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RELIABILITY ANALYSIS OF A TWO NON-IDENTICAL UNIT SYSTEM WITH INSPECTION AND DIFFERENT REPAIRS/REPLACEMENT POLICIES

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ABSTRACT

This paper presents the reliability and availability measures of a two non-identical unit system, one is priority (p) and other is ordinary (o) unit, with inspection and different repairs/replacement policies. The p-unit has three modes (normal, partial failure and total failure) and o-unit has only two modes (normal and total failure). Upon partial failure of p-unit, it first goes for inspection to decide that whether the unit goes for online repair or offline repair while if o-unit fails it replaced by the new one since it is non-reparable. A single repair facility is always available for inspection, repair (online/offline and total failed) of p-unit and to replace the failed o-unit. After giving basic definitions and notations, we obtain various measures of system effectiveness, reliability, availability due to a unit in N-mode and P-mode, mean time to system failure, busy period of repair facility due to inspection of failed unit, repair of totally failed unit, online/offline repair of partially failed unit and replacement of a failed unit and the expected profit function, of interest to system designers and operation managers by using regenerative point technique. The graphical behaviors of MTSF and profit function have also been studied.

Keywords: *Cold Standby, Mean sojourn time, Reliability, Availability, Busy period, Regenerative Point Technique.*

1. INTRODUCTION

In the field of reliability, reparable systems have been studied due to their frequent and imperative use in modern industry. Various authors including [1-5] have studied two unit standby systems under the assumptions:
(i) Repair is infallible and after repair the failed unit becomes new.
(ii) Each unit of the system is repairable.
However, it is not always possible to impose off the above assumptions on every system due to different operating and repair environment. Sometimes replacement of the failed unit is cheaper than to repair of the failed unit due to the non-availability of parts or the unit is non-reparable which is not cost-effective for the system.

Keeping the above facts into consideration we in the present chapter analyse a two non-identical unit system, one is priority (p) and other is ordinary (o) unit, with inspection and different repairs/replacement policies. The p-unit has three modes (normal, partial failure and total failure) and o-unit has only two modes (normal and total failure). Upon partial failure of p-unit, it first goes for inspection to decide that whether the unit goes for online repair or offline repair while if o-unit fails it replaced by the new one since it is non-reparable. A single repair facility is always available for inspection, repair (online/offline and total failed) of p-unit and to replace the failed o-unit.

The following system characteristics are obtained by identifying the system at suitable regenerative epochs with the help of regenerative point technique:
(i) State transition probabilities and mean sojourn times in different states.
(ii) Reliability of the system and mean time to system failure (MTSF).
(iii) Pointwise and steady state availabilities of the system during (0, t).
(iv) Expected busy period of repair facility during (0, t) and in steady state.
(v) Expected profit incurred by the system during (0, t) and in steady state.

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2. SYSTEM DESCRIPTION AND ASSUMPTIONS

The assumptions about the model under study are as under:

(i) The system consists of two non-identical units namely p-unit and o-unit. Initially p-unit is operative and o-unit is kept as cold standby.
(ii) The p-unit has three modes – normal (N), partial failure (P) and total failure (F) while o-unit has only two modes – normal (N) and total failure (F).
(iii) When p-unit fails partially, it is first inspected for online/offline repair. The inspection is performed during its operation. In inspection, if a minor fault is detected after inspection, the failed unit goes for online repair otherwise it goes for offline repair.
(iv) The o-unit is non-reparable, hence after failure it goes for replacement.
(v) p-unit in partially failure mode may fail totally during the inspection/on-line repair.
(vi) p-unit can't enter into F-mode without passing through P-mode.
(vii) A single repair facility is always available with the system for inspection and to repair (online, offline and total failed unit) and to replace the failed o-unit on FCFS basis.
(viii) The switching device, used to detect the failed unit and to switch the standby unit into operation, is perfect and instantaneous.
(ix) After each type of repair, the repaired unit works as good as new.
(x) The failure and repair times of the units are assumed to be independent and uncorrelated random variables.
(xi) The failure time distributions of both the units, time to inspection and replacement rate of o-unit are taken as exponential with different parameters while all the repair time distributions of p-unit is taken as general.
(xii) The system failure occurs when it breaks down in any way.

NOTATIONS AND STATES OF THE SYSTEM

(a) Notations:

α	:	Constant failure rate of p-unit from N to P-mode.
θ_1	:	Constant rate of inspection of the partially failed p-unit.
p/q	:	Probability that the partially failed unit goes for online/offline repair (p+q=1).
θ_2	:	Constant failure rate of p-unit from P to F-mode.
$G_1(\cdot), G_2(\cdot)$	:	c.d.f. of time to online/offline repair of a partially failed unit.
θ_3	:	Constant failure rate of o-unit.
λ	:	rate of replacement of o-unit.
$H(\cdot)$	:	c.d.f. of time to repair of totally failed p-unit.

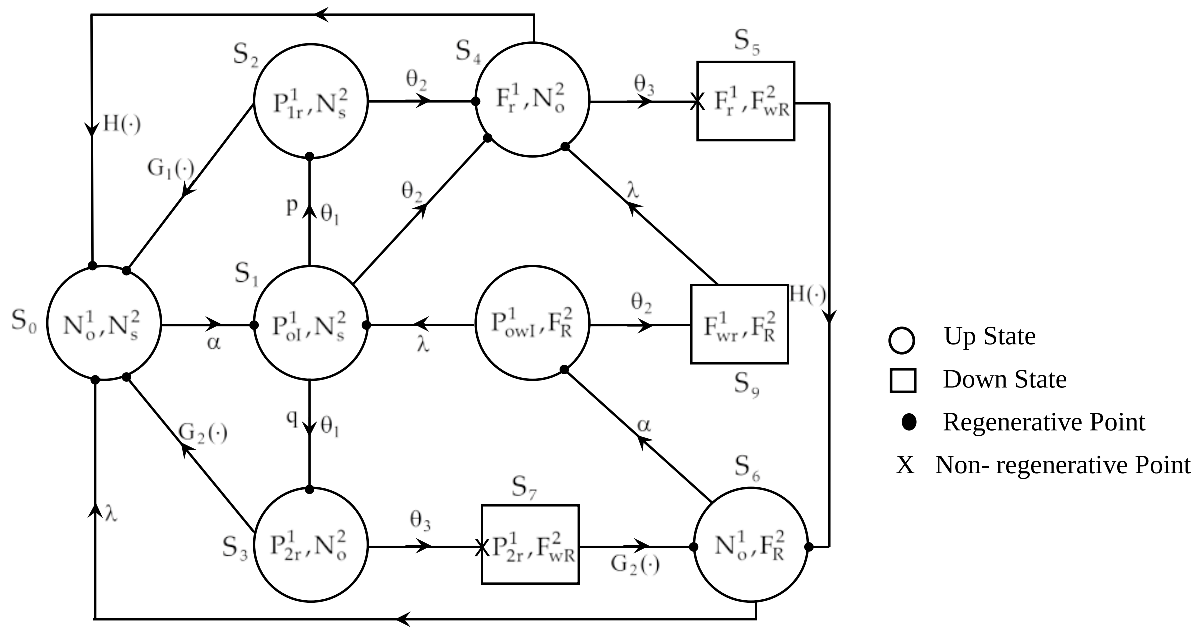
(b) Symbols for the states of the system

We define the following symbols for the states of the system:

N_O^1	:	p-unit is in N-mode and operative.
P_{OI}^1	:	p-unit is in P-mode, operative and under inspection.
P_{1r}^1 / P_{2r}^1	:	p-unit is in P-mode and under online/offline repair.
N_O^2 / N_S^2	:	o-unit is in N-mode and operative/standby.
F_r^1	:	p-unit is in F-mode and under repair.
F_R^2 / F_{WR}^2	:	o-unit is in F-mode and under replacement/waiting for replacement.

Using the above symbols, the transition diagram of the system model, along with transition rates, is shown in figure – 1. From the figure, we observe that the epochs of entrance from S_3 to S_7 and S_4 to S_5 are non-regenerative as the future probabilistic behaviour of the system at these epochs depends upon the previous states. The all other entrance epochs are regenerative.

Figure-1: Transition Diagram



3. TRANSITION PROBABILITIES AND SOJOURN TIMES

By simple probability arguments, we have the following steady state transition probabilities

$p_{01}=\alpha\int e^{-\alpha t}\,dt=1$; $\quad p_{12}=\dfrac{p\theta_1}{\theta_1+\theta_2}$; $\quad p_{13}=\dfrac{q\theta_1}{\theta_1+\theta_2}$;

$p_{14}=\dfrac{\theta_2}{\theta_1+\theta_2}$; $\quad p_{20}=\int e^{-\theta_2 u}\,dG_1(u)=\widetilde{G}_1(\theta_2)$;

$p_{24}=\theta_2\int e^{-\theta_2 u}\,\overline{G}_1(u)\,du=1-\widetilde{G}_1(\theta_2)$; $\quad p_{30}=\int e^{-\theta_3 u}\,dG_2(u)=\widetilde{G}_2(\theta_3)$;

$p_{37}=\theta_3\int e^{-\theta_3 u}\,\overline{G}_2(u)\,du=1-\widetilde{G}_2(\theta_3)$; $\quad p_{40}=\int e^{-\theta_3 u}\,dH(u)=\widetilde{H}(\theta_3)$;

$p_{56}=\int dH(u)=1$; $\quad p_{60}=\dfrac{\lambda}{\alpha+\lambda}$; $\quad p_{68}=\dfrac{\alpha}{\alpha+\lambda}$;

$p_{76}=\int dG_2(u)=1$; $\quad p_{81}=\dfrac{\lambda}{\lambda+\theta_2}$; $\quad p_{89}=\dfrac{\theta_2}{\lambda+\theta_2}$;

$p_{94}=1$; $\quad p_{36}^{(7)}=\int\left(1-e^{-\theta_3 v}\right)dG_2(v)=1-\widetilde{G}_2(\theta_3)$;

$p_{46}^{(5)}=\int\left(1-e^{-\theta_3 v}\right)dH(v)=1-\widetilde{H}(\theta_3)$.

Thus we have the following relations,

$p_{01}=1$; $\quad p_{12}+p_{13}+p_{14}=1$; $\quad p_{20}+p_{24}=1$;

$p_{30}+p_{36}^{(7)}=1$; $\quad p_{40}+p_{46}^{(5)}=1$; $\quad p_{60}+p_{68}=1$;

$p_{81}+p_{89}=1$; $\quad p_{94}=1$.

The mean sojourn times in various states are as follows:

$\psi_0=\int e^{-\alpha t}\,dt=\dfrac{1}{\alpha}$; $\quad \psi_1=\dfrac{1}{\theta_1+\theta_2}$; $\quad \psi_2=\int e^{-\theta_2 t}\,\overline{G}_1(t)\,dt$;

$\psi_3=\int e^{-\theta_3 t}\,\overline{G}_2(t)\,dt$; $\quad \psi_4=\int e^{-\theta_3 t}\,\overline{H}(t)\,dt$; $\quad \psi_5=\int\overline{H}(t)\,dt$;

$\psi_6=\dfrac{1}{\lambda}$; $\quad \psi_7=\int\overline{G}_2(t)\,dt$; $\quad \psi_8=\dfrac{1}{\lambda+\theta_2}$;

$\psi_9=\dfrac{1}{\lambda}$;

4. RELIABILITY OF THE SYSTEM AND MTSF

Using the technique of regenerative point, the expression of reliability, in terms of its Laplace transform, is given by

$$R_0^*(s)=\frac{Z_0^*+q_{01}^*\left[Z_1^*+q_{12}^*Z_2^*+q_{13}^*Z_3^*+\left(q_{12}^*q_{24}^*+q_{14}^*\right)Z_4^*\right]}{1-q_{01}^*\left[q_{12}^*q_{20}^*+q_{13}^*q_{30}^*+\left(q_{12}^*q_{24}^*+q_{14}^*\right)q_{40}^*\right]}$$

where $Z_0^*, Z_1^*, Z_2^*, Z_3^*$ and Z_4^* are the Laplace Transform of

$$Z_0(t)=e^{-\alpha t};\qquad Z_1(t)=e^{-(\theta_1+\theta_2)t};\qquad Z_2=e^{-\theta_2 t}\,\overline{G}_1(t);\quad Z_3=e^{-\theta_3 t}\,\overline{G}_2(t) \text{ and } Z_4=e^{-\theta_3 t}\,\overline{H}(t).$$

and the expression of mean time to system failure is given by

$$E(T_0)=\lim_{s\to 0}R_0^*(s)=\frac{\psi_0+p_{01}\left[\psi_1+p_{12}\psi_2+p_{13}\psi_3+\left(p_{12}p_{24}+p_{14}\right)\psi_4\right]}{1-p_{01}\left[p_{12}p_{20}-p_{13}p_{30}-\left(p_{12}p_{24}+p_{14}\right)p_{40}\right]}$$

5. AVAILABILITY ANALYSIS

Let $A_i^N(t)$ and $A_i^P(t)$ be the probability that the system is up, due to a unit in N-mode and P–mode respectively at epoch t, when it initially starts from state $S_i\in E$ .Using the method of L.T., the value of $A_0^N(t)$ and $A_0^P(t)$ in terms of its L.T. is as follows:

$$A_0^{N*}(s)=\frac{Z_0^*+q_{01}^*\left[q_{13}^*Z_3^*+\left(q_{12}^*q_{24}^*+q_{13}^*q_{36}^{(7)*}+q_{14}^*\right)\left(Z_4^*+q_{46}^{(5)*}Z_6^*\right)\right]}{1-q_{01}^*q_{12}^*q_{20}^*-q_{01}^*q_{13}^*q_{30}^*-q_{01}^*\left(q_{12}^*q_{24}^*+q_{13}^*q_{36}^{(7)*}+q_{14}^*\right)\left(q_{40}^*+q_{46}^{(5)*}q_{60}^*\right)}$$

and
$$A_0^{P*}(s)=\frac{q_{01}^*\left(Z_1^*+q_{12}^*Z_2^*+q_{13}^*Z_3^*\right)}{1-q_{01}^*q_{12}^*q_{20}^*-q_{01}^*q_{13}^*q_{30}^*-q_{01}^*\left(q_{12}^*q_{24}^*+q_{13}^*q_{36}^{(7)*}+q_{14}^*\right)\left(q_{40}^*+q_{46}^{(5)*}q_{60}^*\right)}$$

The steady state probability that the system will be operative due to a unit in N-mode and P-mode respectively is given by,

$$A_0^N=\lim_{t\to\infty}A_0^N(t)=\lim_{s\to 0}s\,A_0^{N*}(s)=\frac{\psi_0+p_{01}\left[p_{13}\psi_3+\left(p_{12}p_{24}+p_{13}p_{36}^{(7)}+p_{14}\right)\left(\psi_4+p_{46}^{(5)}\psi_6\right)\right]}{\psi_0+\psi_1+p_{12}\psi_2+p_{13}n_1+\left(p_{12}p_{24}+p_{13}p_{36}^{(7)}+p_{14}\right)\left(n_2+p_{46}^{(5)}\psi_6\right)}$$

Similarly, $$A_0^N=\frac{p_{01}\left(\psi_1+p_{12}\psi_2+p_{13}\psi_3\right)}{\psi_0+\psi_1+p_{12}\psi_2+p_{13}n_1+\left(p_{12}p_{24}+p_{13}p_{36}^{(7)}+p_{14}\right)\left(n_2+p_{46}^{(5)}\psi_6\right)}$$

where $n_1=\int t\,dG_2(t)$ and $n_2=\int t\,dH(t)$.

The expected up time of the system during (0, t) in N-mode and P-mode respectively is given by,

$$\mu_{up}^N(t)=\int_0^t A_0^N(u)\,du;\qquad \text{So that}\qquad \mu_{up}^{N*}(s)=\int_0^t\frac{A_0^{N*}(s)}{s}$$

and
$$\mu_{up}^P(t)=\int_0^t A_0^P(u)\,du;\qquad \text{So that,}\quad \mu_{up}^{P*}(s)=\int_0^t\frac{A_0^{P*}(s)}{s}$$

6. BUSY PERIOD OF REPAIR FACILITY

Let $B_i^i(t), B_i^F(t), B_i^p(t)$ and $B_i^R(t)$ be the probability that the repair facility is busy in the inspection of partially failed unit, repair of totally failed unit, online/offline repair of partially failed unit and replacement of a failed unit respectively at time t, when the system initially starts from state $S_i\in E$. Using the technique of Laplace transforms, one can obtain the value of $B_0^i(t), B_0^F(t), B_0^p(t)$ and $B_0^R(t)$ in terms of their Laplace transforms

i.e. $B_0^{i*}(s), B_0^{F*}(s), B_0^{p*}(s)$ and $B_0^{R*}(s)$ respectively.

In a long run, the probability that the repair facility will be busy in the inspection of partially failed unit, repair of totally failed unit, online/offline repair of partially failed unit and replacement of a failed unit respectively is given by,

$B_0^i=N_1/D_1$, $\qquad$ $B_0^F=N_2/D_1$,

$B_0^p=N_3/D_1$ $\qquad$ $B_0^R=N_4/D_1$.

Where, $N_1 = p_{01}\,\psi_1$; $\quad N_2 = p_{01}\left(p_{12}\,p_{24} + p_{13}\,p_{36}^{(7)} + p_{14}\right)\psi_4$;

$N_3 = p_{01}\left(p_{12}\,\psi_2 + p_{13}\,\psi_3\right)$; $\quad N_4 = p_{01}\left(p_{12}\,p_{24} + p_{13}\,p_{36}^{(7)} + p_{14}\right)p_{46}^{(5)}\,\psi_6$

and $D_1 = \psi_0 + \psi_1 + p_{12}\,\psi_2 + p_{13}\,n_1 + \left(p_{12}\,p_{24} + p_{13}\,p_{36}^{(7)} + p_{14}\right)\left(n_2 + p_{46}^{(5)}\,\psi_6\right)$

The expected busy period of the repair facility in (0, t) is given by,

$$\mu_b^i(t) = \int_0^t B_0^i(u)\,du\,,\quad \mu_b^F(t) = \int_0^t B_0^F(u)\,du\,,\quad \mu_b^p(t) = \int_0^t B_0^p(u)\,du\,,\quad \mu_b^R(t) = \int_0^t B_0^R(u)\,du\,.$$

So that, $\mu_b^{i*}(s) = \dfrac{B_0^{i*}(s)}{s}$, $\quad \mu_b^{F*}(s) = \dfrac{B_0^{F*}(s)}{s}$, $\quad \mu_b^{p*}(s) = \dfrac{B_0^{p*}(s)}{s}$, $\quad \mu_b^{R*}(s) = \dfrac{B_0^{R*}(s)}{s}$.

7. PROFIT FUNCTION ANALYSIS

The expected profit incurred by the system during (0, t) is given by,

P(t) = Expected total revenue in (0, t) – Expected total repair cost in (0, t)

$$= K_0\,\mu_{up}^N(t) + K_1\,\mu_{up}^P(t) - K_2\,\mu_b^i(t) - K_3\,\mu_b^F(t) - K_4\,\mu_b^p(t) - K_5\,\mu_b^R(t)$$

where, K_0 and K_1 are the revenues per unit up time by the system when it is up due to a unit operative in N–mode and in P–mode respectively, K_2 is the cost per unit time when the system is under inspection of a partially failed unit and K_3, K_4 and K_5 are the per unit time amount that is paid to the repair facility for repairing of a totally failed unit, online/offline repair and replacement of a unit respectively.

The expected profit per unit time in a steady state is given by,

$$P = K_0\,A_0^N + K_1\,A_0^P - K_2\,B_0^i - K_3\,B_0^F - K_4\,B_0^p - K_5\,B_0^R$$

8. GRAPHICAL PRESENTATION

For a more concrete study of the system behavior, we plot curves for MTSF and profit function w.r.t. α for different values of α_2.

Figure-2 shows the variation in MTSF w.r.t. α for different values of α_2 = 0.01, 0.03 and 0.05 when other parameters are kept fixed as p=0.5, q = 0.5, $\beta = 0.06$, $\theta_1 = 0.5$, $\theta_2 = 0.15$, $\theta_3 = 0.04$, $\alpha_1 = 0.5$. It is observed from the graph that MTSF initially decreases rapidly and then tends to vanish as α increases. Also with the increase in the values of α_2, MTSF of the system decreases.

Figure-3 shows the curves for profit function. From the curve it is clear that profit decreases almost with linear trend as the values of α increases when other parameters are kept fixed as K_0 = 5000, K_1 = 4000, K_2 = 2500, K_3 = 2000, K_4 = 1500 and K_5 = 1000.

Also the profit tends to decrease as we increase the values of failure rate α_2.

Thus we conclude that better understanding of failure phenomenon by the repair facility results in better system performance.

Figure – 2

Behavior of MTSF w.r.t. α for different values of α_2

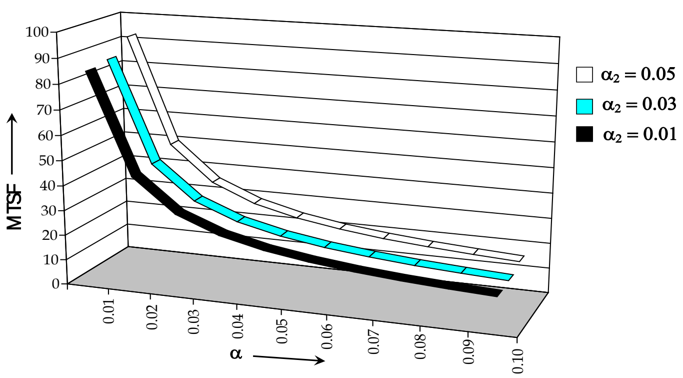
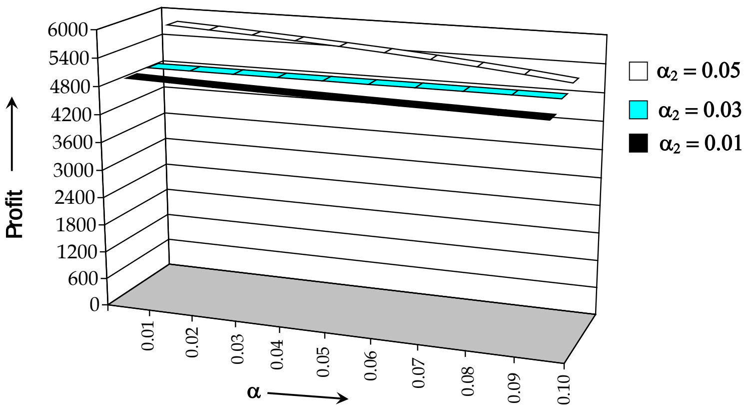


Figure – 3

Behavior of profit w.r.t. α for different values of α_2



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