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NEW ESTIMATION STRATEGY OF POPULATION MEAN USING KNOWN MEDIAN OF THE STUDY VARIABLE

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ABSTRACT

In the present paper, we have suggested a generalized ratio type estimator of population mean of study variable using known population median of study variable. Up to the first order of approximation, the expressions for the bias and mean squared error of the proposed estimator have been derived. The optimum value of the characterizing scalar has been obtained. The minimum value of the mean squared error of the proposed estimator has also been obtained for this optimum value of the characterizing scalar. The proposed estimator has been theoretically compared with the competing estimators, the mean per unit estimator, usual regression estimator of Watson [1937] and usual ratio of Cochran [1940] also with the Bahl and Tuteja [1991], Srivastava [1967], Reddy [1974], Kadilar [2016] and Subramani [2016] estimators. The theoretical findings are validated with the numerical illustrations and it has been shown that proposed estimator is better than the competing estimators as it has least mean square error.

Key words: *Study variable, Bias, Ratio estimator, Mean squared error, Simple random sampling, Efficiency.*

2010 AMS Subject Classification: *62D05.*

INTRODUCTION

Auxiliary information is useful for improved estimation of population parameters but it is collected on additional cost of the survey. Many times in practice we see that even population mean of the study variable is not known but the population median of the main variable under study is known. For example if we ask for the exact number in grade system or exact blood pressure of a person, it is very hard to get the exact value but we get the information in terms of intervals. Here the median of the study variable is easily available which can be used for improved estimation of population mean of study variable. The use of auxiliary variable which is highly correlated with study variable also improves the efficiency of the estimator but it is collected on additional cost of the survey. In this manuscript we have proposed a generalized ratio type estimator of population mean of the study variable using median of the study variable.

Let us consider the finite population consisting of *N* distinct and identifiable units and let $(x_i, y_i), i = 1, 2, ..., n$ be a bivariate sample of size *n* taken from (*X*, Y) using a simple random sampling without replacement (*SRSWOR)* scheme. Let $\overline{X}$ and $\overline{Y}$ respectively be the population means of the auxiliary and the study variables, and let $\overline{x}$ and $\overline{y}$ be the corresponding sample means. In simple random sampling without replacement, it is well known that sample means $\overline{x}$ and $\overline{y}$ are unbiased estimators of population means of $\overline{X}$ and $\overline{Y}$ respectively.

The problem under consideration has been demonstrated more effectively through the following two real world examples. Let us consider these examples of estimation of population mean of study variable using median of study variable given by Subramani [2016]. The tables representing examples have been used with the permission of the author.

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Example1. In an Indian university 5000 students entered for the university examination. The results are given below. The problem is to estimate the average marks scored by the students (population mean). Here, it is reasonable to assume that the median of the marks is known since we have the following information.

Table - 1: Results of the University Examination

Passed with	Percentage of marks	Number of Students	Cumulative total
Distinction	75-100	850	850
First Class	60-75	3100	3950
Second Class	50-60	600	4550
Failed	0-50	450	5000
Total		5000	5000

The median value will be between 60 and 75. Approximately one can assume the population median value as 67.5.

Example 2: In the problem of estimating the blood pressure of the 202 patients of a hospital, it is reasonable to assume that the median of the blood pressure is known based on the information available in Table 2.

Table – 2: Blood pressure of 202 patients of a hospital

Category	Systolic, mmHg	Number of patients	Cumulative No. of patients
Hypotension	< 90	10	10
Desired	90–119	112	122
Pre-hypertension	120–139	40	162
Stage 1 Hypertension	140–159	20	182
Stage 2 Hypertension	160–179	13	195
Hypertensive Emergency	≥ 180	7	202
	Total	202	202

The median value will be between 90 and 119. Approximately one can assume the population median value as 104.5.

REVIEW OF EXISTING ESTIMATORS

The natural and the most suitable estimator of population mean of the study variable, given by,

$$t_o = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{1}$$

Sample mean is an unbiased estimator of population mean and up to the first order of approximation; its variance is given by,

$$V(t_0) = \frac{1-f}{n} S_y^2 = \frac{1-f}{n} \bar{Y}^2 C_y^2 \tag{2}$$

where, $C_y = \frac{S_y}{\bar{Y}}$, $S_y^2 = \frac{1}{N-1}\sum_{i-1}^{N}(Y_i - \bar{Y})^2 = \frac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(\bar{y}_i - \bar{Y})^2$, $f = \frac{n}{N}$.

Watson [1937] used the highly correlated auxiliary variable with the study variable and proposed the usual linear regression estimator of population mean as,

$$t_1 = \bar{y} + \beta_{yx}(\bar{X} - \bar{x}) \tag{3}$$

where β_{yx} is the regression coefficient of the line Y on X.

The regression estimator is also unbiased for population mean and its variance up to the first order of approximation, is given by,

$$V(t_1) = \frac{1-f}{n} \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2) \,. \tag{4}$$

Cochran [1940] utilized the highly positively correlated auxiliary variable with the study variable and proposed the following usual ratio estimator as,

$$t_2 = \bar{y}\frac{\bar{X}}{\bar{x}} \tag{5}$$

The usual ratio estimator is a biased estimator of population mean and its bias and mean squared error, up to the first order of approximation are respectively given by,

$$B(t_2) = \frac{1-f}{n}\bar{Y}[C_x^2 - C_{yx}] \text{ and}$$

$$MSE(t_2) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + C_x^2 - 2C_{yx}], \quad (6)$$

where, $C_x = \frac{S_x}{\bar{X}}$, $S_x^2 = \frac{1}{N-1}\sum_{i-1}^{N}(X_i - \bar{X})^2 = \frac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(\bar{x}_i - \bar{X})^2$, $\rho_{yx} = \frac{Cov(x,y)}{S_x S_y}$,

$$Cov(x,y) = \frac{1}{N-1}\sum_{i-1}^{N}(Y_i - \bar{Y})(X_i - \bar{X}), \text{ and } C_{yx} = \rho_{yx} C_y C_x.$$

Bahl and Tuteja [1991] suggested the exponential ratio type estimator of population mean using positively correlated auxiliary variable as,

$$t_3 = \bar{y}\exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right] \quad (7)$$

This estimator is biased and the bias and the mean squared error of this estimator, up to the first order of approximation, are respectively given by,

$$B(t_3) = \frac{1-f}{8n}\bar{Y}[3C_x^2 - 4C_{yx}] \text{ and}$$

$$MSE(t_3) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + \frac{C_x^2}{4} - C_{yx}]. \quad (8)$$

Srivastava [1967] suggested the generalized ratio type estimator of population mean using positively correlated auxiliary variable as,

$$t_4 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \quad (9)$$

where α is a suitably chosen constant such that MSE of above estimator is minimum.

The above estimator is a biased estimator and its bias and the mean squared error up to the first order of approximation are respectively given by,

$$B(t_4) = \frac{1-f}{n}\bar{Y}[\frac{\alpha(\alpha-1)}{2}C_x^2 + \alpha C_{yx}] \text{ and}$$

$$MSE(t_4) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + \alpha^2 C_x^2 + 2\alpha C_{yx}]$$

The optimum value of the constant α is, $\alpha = -C_{yx}/C_x^2$.

The minimum value of $MSE(t_4)$ for optimum value of α is given by,

$$MSE_{\min}(t_4) = \frac{1-f}{n}\bar{Y}^2 C_y^2(1-\rho_{yx}^2). \quad (10)$$

Reddy [1974] proposed a class of ratio type estimators for population mean of study variable using positively correlated auxiliary variable as,

$$t_5 = \bar{y}\left[\frac{\bar{X}}{\bar{X} + \alpha(\bar{x} - \bar{X})}\right] \quad (11)$$

This estimator is biased estimator and the bias and the mean squared error of this estimator, up to the first order of approximation are respectively given by,

$$B(t_5) = \frac{1-f}{n}\bar{Y}[\alpha^2 C_x^2 - \alpha C_{yx}] \text{ and}$$

$$MSE(t_5) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + \alpha^2 C_x^2 - 2\alpha C_{yx}]$$

The MSE of the above estimator is minimum for optimum value of $\alpha = C_{yx} \big/ C_x^2$ and the minimum MSE is given by,

$$MSE_{\min}(t_5) = \frac{1-f}{n}\bar{Y}^2 C_y^2 (1-\rho_{yx}^2)\ . \tag{12}$$

Kadilar [2016] proposed an exponential type estimator of population mean using positively correlated auxiliary variable as,

$$t_6 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\delta} \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \tag{13}$$

where δ is a constant to be determined such the MSE of above estimator is minimum.

The bias and the mean squared error of the above estimator up to the first order of approximation respectively are,

$$B(t_6) = \frac{1-f}{n}\bar{Y}\left[\left\{\frac{\delta(\delta-1)}{2}+\frac{3}{8}\right\}C_x^2 + \left(\delta+\frac{1}{2}\right)C_{yx}\right]$$

$$MSE(t_6) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + \left(\delta^2+\delta+\frac{1}{4}\right)C_x^2 + (2\delta+1)\,C_{yx}\right] \tag{14}$$

The optimum value of the characterizing scalar δ which minimizes the mean squared error of t_6 is,

$$\delta_{opt} = \left(\frac{1}{2} - \rho_{yx} C_y / C_x\right)$$

The minimum mean squared error of above estimator is,

$$MSE_{\min}(t_6) = \frac{1-f}{n}\bar{Y}^2 C_y^2 (1-\rho_{yx}^2) \tag{15}$$

This minimum mean squared error is equal to the variance of the usual regression estimator of Watson [1937].

Subramani [2016] used the population median of the study variable and proposed the ratio estimator of population mean of the study variable as,

$$t_7 = \bar{y}\left(\frac{M}{m}\right) \tag{16}$$

where M and m are the population and sample medians of study variable respectively.

This estimator is biased estimator and its bias and the mean squared error, up to the first order of approximation, are respectively given by,

$$B(t_7) = \frac{1-f}{n}\bar{Y}[C_m^2 - C_{ym} - \frac{Bias(m)}{M}] \text{and}$$

$$MSE(t_7) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + R_7^2 C_m^2 - 2R_7 C_{ym}]\ , \tag{17}$$

where, $R_7 = \dfrac{\bar{Y}}{M}$, $C_m = \dfrac{S_m}{M}$, $S_m^2 = \dfrac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(m_i - M)^2$, $S_{ym} = \dfrac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(\bar{y}_i - \bar{Y})(m_i - M)$ and $C_{ym} = \dfrac{S_{ym}}{\bar{Y}M}$.

Various modified estimators of population mean have been given by various authors in the literature. The latest references can be found in Subramani (2013), Subramani and Kumarapandiyan [2012, 2013], Tailor and Sharma [2009], Yan and Tian [2010], Yadav *et al. [*2014, 2015], Yadav *et al.* [2016], and Abid *et al.* [2016].

PROPOSED ESTIMATORS

Motivated by Jerajuddin and Kishun [2016], we propose a generalized ratio type estimator of population mean using known population median of study variable as,

$$t_p = \bar{y}\left[\alpha + (1-\alpha)\left(\frac{M+n}{m+n}\right)\right] \tag{18}$$

where α is a characterizing scalar to be determined such that the MSE of the proposed estimator t_p is minimum.

To know the properties of the proposed estimator, we have made the following assumptions as,

$\bar{y} = \bar{Y}(1+e_0)$ and $m = M(1+e_1)$ such that $E(e_0) = 0$, $E(e_1) = \dfrac{\overline{M} - M}{M} = \dfrac{Bias(m)}{M}$ and

$E(e_0^2) = \dfrac{1-f}{n}C_y^2$, $E(e_1^2) = \dfrac{1-f}{n}C_m^2$, $E(e_0 e_1) = \dfrac{1-f}{n}C_{ym}$,

where, $\overline{M} = \dfrac{1}{n}\sum_{i=1}^{n} m_i$

The proposed estimator t_p can be rewritten in terms of e_i's ($i = 1, 2$) as,

$$t_p = \bar{Y}(1+e_0)\left[\alpha + (1-\alpha)\left(\frac{M+n}{M(1+e_1)+n}\right)\right]$$

$$= \bar{Y}(1+e_0)[\alpha + (1-\alpha)(1+\theta e_1)^{-1}], \text{ where } \theta = \frac{M}{M+n}$$

Expanding the right hand side of the above equation and up to the first order of approximations, we get,

$$t_p = \bar{Y}(1+e_0)[1-(1-\alpha)\theta e_1 + (1-\alpha)\theta^2 e_1^2]$$

$$t_p - \bar{Y} = \bar{Y}[e_0 - \alpha_1\theta e_1 - \alpha_1\theta e_0 e_1 + \alpha_1\theta^2 e_1^2]. \tag{19}$$

Taking expectation on both sides of equation and putting the values of various expectations, we get the bias of the proposed estimator t_p, up to the first order of approximation as,

$$B(t_p) = \bar{Y}\left[\frac{1-f}{n}\alpha_1\theta^2 C_m^2 + \alpha_1\theta\frac{Bias(m)}{M}\right]$$

From equation (19), up to the first order of approximation, we have,

$$t_p - \bar{Y} \cong \bar{Y}[e_0 - \alpha_1\theta e_1]$$

Squaring both sides of above equation and taking expectations on both sides, we get mean squared error of the proposed estimator t_p as

$$MSE(t_p) = \bar{Y}^2 E(e_0^2 + \alpha_1^2\theta^2 e_1^2 - 2\alpha_1\theta e_0 e_1)$$

$$= \bar{Y}^2[E(e_0^2) + \alpha^2\theta^2 E(e_1^2) - 2\alpha\theta E(e_0 e_1)]$$

Putting values of different expectations in above equation, we have,

$$MSE(t_p) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + \alpha_1^2\theta^2 C_m^2 - 2\alpha_1\theta C_{ym}] \tag{20}$$

which is minimum for,

$$\alpha_{1(opt)} = C_{ym} \big/ \theta C_m^2 .$$

The minimum mean squared error of the proposed estimator t_p for the optimum value of α_{opt} is,

$$MSE_{\min}(t_p) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2 - \frac{C_{ym}^2}{C_m^2}\right]. \tag{21}$$

EFFICIENCY COMPARISON

In this section proposed estimator is theoretically compared with the competing estimator of population mean.

From equation (21) and equation (2), we have,

$$V(t_0) - MSE_{\min}(t_p) > 0 \text{ if}$$

$$\frac{C_{ym}^2}{C_m^2} > 0 \text{, or if } C_{ym}^2 > 0$$

Thus the proposed estimator is better than the usual mean per unit estimator of population mean.

From equation (21) and equation (4), we have,

$$MSE(t_1) - MSE_{\min}(t_p) > 0 \text{ if}$$

$$\frac{C_{ym}^2}{C_m^2} - C_y^2 \rho_{yx}^2 > 0$$

The proposed estimator is better than the usual regression estimator of Watson [1937] under above condition.

From equation (21) and equation (6), we have,

$$MSE(t_2) - MSE_{\min}(t_p) > 0 \text{ if}$$

$$C_x^2 - 2C_{yx} + \frac{C_{ym}^2}{C_m^2} > 0 \text{, if}$$

$$C_x^2 + \frac{C_{ym}^2}{C_m^2} > 2C_{yx}$$

The proposed estimators are better than the usual ratio estimator given by Cochran [1940] with the above condition.

From equation (21) and equation (8), we have,

$$MSE(t_3) - MSE_{\min}(t_p) > 0 \text{ if}$$

$$\frac{C_x^2}{4} - C_{yx} + \frac{C_{ym}^2}{C_m^2} > 0 \text{, or}$$

$$\frac{C_x^2}{4} + \frac{C_{ym}^2}{C_m^2} > C_{yx}$$

The proposed estimator performs better than Bahl and Tuteja [1991] ratio type estimator of population mean under above condition.

From equation (21) and equation (10), we have,

$$MSE(t_4) - MSE_{\min}(t_p) > 0 \text{, if}$$

$$\frac{C_{ym}^2}{C_m^2} - C_y^2 \rho_{yx}^2 > 0$$

With the above condition, the proposed estimator performs better than the Srivastava (1967) estimator.

It is also better than Reddy [1974] and Kadilar [2016] estimators of population mean using auxiliary information under the above condition as for Srivastava [1967] estimator in above equation.

From equation (21) and equation (17), we have,

$$MSE(t_7) - MSE_{\min}(t_p) > 0 \text{, if}$$

$$R_7^2 C_m^2 - 2R_7 C_{ym} + \frac{C_{ym}^2}{C_m^2} > 0 \text{, or}$$

$$R_7^2 C_m^2 + \frac{C_{ym}^2}{C_m^2} > 2R_7 C_{ym}$$

The proposed estimator performs better than the Subramani [2016] competing estimator of population mean using information on median of the study variable with the above condition.

NUMERICAL STUDY

For the numerical illustration, we have considered the natural populations given in Subramani [2016]. He has used three natural populations. The population 1 and 2 have been taken from Singh and Chaudhary (1986, page no. 177) and the population 3 has been taken from Mukhopadhyay [2005]. In populations 1 and 2, the study variable is the estimate the area of cultivation under wheat in the year 1974, whereas the auxiliary variables are the cultivated areas under wheat in 1971 and 1973 respectively. In population 3, the study variable is the quantity of raw materials in lakhs of bales and the number of labourers as the auxiliary variable, in thousand for 20 jute mills. Tables 3-5 represent the parameter values along with constants, biases of various estimators along with proposed estimator and variances and mean squared errors of existing and proposed estimator

Table-3: Parameter values and constants for three natural populations

Parameter	Population-1	Population-2	Population-3
N	34	34	20
n	5	5	5
${}^{N}C_{n}$	278256	278256	15504
$\bar{Y}$	856.4118	856.4118	41.5
$\bar{M}$	736.9811	736.9811	40.0552
M	767.5	767.5	40.5
$\bar{X}$	208.8824	199.4412	441.95
R_7	1.1158	1.1158	1.0247
C_y^2	0.125014	0.125014	0.008338
C_x^2	0.088563	0.096771	0.007845
C_m^2	0.100833	0.100833	0.006606
C_{ym}	0.07314	0.07314	0.005394
C_{yx}	0.047257	0.048981	0.005275
ρ_{yx}	0.4491	0.4453	0.6522

Table-4: Bias of various estimators

Estimator	Popln-1	Popln-2	Popln-3
t_2	35.3748	40.9285	0.1067
t_3	1.39995	1.72380	0.0019
t_4	-1.60997	1.76775	0.0054
t_5	2.07309	1.85541	0.0167
t_6	27.4137	27.4137	0.3743
t_7	57.7705	57.7705	0.5061

Table-5: Mean squared error of various estimators

Estimator	Popln-1	Popln-2	Popln-3
t_0	15640.97	15640.97	2.15
t_1	12486.75	12539.30	1.24
t_2	14895.27	15492.08	1.48
t_3	12498.01	12539.30	1.30
t_4	12486.75	12539.30	1.24
t_5	12486.75	12539.30	1.24
t_6	12486.75	12539.30	1.24
t_7	10926.53	10926.53	1.09
t_p	9002.22	9002.22	0.98

RESULTS AND CONCLUSION

In this manuscript, we have proposed a generalized ratio type estimator of population mean of study variable using population median of study variable. The expressions for the bias and mean squared error for the proposed estimator have been obtained up to the first order of approximation. The optimum value of the constant for which the MSE is minimum, is obtained. The minimum value of the mean squared error of the proposed estimator has also been obtained for the optimum value of the constant. The proposed estimator is theoretically compared with the competing estimators of population mean under simple random sampling scheme. The conditions with which the proposed estimator performs better than the competing estimators have also been discussed. These theoretical conditions are verified through the numerical examples from some natural populations. From Table-5, it can be easily seen that the proposed estimator has least mean squared error among other competing estimators of population mean of study character. Thus proposed estimator is better than the competing estimators of Watson [1937] usual regression estimator, Cochran [1940] usual ratio estimator, Bahl and Tuteja [1991] exponential ratio type estimator, Srivastava [1967] estimator, Reddy [1974] estimator, Kadilar [2016] estimator and Subramani [2016] estimator. Therefore it is advisable to use the proposed estimator for improved estimation of population mean under simple random sampling scheme.

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