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FIXED POINT RESULTS FOR EXPANSIVE MAPPINGS IN DIGITAL METRIC SPACES

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ABSTRACT

In this paper we present fixed point results for generalized $\alpha-\psi$ −expansive type mappings in digital metric spaces. We also provide some examples to illustrate our results.

Keywords: *fixed point; digital metric space; $\alpha-\psi$ −expansive mapping.*

1. INTRODUCTION

Fixed point theory is a beautiful subject for dynamic research in non-linear analysis. In 1912, Brouwer [2] proved a result that a unit closed ball in $\mathbb{R}^n$ has a fixed point. The most remarkable result in the fixed point theory was given by Banach [1] in 1922. He proved that each contraction in a complete metric space has a unique fixed point. Later on, many authors generalized the Banach fixed point theorem in various ways [13, 14, 15, 18, 19, 22, 24]. In 1984, Wang [23] introduced the concept of expansive mappings in complete metric spaces. Recently, Samet *et al.* [21] introduced the notion of α-ψ contractive mappings and proved the related fixed point theorems.

Digital topology is a developing area based on general topology and functional analysis which studies features of 2D and 3D digital images. Rosenfield [20] was the first to consider digital topology as the tool to study digital images. Kong [17], then introduced the digital fundamental group of a discrete object. The digital versions of the topological concepts were given by Boxer [3], who later studied digital continuous functions [4]. Later, he gave results of digital homology groups of 2D digital images in [6] and [7]. Ege and Karaca [9, 10] give relative and reduced Lefschetz fixed point theorem for digital images. They also calculate degree of antipodal map for the sphere like digital images using fixed point properties. Ege and Karaca [11] then defined a digital metric space and proved the famous Banach Contraction Principle for digital images.

In this paper, we generalize the concept of α-ψ mappings as d-α-ψ-expansive mappings in the setting of dislocated metric spaces.

2. PRELIMINARIES

In 1984, Wang *et al.* [2] defined expansive mappings in the form of the following theorem:

Theorem 2.1: Let T be a self map on a complete metric space X such that
(i) T is onto,
(ii) $d(Tx,Ty) \geq kd(x,y), k \geq 1$.
Then T has a unique fixed point in X.

Recently, Samet et al. [3] Introduced the following concepts.

Definition 2.2: Let ψ be a family of functions $\psi: [0,\infty) \to [0,\infty)$ satisfying the following conditions:
(i) ψ is nondecreasing;
(ii) $\sum_{n=1}^{\infty} \psi^n < \infty$ for each $t > 0$, where ψ^n is the nth iterate of ψ;

Definition 2.3: Let $T: X \to X$ and $\alpha : X \times X \to [0,\infty)$. we say that T is α-admissible if for all$x, y \in X$, we have
$$\alpha(x,y) \geq 1 \quad \Rightarrow \quad \alpha(Tx,Ty) \geq 1$$

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Definition 2.4: Let (X,d) be a metric space and let $T: X \to X$ be a given mapping. We say that T is an $\alpha - \psi$-contractive mapping if there exist two functions $\alpha : X \times X \to [0,\infty)$ and $\psi \in \Psi$ such that

$$\alpha(x,y)d(Tx,Ty) \le \psi(d(x,y))$$

For all $x, y \epsilon X$

Let X be a subset of $\mathbb{Z}^n$ for a positive integer n where $\mathbb{Z}^n$ is the set of lattice points in the n- dimensional Euclidean space and ρ represent an adjacency relation for the members of X. A digital image consists of (X,ρ).

Definition 2.5[9]: Let l, n be positive integers, $1 \le l \le n$ and two distinct points

$$a = (a_1, a_2, \dots, a_n), b = (b_1, b_2, \dots, b_n) \in \mathbb{Z}^n$$

a and b are k_l- adjacent if there are at most l indices i such that $|a_i - b_i| = 1$ and for all other indices j such that $|a_j - b_j| \neq 1$, $a_j = b_j$.

There are some statements which can be obtained from definition 2.1:

- a and b are 2- adjacent if $|a - b| = 1$.
- a and b in $\mathbb{Z}^2$ are 8- adjacent if they are distinct and differ by at most 1 in each coordinate.
- a and b in $\mathbb{Z}^3$ are 26- adjacent if they are distinct and differ at most 1 in each coordinate.
- a and b in $\mathbb{Z}^3$ are 18- adjacent if are 26- adjacent and differ by at most two coordinates.
- a and b are 6- adjacent if they are 18- adjacent and differ in exactly one coordinate.

A ρ-neighbour [9] of $a \in \mathbb{Z}^n$ is a point of $\mathbb{Z}^n$ that is ρ- adjacent to a where $\rho \in \{2,4,8,6,18,26\}$ and $n \in 1,2,3$. The set

$$\mathrm{N}_\rho(a) = \{b | b \ is \ \rho - adjacent \ to \ a\}$$

is called the ρ- neighbourhood of a. A digital interval [9] is defined by

$$[p,q]_{\mathbb{Z}} = \{z \in \mathbb{Z} | p \le z \le q\}$$

where $p, q \in \mathbb{Z}$ and $p < q$.

A digital image $X \subset \mathbb{Z}^n$ is ρ- connected [10] if and only if for every pair of different points $u, v \in X$, there is a set $\{u_0, u_1, \dots, u_r\}$ of points of digital image X such that $u = u_0, v = u_r$ and u_i and u_{i+1} are ρ- neighbours where $i = 0,1,\dots,r-1$.

Definition 2.6: Let $(X,\rho_0) \subset \mathbb{Z}^{n_0}$, $(Y,\rho_1) \subset \mathbb{Z}^{n_1}$ be digital images and $T: X \to Y$ be a function.

- T is said to be (ρ_0,ρ_1)- continuous[9], if for all ρ_0- connected subset E of X, $f(E)$ is a ρ_1- connected subset of Y.
- For all ρ_0- adjacent points $\{u_0, u_1\}$ of X, either $T(u_0) = T(u_1)$ or $T(u_0)$ and $T(u_1)$ are a ρ_1- adjacent in Y if and only if T is (ρ_0,ρ_1)- continuous [9].
- If f is (ρ_0,ρ_1)- continuous, bijective and T^{-1} is (ρ_1,ρ_0)- continuous, then T is called (ρ_0,ρ_1)- isomorphism [11] and denoted by $X \cong_{(\rho_0,\rho_1)} Y$.

A $(2,\rho)$- continuous function T, is called a digital ρ- path [9] from u to v in a digital image X if $T: [0,m]_{\mathbb{Z}} \to X$ such that $T(0) = u$ and $T(m) = v$. A simple closed ρ- curve of $m \ge 4$ points [12] in a digital image X is a sequence $\{T(0), T(1), \dots, T(m-1)\}$ of images of the ρ- path $T: [0, m-1]_{\mathbb{Z}} \to X$ such that $T(i)$ and $T(j)$ are ρ- adjacent if and only if $j = i \pm mod \ m$.

Definition 2.7[8]: A sequence $\{x_n\}$ of points of a digital metric space (X,d,ρ) is a Cauchy sequence if for all $\in > 0$, there exists $\delta \in \mathbb{N}$ such that for all $n, m > \delta$, then

$$d(x_n, x_m) < \in.$$

Definition 2.8[8]: A sequence $\{x_n\}$ of points of a digital metric space (X,d,ρ) converges to a limit $p \in X$ if for all $\in > 0$, there exists $\alpha \in \mathbb{N}$ such that for all $n > \delta$, then

$$d(x_n, p) < \in.$$

Definition 2.9[8]: A digital metric space (X,d,ρ) is a digital metric space if any Cauchy sequence $\{x_n\}$ of points of (X,d,ρ) converges to a point p of (X,d,ρ).

Definition 2.10[8]: Let, (X,d,ρ) be any digital metric space and $T: (X,d,\rho) \to (X,d,\rho)$ be a self digital map. If there exists $\alpha \in (0,1)$ such that for all $x, y \in X$,

$$d\big(f(x), f(y)\big) \le \ \alpha d(x,y),$$

then T is called a digital contraction map.

Proposition 2.11[8]: Every digital contraction map is digitally continuous.

Theorem 2.12[8]: (Banach Contraction principle) Let (X,d,ρ) be a complete metric space which has a usual Euclidean metric in $\mathbb{Z}^n$. Let, $T: X \to X$ be a digital contraction map. Then T has a unique fixed point, i.e. there exists a unique $p \in X$ such that $f(p) = p$.

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3. MAIN RESULTS

We introduce the concept of generalized α-ψ-expansive mapping in digital metric spaces as follows:

Definition 3.1: Let (X, d, ρ) be a digital metric space and let $T : X \to X$ be a given mapping. We say that T is a generalised α-ψ-expansive mapping, if there exist two functions $\alpha : X \times X \to [0, \infty)$ and $\psi \in \Psi$ such that for all $x, y \in X$, we have

$$\psi(d(Tx, Ty)) \geq \alpha(x, y)M(x, y). \tag{1}$$

where $M(x, y) = max\left\{d(x, y), \frac{d(x,Tx)+d(y,Ty)}{2}, \frac{d(x,Ty)+d(y,Tx)}{2}\right\}$

Remark 3.2: Clearly, any expansive mapping is a generalized α-ψ-expansive mapping with $\alpha(x, y) = 1$ for all $x, y \in X$ and $\psi(t) = kt$, for all $t \geq 0$ and $k \in (0,1)$.

Definition 3.3: Let $T : X \to X$ and $\alpha : X \times X \to [0, \infty)$. We say that T is α-admissible if for all $x, y \in X$, we have

$$\alpha(x, y) \geq 1 \quad \Longrightarrow \quad \alpha(Tx, Ty) \geq 1$$

Now, we prove our main results.

Theorem 3.4: Let (X, d, ρ) be a complete digital metric space and let $T : X \to X$ is a bijective and generalised α-ψ-expansive mapping and satisfies the following conditions:

(i) T^{-1} is α-admissible

(ii) there exist $x_0 \in X$ such that $\alpha(x_0, T^{-1}x_0) \geq 1$;

(iii) T is digital continuous.

Then T has a fixed point.

Proof: Let $x_0 \in X$ such that $\alpha(x_0, T^{-1}x_0) \geq 1$ (given by condition (ii)). Define the sequence $\{x_n\}$ in X by $x_n = Tx_{n+1}$ for all $n \geq 0$. If $x_n = x_{n+1}$ for some n, then x_n is a fixed point of T. So, we can assume that $x_n \neq x_{n+1}$ for all n. Since T^{-1} is α-admissible, we have

$$\alpha(x_0, x_1) = \alpha(x_0, T^{-1}x_0) \geq 1 \tag{2}$$

$$\Longrightarrow \quad \alpha(T^{-1}x_0, T^{-1}x_1) = \alpha(x_1, x_2) \geq 1.$$

Inductively, we have

$$\alpha(x_n, x_{n+1}) \geq 1 \quad \text{for all } n = 0,1,2 \dots \dots. \tag{3}$$

From (1) and (3), it follows that for all $n \geq 1$, we have

$$\psi\big(d(x_{n-1}, x_n)\big) = \psi\big(d(Tx_n, Tx_{n+1})\big) \geq \alpha(x_n, x_{n+1})M(x_n, x_{n+1}) \geq M(x_n, x_{n+1})$$

$$\psi(d(Tx_{n-1}, Tx_n) \geq M(x_n, x_{n+1}) \tag{4}$$

where
$$\begin{aligned} M(x_n, x_{n+1}) &= max\left\{d(x_n, x_{n+1}), \frac{d(x_n,Tx_n)+d(x_{n+1},Tx_{n+1})}{2}, \frac{d(x_n,Tx_{n+1})+d(Tx_n,x_{n+1})}{2}\right\} \\ &= max\left\{d(x_n, x_{n+1}), \frac{d(x_n,x_{n-1})+d(x_{n+1},x_n)}{2}, \frac{d(x_{n-1},x_{n+1})}{2}\right\} \\ &\leq max\left\{d(x_n, x_{n+1}), \frac{d(x_n,x_{n-1})+d(x_{n+1},x_n)}{2}\right\} \\ &\leq max\{d(x_n, x_{n+1}), d(x_n, x_{n-1})\} \end{aligned} \tag{5}$$

From (4) and taking in consideration that ψ is a non-decreasing function, we get that

$$\psi\big(d(x_{n+1}, x_n)\big) \geq (max\{d(x_n, x_{n-1}), d(x_n, x_{n+1})\}). \tag{6}$$

for all $n \geq 1$. If for some $n \geq 1$, we have $d(x_n, x_{n+1}) \leq d(x_n, x_{n-1})$, from (6), we obtain that

$$d(x_n, x_{n-1}) \geq \psi\big(d(x_n, x_{n-1})\big) > d(x_n, x_{n-1}), \tag{7}$$

a contradiction. Thus, for all $n \geq 1$, we have

$$max\{d(x_n, x_{n-1}), d(x_n, x_{n+1})\} = d(x_n, x_{n+1}). \tag{8}$$

Using (6) and (8), we get that

$$d(x_{n+1}, x_n) \leq \psi\big(d(x_n, x_{n-1})\big), \tag{9}$$

for all $n \geq 1$. By induction, we get

$$d(x_{n+1}, x_n) \leq \psi^n\big(d(x_1, x_0)\big), \text{ for all } n \geq 1 \tag{10}$$

From (10) and using the triangular inequality, for all $k \geq 1$, we have

$$\begin{aligned} d(x_n, x_{n+k}) &\leq d(x_n, x_{n+1}) + \cdots .. + d(x_{n+k-1}, x_{n+k}) \\ &\leq \textstyle\sum_{p=n}^{n+k-1} \psi^p\big(d(x_1, x_0)\big), \\ &\leq \textstyle\sum_{p=n}^{\infty} \psi^p\big(d(x_1, x_0)\big) \to 0 \text{ as } n \to \infty \end{aligned}$$

This implies that $\{x_n\}$ is a Cauchy sequence in the digital metric space (X, d, ρ). since (X, d, ρ) is complete, there exist $u \in X$ such that $\{x_n\}$ is digital convergent to u. since T is digital continuous, it follows that $\{Tx_n\}$ is digital convergent to Tu. By the uniqueness of the limit, we get $u = Tu$, that is, u is a fixed point of T.

The next theorem does not require continuity of T.

Theorem 3.5: Let (X, d, ρ) be a complete digital metric space. Suppose that $T: X \to X$ is a generalized α-ψ-expansive mapping and the following conditions hold:
(i) T^{-1} is α-admissible;
(ii) there exists $x_0 \in X$ such that $\alpha(x_0, T^{-1}x_0) \geq 1$;
(iii) if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $\{x_n\}$ is digital convergent to $x \in X$, then $\alpha(T^{-1}x_n, T^{-1}x) \geq 1$ for all n.
Then there exist $u \in U$ such that $Tu = u$.

Proof: Following the proof of theorem 3.4, we know that the sequence $\{x_n\}$ is a digital Cauchy sequence in the complete metric space (X, d, ρ) such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to u$ as $n \to \infty$.

From (iii), we have

$$\alpha(T^{-1}x_n, T^{-1}u) \geq 1 \quad \text{for all } n \geq 0. \tag{11}$$

Using triangle inequality of digital metric space, we have

$$\begin{aligned} d(T^{-1}u, u) &\leq d(T^{-1}u, x_{n+1}) + d(x_{n+1}, u) \\ &= d(T^{-1}u, T^{-1}x_n) + d(x_{n+1}, u) \\ &\leq M(T^{-1}u, T^{-1}x_n) + d(x_{n+1}, u) \\ &\leq \alpha(T^{-1}u, T^{-1}x_n)M(T^{-1}u, x_{n+1}) + d(x_{n+1}, u) \\ &\leq \psi\big(d(u, x_n)\big) + d(x_{n+1}, u). \end{aligned}$$

Since ψ is continuous at $t = 0$, implies that $d(T^{-1}u, u) = 0$ as $n \to \infty$. That is, $T^{-1}u = u$.

consider $Tu = T(T^{-1}u) = u$, which implies that, u is a fixed point of T.

We now provide some examples in support of our results and show that hypotheses of Theorem 3.4 and 3.5 do not guarantees uniqueness of fixed point.

Example 3.6: let $X = [0, \infty)$ be the digital metric space, where $d(x, y) = |x - y|$ for all $x, y \in X$. Consider the self-mapping $T : X \to X$ given by

$$Tx = \begin{cases} 2x - \dfrac{11}{6} & if\ x > 1, \\ \dfrac{x}{6} & if\ 0 \leq x \leq 1. \end{cases}$$

Define $\alpha: X \times X \to [0, \infty)$, as

$$\alpha(x, y) = \begin{cases} 0 & if\ x, y \in [0,1], \\ 1 & otherwise. \end{cases}$$

Let $\psi(t) = \frac{t}{6}$ for $t \geq 0$. Then we conclude that T is a generalised-α-ψ-expansive mapping. In fact, for all $x, y \in X$, we have $\frac{1}{6}d(Tx, Ty) \geq \alpha(x, y)M(x, y)$.

On the other hand, there exists $x_0 \in X$ such that $\alpha(x_0, T^{-1}x_0) \geq 1$. Indeed, for $x_0 = 1$, we have $\alpha(1, T^{-1}1) = 1$.

Notice also that T is digital continuous. It is sufficient to show that T^{-1} is α-admissible. For this purpose, let $x, y \in X$ such that $\alpha(x, y) \geq 1$. This implies that $x \geq 1$ and $y \geq 1$, and by definition of $T^{-1}$and α, we have

$$T^{-1}x = \frac{x}{2} + \frac{11}{12} \geq 1, \qquad T^{-1}y = \frac{y}{2} + \frac{11}{12} \in [0,1].$$

Hence, $\alpha(T^{-1}x, T^{-1}y) \geq 1$. Then T^{-1} is α-admissible.

As a result, all the conditions of theorem 3.4 are satisfied. Consequently, T has a fixed point. In this example 0 and $\frac{11}{6}$are two fixed points of T.

In the following example T is not continuous.

Example 3.7: Let X, G and β be defined as in example 3.6. Let $T: X \to X$

$$Tx = \begin{cases} 2x - \frac{5}{2} & if\ x > 1, \\ \frac{x}{5} & if\ 0 \leq x \leq 1. \end{cases}$$

let $\psi(t) = \frac{t}{5}$ for $t \geq 0$. Then we conclude that T is a generalised-α-ψ-expansive mapping. In fact, for all $x, y \in X$, we have

$$\alpha(x,y)d(Tx,Ty) \leq \frac{1}{2}M(x,y)$$

Furthermore, there exist $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. For $x_0 = 1$, we have $\alpha(1, T1) = \alpha\left(1, \frac{1}{3}\right) = 1$.

Let $\{x_n\}$ be a sequence such that $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and as $x_n \to x$ as $n \to \infty$. By the definition of α, we have $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$, then we see that $x_n \in [0,1]$. Thus, $\alpha(T^{-1}x_n, T^{-1}x) \geq 1$.

To show that T satisfies all the hypotheses of Theorem 3.5, it is sufficient to observe that T is α –admissible. For this purpose let$x, y \in X$ such that $\alpha(x,y) \geq 1$. It is equivalent to saying that $x, y \in [0,1]$. Due to the definition of α and T, we have

$$Tx = \frac{x}{3} \in [0,1], \qquad Ty = \frac{y}{3} \in [0,1].$$

Hence $\alpha(Tx, Ty) \geq 1$.

As a result, all the conditions of the theorem 3.5 are satisfied. In this example, 0 and $\frac{7}{4}$ are two fixed points of T.

Theorem 3.8: Adding the following condition to the hypotheses of theorem 3.5 (resp. the theorem 3.4) we obtain the uniqueness of a fixed point of T.

(iv) For all $u, u^* \in X$, there exist $v \in X$ such that $\alpha(u,v) \geq 1$ and $\alpha(u^*, v) \geq 1$.

Proof: From the theorem 3.4 and 3.5 the set of fixed points is non-empty. We shall show that if $u, u^*$are two fixed points of T, that is, $T(u) = u$ and $T(u^*) = u^*$, then $u = u^*$.

From the condition (iv) we have, $\alpha(u,v) \geq 1$ and $\alpha(u^*, v) \geq 1$. (12)

We know α-admissible property of T^{-1}, so we obtain from (12)
$\alpha(u, T^{-1}v) \geq 1$ and $\alpha(u^*, T^{-1}v) \geq 1$.

Repeatedly applying the α-admissible property of T^{-1}, we get

$$\alpha(u, T^{-n}v) \geq 1 \quad and \quad \alpha(u^*, T^{-n}v) \geq 1 \qquad \text{for all } n = 1,2,3, \ldots \tag{13}$$

From (1) and (13), we have

$$\begin{aligned} d(u, T^{-n}v) \leq M(u, T^{-n}v) &\leq \alpha(u, T^{-n}v)M(u, T^{-n}, v) \\ &\leq \psi\left(d(Tu, T^{-n+1}v)\right) = \psi\left(d(u, T^{-n+1}v)\right). \end{aligned}$$

Thus, we get by induction that

$$d(u, T^{-n}v) \leq \psi\left(d(u,v)\right) \quad \text{for all } n = 1,2,3, \ldots$$

Letting $n \to \infty$, and since $\psi \in \Psi$, we have

$$d(u, T^{-n}v) \to 0.$$

This implies that $\{T^{-n}v\}$ is digital convergent to u. Similarly, we get $\{T^{-n}v\}$ is digital convergent to u^*. By the uniqueness of the limit of T^{-1}, we get $u = u^*$, that is, the fixed point of T is unique.

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