$  and  $\alpha = 0.1$ . Furthermore, since most routine upper gastrointestinal endoscopes are between 8–11mm in diameter as reported by Cotton and Williams (5) and radius ratio 1.25cm reported by Srivastava and Srivastava (7).

Fig. 1: shows the pressure rise against the flow rate; here it is observed that the pressure decreases with the increase of flow rate for different values of radius ratio  $\epsilon = 0.33, \epsilon = 0.66$  and  $\epsilon = 0.88$  and pressure increases for the viscosity  $\alpha = 0.07$  and  $\alpha = 0.1$ . Fig2: shows the pressure rise against the flow rate; here it is observed that the pressure increases with the increase of flow rate for different values of magnetic  $M = 1.25, M = 1.5$  and  $M = 2$  and pressure increases for the viscosity  $\alpha = 0.8$  and  $\alpha = 0.9$ . Fig. 3: shows that as the viscosity  $\alpha$  increases the pressure increases. And for the different values of amplitude ratio  $\phi = 0.0, \phi = 0.05$  and  $\phi = 1$ , the pressure decreases. Fig. 4: it is noticed that the pressure decreases for different values of porous media  $K = 1, K = 5$  and  $K = 10$ . Fig. 5: it is noticed that the pressure decreases for different values of couple stress parameter  $\gamma = 0.4, \gamma = 0.6$  and  $\gamma = 0.8$ . In Fig. 6: it is noticed that the friction force on the inner tube (endoscope) for different values of radius ratio  $\epsilon = 0.32, \epsilon = 0.38$  &  $\epsilon = 0.44$  and for the values of viscosity  $\alpha = 0.07$  and  $\alpha = 0.1$ . It is noticed that as the radius ratio  $\epsilon$  increases the friction force on the inner tube increases and as the viscosity increases the friction force on the inner tube decreases. Fig7: shows the friction force on the inner tube against the flow rate; here it is observed that the pressure increases with the increase of flow rate for different values of magnetic  $M = 1.25, M = 1.5$  and  $M = 2$  and pressure increases for the viscosity  $\alpha = 0.8$  and  $\alpha = 0.9$ . In Fig.8: it is noticed that the friction force on the inner tube (endoscope) for different values of amplitude ratio  $\phi = 0.0, \phi = 0.05$  &  $\phi = 1$  and for the values of viscosity  $\alpha = 0.07$  and  $\alpha = 0.1$ . It is noticed that the amplitude ratio  $\phi$  increases the friction force on the inner tube decreases and as the viscosity increases the friction force on the inner tube increases. Figures 9 and 14: it is noticed that the viscosity increases with friction force on the inner and outer tube decreases in porous media  $K$ . Figures 10 and 15: it is noticed that the viscosity decreases with friction force on the inner and outer tube increases in couple stress parameter  $\gamma$ . From Figures 11, 12 and 13: show the friction force on the outer tube for different values of radius ratio, amplitude ratio and Magnetic field; here it is observed that as radius ratio, amplitude ratio & magnetic increases the friction force increases.

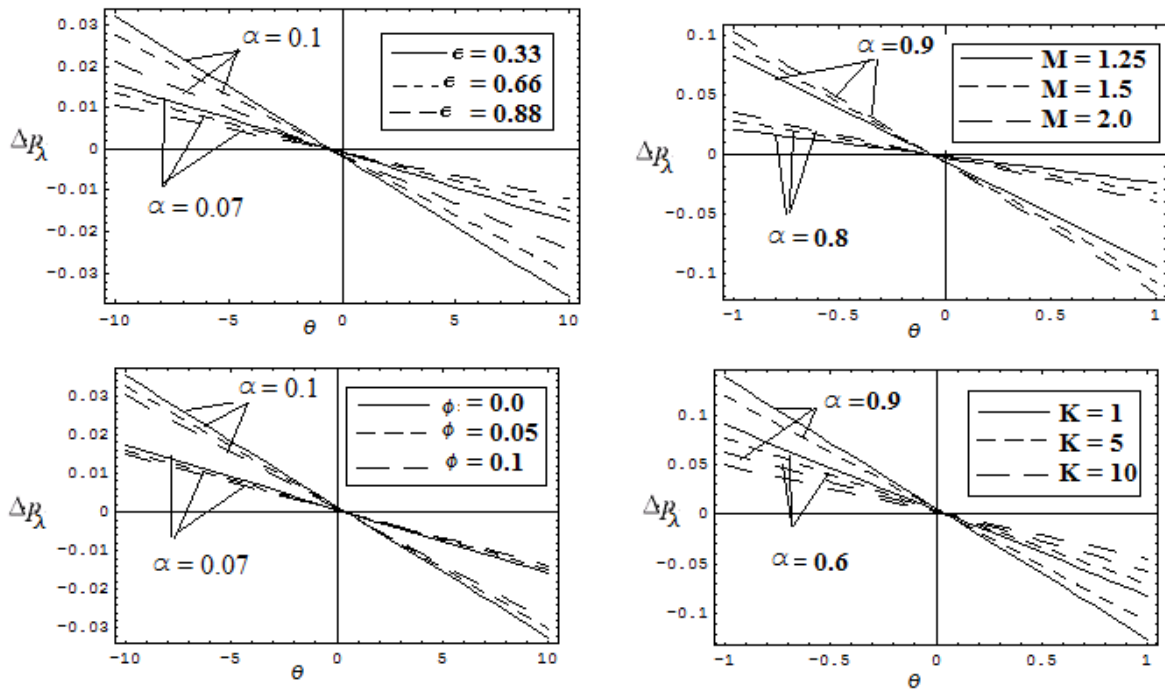


Fig1: Variation of Pressure rise over the flow rate for  $\gamma = 0.4, K=5, M=2, \phi=0, z=0.2$  □□ different values of  $\epsilon$ . Fig 2: Variation of Pressure rise over the flow rate for  $\gamma = 0.4, K=5, \epsilon=0.32, z=0.2, \phi=0.4$  & different values of  $M$ . Fig 3: Variation of Pressure rise over the flow rate for  $\gamma = 0.4, K=5, M=2, \epsilon=0.32, z=0.2$  & different values of  $\phi$ . Fig 4: Variation of Pressure rise over the flow rate for  $\gamma = 0.4, \epsilon=0.44, \phi=0.4, z=0.2, M = 0.25$  & different values of  $K$ .

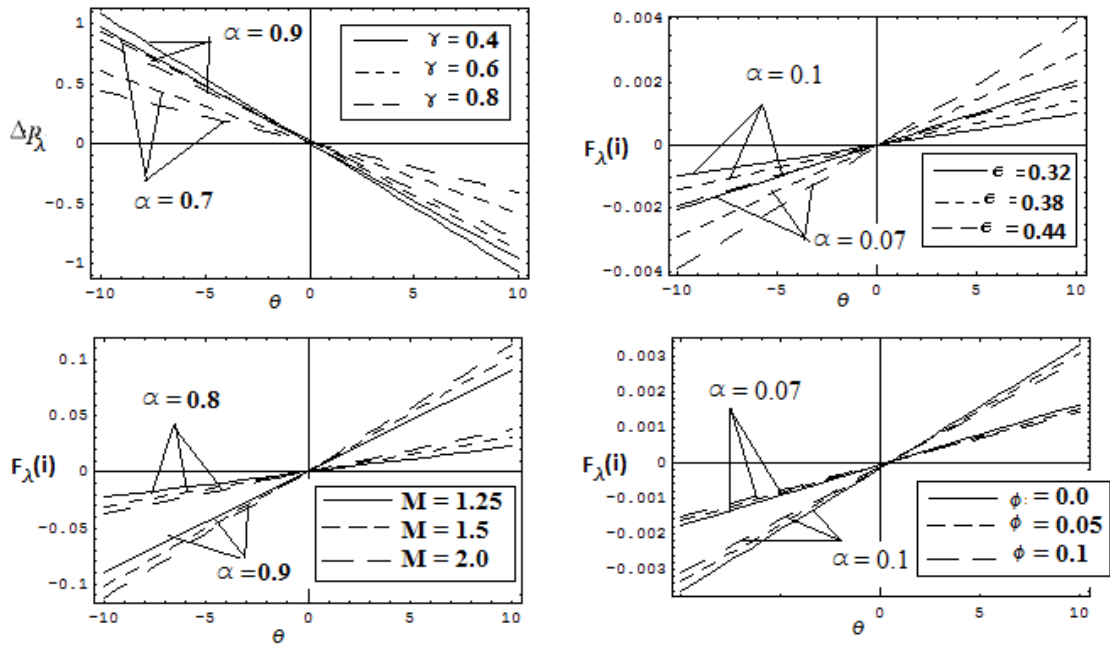


Fig 5: Variation of Pressure rise over the flow rate for  $\kappa=5$ ,  $\varepsilon = 0.44$ ,  $\phi = 0.4$ ,  $z=0.2$ ,  $M = 1.25$  & different values of  $\gamma$ . Fig 6: Variation of Friction on the inner tube (endoscope) over the flow rate for  $\gamma = 0.4$ ,  $\kappa = 5$ ,  $\phi = 0.4$ ,  $z = 0.2$ ,  $M=2$  & different values of  $\varepsilon$ . Fig 7: Variation of Friction on the inner tube (endoscope) over the flow rate for  $\gamma = 0.4$ ,  $\kappa = 5$ ,  $\varepsilon = 0.32$ ,  $z = 0.2$ ,  $\phi = 0.4$  & different values of  $M$ . Fig 8: Variation of Friction on the inner tube (endoscope) over the flow rate for  $\gamma = 0.4$ ,  $\kappa = 5$ ,  $\varepsilon = 0.32$ ,  $z = 0.2$ ,  $M = 2$  & different values of  $\phi$ .

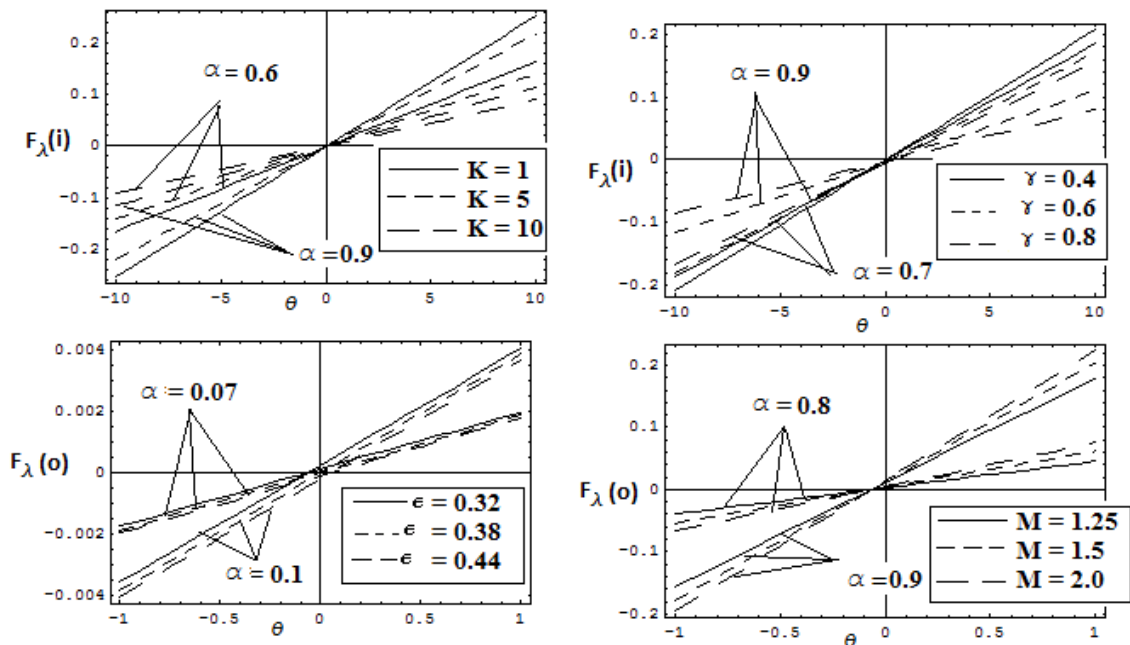


Fig 9: Variation of Friction on the inner tube (endoscope) over the flow rate for  $\gamma = 0.4$ ,  $\varepsilon = 0.44$ ,  $\phi = 0.4$ ,  $z = 0.2$ ,  $M = 0.25$  & different values of  $\kappa$ . Fig 10: Variation of Friction on the inner tube (endoscope) over the flow rate for  $\kappa = 5$ ,  $\varepsilon = 0.44$ ,  $\phi = 0.4$ ,  $z = 0.2$ ,  $M = 1.25$  & different values of  $\gamma$ . Fig 11: Variation of Friction on the outer tube over the flow rate for  $\gamma = 0.4$ ,  $\kappa = 5$ ,  $\phi = 0.4$ ,  $z = 0.2$ ,  $M = 2$  & different values of  $\varepsilon$ . Fig 12: Variation of Friction on the outer tube over the flow rate for  $\gamma = 0.4$ ,  $\kappa = 5$ ,  $\varepsilon = 0.32$ ,  $z = 0.2$ ,  $\phi = 0.4$  & different values of  $M$ .



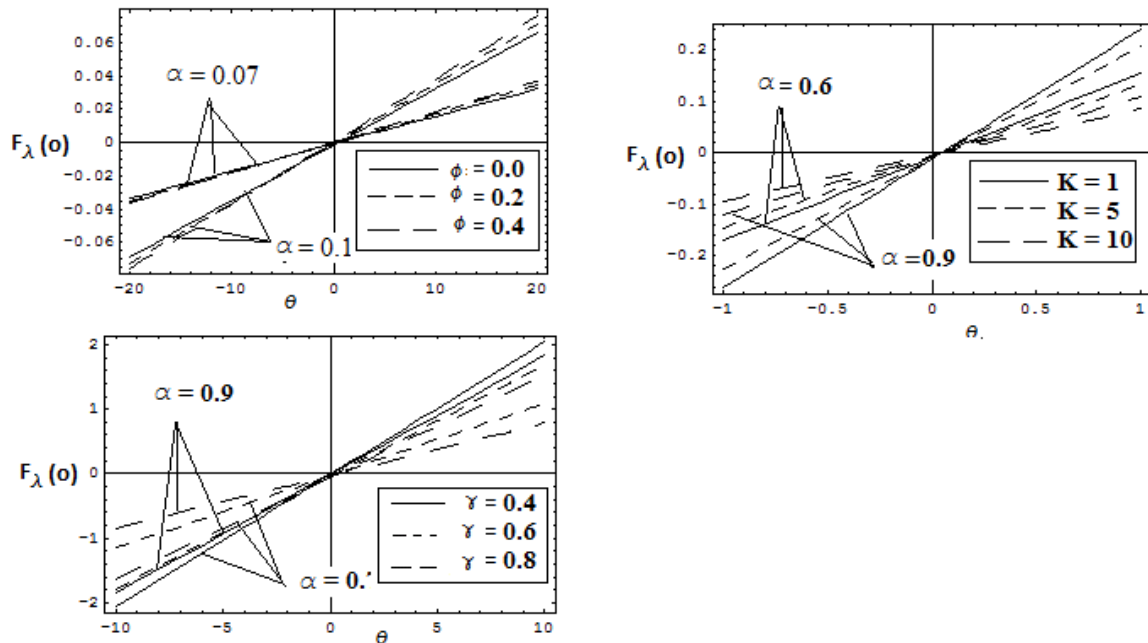


Fig13: Variation of Friction on the outer tube over the flow rate for  $\gamma = 0.4$ ,  $K = 5$ ,  $\varepsilon = 0.32$ ,  $z = 0.2$ ,  $M = 2$  & different values of  $\phi$ . Fig 14: Variation of Friction on the outer tube over the flow rate for  $\gamma = 0.4$ ,  $\varepsilon = 0.44$ ,  $\phi = 0.4$ ,  $z = 0.2$ ,  $M = 0.25$  & different values of  $K$ . Fig 15: Variation of Friction on the outer tube over the flow rate for  $K = 5$ ,  $\varepsilon = 0.44$ ,  $\phi = 0.4$ ,  $z = 0.2$ ,  $M = 1.25$  & different values of  $\gamma$ .

#### 4. CONCLUSION

In this analysis peristaltic transport of a couple stress fluid through a porous media in variable viscosity with magnetic field has been Studied. The viscosity  $\alpha$  increases with pressure rise increases for radius ratio  $\varepsilon$ , amplitude ratio  $\phi$ , porous media  $K$ , magnetic field  $M$  and couple stress parameter  $\gamma$ . The viscosity  $\alpha$  increases with frictional forces of inner and outer tube increases for radius ratio  $\varepsilon$ , amplitude ratio  $\phi$ , porous media  $K$ , magnetic field  $M$  and couple stress parameter  $\gamma$ .

#### REFERENCES

1. A.Alsaedi, N.Ali, D.Tripathi, T.Hayat, Peristaltic flow of couple stress fluid through uniform porous medium, Applied Mathematics and Mechanics, (4), Vol. 35, (2014), 469-480.
2. Ayman and M.F.Sobh, Peristaltic Transport of a Magneto-Newtonian Fluid Through a porous medium, J. Islamic University of Gaza, Vol. 12, No.1, (2004), 37-49.
3. B. Jayarami Reddy, M. V. Subba Reddy, C. Nadhamuni Reddy and P. Yogeswar Reddy, Peristaltic flow of a Williamson fluid in an inclined planar channel under the effect of a magnetic field, Adv. Appl. Sci. Res., 3 (1) (2012), 452-461.
4. B.B.Gupta and V.Sheshadri, "Peristaltic Pumping in Non-Uniform Tubes", J.Biomech.9 (1976), 105-109.
5. El-dabe, N.T. and El-Mohandis, S., Magneto hydrodynamic flow of second order fluid through a porous medium on an inclined porous plane, The Arabian J.for Sci. and Eng., 20, No. 3, (1995), 571-580.
6. Habtu alemaychu and G.Radhakrishnamacharya, Dispersion of a Solute in Peristaltic Motion of a couple stress fluid through a porous medium with slip condition, World Aca. Of Sci., Eng. And Tech., 75, (2011), 804-809.
7. Kh.S.Mekhemier, " Non Linear Peristaltic Transport a Porous Medium in an Inclined Planar Channel", J.Porous Media, 6(3), (2003), 189-201.
8. Kh.S.Mekhemier, Y.Abd elmaboud, Peristaltic flow of a couple stress fluid in an annulus : Application of an endoscope, Science Direct, Physica A, 387, (2008), 2403-2415.
9. L. M. Srivastava and V. P. Srivastava, "Peristaltic transport of a non-newtonian fluid: applications to the vas deferens and small intestine," Annals of Biomedical Engineering, vol. 13, (1985), 137-153.
10. L.M.Srivastava and V.P.Srivastava, and S.K.Sinha, "Peristaltic Transport of a Physiological Fluid: Part I Flow in Non- Uniform Geometry" Biorheol. 20 (1983), 428-433.
11. N.G.Sridhar, Effect of thickness of the porous material using porous media on the peristaltic pumping of couple stress fluid through non - erodible porous lining tube., Transactions on Mathematics, Vol.3, No.2, (2017), 1-13.

12. P.B. Cotton and C.B. Williams, Practical Gastrointestinal Endoscopy, London: Oxford University Press Third Edition, (1990). Paper Received 18 March 2003, Revised 12 October 2003; Accepted 13 January (2004).
13. R.A.Ramachandra and S.Usha, "Peristaltic Transport of Two Immiscible Viscous Fluids in a Circular Tube", J.Fluid Mech., 298 (1995), 271.
14. Raptis. A., Peridikis.C. , Flow of a viscous fluid through a porous medium bounded by a vertical surface, Int. J. Engng. Sci., 21, No 11, (1983), 1327-1336.
15. T.W.Latham , " Fluid Motion in a Peristaltic Pump", M.Sc.Thesis, MIT, Cambridge MA (1966).
16. V. P. Rathod and N. G. Sridhar, Peristaltic transport of couple stress fluid in uniform and non-uniform annulus through porous medium, international journal of mathematical archive, 3(4), (2012), 1561-1574.
17. V. P. Rathod and S.K.Asha, Effect of couple stress fluid and an endoscope in peristaltic motion, Ultra Science, vol.21, no.1, (2009), 83-90.
18. V. P. Rathod and S.K.Asha, Effects of Magnetic Field and an Endoscope on Peristaltic Motion, Journal of Applied Mathematics,2(4), (2011), 102-109.
19. V. P. Rathod, N. G. Sridhar and Mahadev. M. Peristaltic pumping of couple stress fluid through non - erodible porous lining tube wall with thickness of porous material, Advances in Applied Science Research, 3 (4): (2012), 2326-2336.
20. V.P. Rathod and Laxmi Devindrappa: Effects of heat transfer on the peristaltic MHD flow of a Bingham Fluid through a porous medium in an inclined channel. Mathematical sciences international research journal (This was chandigarh conference they published in their own journal), 3(1), 2(014).
21. V.P. Rathod And Laxmi Devindrappa: Effects of heat transfer on the peristaltic mhd flow of a bingham fluid through a porous medium in a channel, International Journal of Biomathematics, Vol. 7, No. 6 (2014), 1450060-1450080.
22. V.P. Rathod And Laxmi Devindrappa: peristaltic transport of a conducting fluid in an asymmetric vertical channel with heat and mass transfer, j.Chem.Biol. & Physical Sci.,4(2), (2015), 1452-1470.
23. V.P. Rathod And Laxmi Devindrappa: Slip effect on peristaltic transport of a conducting fluid through a porous medium in an asymmetric vertical channel by Adomian decomposition method.Int.J Mathematical Archive,4(3), (2013), 133-141.
24. V.P. Rathod And Mahadev, A Study of Ureteral Peristalsis in Cylindrical Tube Through Porous Medium, Advance in Applied Science Research, 2(3), (2011), 134-144.
25. V.P. Rathod And Mahadev: A study of ureteral peristalsis with Jeffrey fluid flow. I.J.Mathematical Modeling, Simulation and Application, 5, (2012), 11-22.
26. V.P. Rathod And Mahadev: Slip effects and heat transfer on MHD peristaltic flow of Jeffrey fluid in an inclined channel.J.Chemical, Biological & Physical Sci. 2 , (2012), 1987-97.
27. V.P. Rathod and Mahadev: Effect of magnetic field on ureteral peristalsis in cylindrical tube, Ultra Scientist of Physical Sciences, 23, (2011), 135-142.
28. V.P.Rathod and Asha.S.K," Peristaltic Transport of a Couple Stress fluid In a Uniform and Non-Uniform Annulus through a Porous Medium", International journal Of Mathematical Modeling, Simulation and Applications, ISSN: 0973-8355, Vol 2, No4, (2009), 414-426.
29. V.P.Rathod and Mahadev. M., Effect of thickness of the porous material on the peristaltic pumping of a Jeffry fluid with non - erodible porous lining wall, Int. j. of Mathematical Archive, 2(10), (2011).
30. V.P.Rathod and N.G.Sridhar, Effect of magnetic field on peristaltic transport of a couple stress fluid in a channel, Advances in Applied Science Research, 7(1), (2016), 134-144.
31. V.P.Rathod and N.G.Sridhar, peristaltic flow of a couple stress fluid in an inclined channel, International Journal of Allied Practice Research and Review, Vol. II, Issue VII, (2015), 25-36.
32. V.P.Rathod and Pallavi Kulkarni: The effect of slip condition and heat transfer on MHD peristaltic transport through a porous medium with complaint wall. Int.J.Applied Mathematical Sciences, 5(12), (2011) 7-63.
33. V.P.Rathod and Pallavi Kulkarni: The influence of wall properties on MHD Peristaltic transport of dusty fluid,Advances in Applied Science Research,2(3), ,(2011), 265-279.
34. V.P.Rathod and Pallavi Kulkarni: The influence of wall properties on Peristaltic transport of dusty fluid through porous medium.Int. J. of Mathematical Archive,2 (12), (2011), 1-13.
35. V.P.Rathod, Navrang Manikrao and N.G.Sridhar, peristaltic flow of a couple stress fluid in an inclined channel under the effect of magnetic field, Advances in Applied Science Research, 6(9) (2015), 101-109.
36. V.P.Rathod, Navrang Manikrao and N.G.Sridhar, peristaltic transport of a conducting couple stress fluid through a porous medium in a channel, International Journal of Mathematical Archive-6(9), (2015), 106-113.
37. Y. abd elmaboud, kh.s. mekheimer, non-linear peristaltic transport of a second-order fluid through a porous medium, Applied mathematical modeling, 35 (2011), 2695-2710.

**Source of support: Nil, Conflict of interest: None Declared.**

**[Copy right © 2017. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]**