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RANK AND NULLITY OF TRIANGULAR FUZZY NUMBER MATRICES

C. JAISANKAR[1], S. ARUNVASAN*[2]

[1,2]Department of Mathematics,
AVC College (Autonomous) Mannampandal – 609305, India.

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ABSTRACT

The ranking and comparing of fuzzy numbers have important practical uses such that in risk analysis problem, decision making, fuzzy matrix and other fuzzy application systems. In this paper, we have been studied rank and nullity with aid of triangular fuzzy number matrices. Also we proposed the algorithm for finding the rank of triangular fuzzy matrices ($\rho_f(A)$) and illustrate to the numerical example is given.

Keywords: *Fuzzy Number, Triangular Fuzzy Number (TFN), Triangular Fuzzy Matrix (TFM), Rank and Nullity of TFM.*

1. INTRODUCTION

Fuzzy sets have been introduced by Lofti.A.Zadeh[11] Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0,1]. It can be used in a wide range of domains where information is incomplete and imprecise. Interval arithmetic was first suggested by Dwyer [2] in 1951, by means of Zadeh's extension principle [10, 11]. A fuzzy number is a quantity whose values are imprecise, rather than exact as is the case with single – valued numbers.

The concept of Rank of a matrix with fuzzy numbers as its elements, which may be used to modern uncertain imprecise aspects of real-word problems. We studied main ideas based on rank of fuzzy matrix and arithmetic operations. We give some necessary and sufficient conditions for algorithm to find rank of fuzzy matrices based on Triangular fuzzy number. In Dubosis and Prade [1] arithmetic operations will be employed for the same purpose but with respect to the inherent difficulties which are derived from the positively restriction on Triangular fuzzy number. The concept of Rank and Nullity of a triangular fuzzy matrix and some of the relevant theorems will be revalued. Finally fuzzifing the defuzzified version of the original problem for introducing fuzzy rank.

The fuzzy matrices introduced first time by Thomson [9] and discussed about the convergence of powers of fuzzy matrix. Kim [5] presented some important results on determinant of square fuzzy matrices. A.K.Shyamal and M.Pal [7,8] first time introduced triangular fuzzy matrices . In C.Jaisankar and S.Arunvasan [3, 4] introduced the concept on Hessenberg of Triangular Fuzzy Matrices. In Latha *et.al* [6] proposed Rank of a Type-2 Triangular Fuzzy Matrix.

The paper organized as follows, Firstly in section 2, we recall the definitions of Triangular fuzzy number and some operations on triangular fuzzy numbers (TFNs). In section 3, we have reviewed the definition of triangular fuzzy matrix (TFM) and some operations on Triangular fuzzy matrices (TFMs). In section 4, we defined the notion of Rank and Nullity of triangular fuzzy matrix. Some of the relevant theorems and properties are justified. In section 5, we have been presented the numerical example with the aid of notion. Finally in section 6, conclusion is included.

2. PRELIMINARIES

In this section, we recapitulate some underlying definitions and basic results of fuzzy numbers.

Definition 2.1 (Fuzzy Set): A fuzzy set is characterized by a membership function mapping the element of a domain, space or universe of discourse X to the unit interval [0,1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs

$$A=\{(x,\mu_A(x))\ ;\ x\in X\}$$

Corresponding Author: S. Arunvasan[2]*
[2]Department of Mathematics, AVC College (Autonomous) Mannampandal – 609305, India.

Here μ_A: X → [0,1] is a mapping called the degree of membership function of the fuzzy set A and μ_A(x) is called the membership value of x∈X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

Definition 2.2 (Normal Fuzzy Set): A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exists at least one x∈X such that μ_A(x) =1.

Definition 2.3 (Convex Fuzzy Set): A fuzzy set A={(x,μ_A(x))}⊆ X is called Convex fuzzy set if all $A_\propto$ are Convex set (i.e.,) for every element $x_1 \in A_\propto$ and $x_2 \in A_\propto$ for every α∈[0,1], λx_1+(1-λ) $x_2 \in A_\alpha$ for all λ∈[0,1] otherwise the fuzzy set is called non-convex fuzzy set.

Definition 2.4 (Fuzzy Number): A fuzzy set $\tilde{A}$ defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics

i. $\tilde{A}$ is normal
ii. $\tilde{A}$ is convex
iii. The support of $\tilde{A}$ is closed and bounded then $\tilde{A}$ is called fuzzy number.

Definition 2.5 (Triangular Fuzzy Number): A fuzzy number $\tilde{A}$ = (a_1, a_2, a_3) is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ;\ x \leq a_1 \\ \dfrac{x-a_1}{a_2-a_1} & ;\ a_1 \leq x \leq a_2 \\ 1 & ;\ x = a_2 \\ \dfrac{a_3-x}{a_3-a_2} & ;\ a_2 \leq x \leq a_3 \\ 0 & ;\ x > a_3 \end{cases}$$

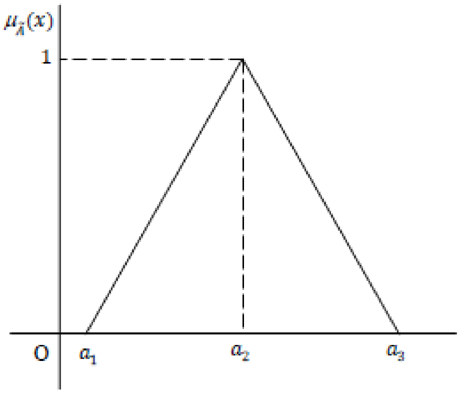

Figure-1: Triangular Fuzzy Number

Definition 2.6 (Ranking Function): We defined a ranking function ℜ: F(R)→R which maps each fuzzy numbers to real line F(R) represent the set of all triangular fuzzy number. If R be any linear ranking function

$$\Re(\tilde{A}) = \left(\frac{a_1+a_2+a_3}{3}\right)$$

Also we defined orders on F(R) by

$\Re(\tilde{A}) \geq \Re(\tilde{B})$ if and only if $\tilde{A} \geq_R \tilde{B}$
$\Re(\tilde{A}) \leq \Re(\tilde{B})$ if and only if $\tilde{A} \leq_R \tilde{B}$
$\Re(\tilde{A}) = \Re(\tilde{B})$ if and only if $\tilde{A} =_R \tilde{B}$

Definition 2.7 (Arithmetic Operations on Triangular Fuzzy Numbers (TFNs)): Let $\tilde{A}$ = (a_1, a_2, a_3) and $\tilde{B}$ = (b_1, b_2, b_3) be triangular fuzzy numbers (TFNs) then we defined,

Addition:

$\tilde{A}+\tilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3)$

Subtraction:

$\tilde{A}-\tilde{B} = (a_1-b_3, a_2-b_2, a_3-b_1)$

Multiplication:

$\tilde{A}\times\tilde{B}$ =($a_1\Re$(B), $a_2\Re$(B), $a_3\Re$(B))

where $\Re(\tilde{B})=\left(\frac{b_1+b_2+b_3}{3}\right)$ or $\Re(\tilde{b})=\left(\frac{b_1+b_2+b_3}{3}\right)$

Division:

$\tilde{A}\div\tilde{B}=\left(\frac{a_1}{\Re(\tilde{B})},\frac{C}{\Re(\tilde{B})},\frac{a_3}{\Re(\tilde{B})}\right)$

where $\Re(\tilde{B})=\left(\frac{b_1+b_2+b_3}{3}\right)$ or $\Re(\tilde{b})=\left(\frac{b_1+b_2+b_3}{3}\right)$

Scalar Multiplication:

$$k\tilde{A}=\begin{cases}(ka_{1,}ka_{2,}ka_3)\ if\ K\geq 0\\ (ka_{3,}ka_2,ka_1) if k<0\end{cases}$$

Definition 2.8 (Zero Triangular Fuzzy Number): If $\tilde{A}$ = (0, 0, 0) then $\tilde{A}$ is said to be zero triangular fuzzy number. It is defined by 0.

Definition 2.9 (Zero Equivalent Triangular Fuzzy Number): A triangular fuzzy number $\tilde{A}$ is said to be a zero equivalent triangular fuzzy number if
$\Re$ ($\tilde{A}$) = 0. It is defined by $\tilde{0}$.

Definition 2.10 (Unit Triangular Fuzzy Number): If $\tilde{A}$ = (1, 1, 1) then $\tilde{A}$ is said to be a unit triangular fuzzy number. It is denoted by 1.

Definition 2.11 (Unit Equivalent Triangular Fuzzy Number): A triangular fuzzy number $\tilde{A}$ is said to be unit equivalent triangular fuzzy number.

If $\Re$ ($\tilde{A}$) = 1. It is denoted by $\tilde{1}$.

3. TRIANGULAR FUZZY MATRICES (TFMS)

In this section, we introduced the triangular fuzzy matrix and the operations of the matrices some examples provided using the operations.

Definition 3.1 (Triangular Fuzzy Matrix (TFM)):A triangular fuzzy matrix of order m×n is defined as A=$(\tilde{a}_{ij})_{m\times n}$, where $\tilde{a}_{ij}$=(a_{ij},b_{ij},c_{ij}) is the ij^{th} element of A.

Definition 3.2 (Operations on Triangular Fuzzy Matrices (TFMs)): As for classical matrices. We define the following operations on triangular fuzzy matrices. Let A = $(\tilde{a}_{ij})$ and B = $(\tilde{b}_{ij})$ be two triangular fuzzy matrices (TFMs) of same order. Then, we have the following

i. A+B = $(\tilde{a}_{ij}+\tilde{b}_{ij})$
ii. A-B = $(\tilde{a}_{ij}-\tilde{b}_{ij})$
iii. For A = $(\tilde{a}_{ij})_{m\times n}$ amd B = $(\tilde{b}_{ij})_{n\times k}$ then AB = $(\tilde{c}_{ij})_{m\times k}$ where $\tilde{c}_{ij}=\sum_{p=1}^{n}\tilde{a}_{ip}\,.\,\tilde{b}_{pj}$, i=1,2,…m and j=1,2,…k
iv. A^T or A^1 = $(\tilde{a}_{ji})$
v. $KA=(K\tilde{a}_{ij})$ where K is scalar.

Definition 3.3 (Diagonal TFM): A square TFM A = $(\tilde{a}_{ij})$ is said to be diagonal TFM is all the elements outside the principal diagonal are 0.

Definition 3.4 (Diagonal-Equivalent TFM): A square TFM A = $(\tilde{a}_{ij})$ is said to be diagonal-equivalent TFM is all the elements outside the principal diagonal are $\tilde{0}$.

Definition 3.5 (Equal Triangular Fuzzy Matrices): Two triangular fuzzy matrices A = $(\tilde{a}_{ij})$ and B = $(\tilde{b}_{ij})$ of the same order are said to be equal if the rank of their elements in the corresponding position are equal. Also it is denoted by A=B.

Notation: Let A = $\tilde{a}_{ij}$ be a triangular fuzzy matrices suppose if we take rank for every $\tilde{a}_{ij}$ in A then A is converted into a classical matrix. It is denoted by A = $(\tilde{a}_{ij})$ C = ($\Re(\tilde{a}_{ij})$).

4. RANK AND NULLITY OF TRIANGULAR FUZZY MATRIX

Definition 4.1: The row space $\Re$(A) of an m× n Triangular Fuzzy Matrix A is the subspace of V_n generated by the rows of A. The row rank ρ_r(A) of A is the smallest possible size of spanning set of $\Re$(A).

Definition 4.2: The column space $\mathbb{C}$(A) of an m× n Triangular Fuzzy Matrix A is the subspace of V_m generated by the columns of A. The column rank ρ_c(A) of A is the smallest possible size of spanning set of $\mathbb{C}$(A).

(i.e) The rank ρ_c(A) of $\mathbb{C}$(A) is number of fuzzy vectors in maximum generating set of $\mathbb{C}$(A) is Dim $\mathbb{C}$(A) =ρ_c(A).

Definition 4.3: A triangular fuzzy matrix A is said to be of rank of TFM if, ρ_r(A)= $\rho_c(A)$= ρ_f(A), written by ρ_f(A).

Definition 4.4: Let $\tilde{v}_1$, $\tilde{v}_2$, …..,$\tilde{v}_k$ be a set of fuzzy vectors in a fuzzy vector space over F_f. Any expression of the form $\alpha_1\tilde{v}_1$+$\alpha_2\tilde{v}_2$+…….+$\alpha_k\tilde{v}_k$ where each $\alpha_i \epsilon F_f$is called a fuzzy linear combination of the fuzzy vectors $\tilde{v}_1$, $\tilde{v}_2$, …..,$\tilde{v}_k$.

Definition 4.5: A set of fuzzy vectors $\{\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_k\}$ in a fuzzy vector space V_f is said to be linearly independent provided that the zero fuzzy vector of V_f is not expressible as a non trivial linear combination of the fuzzy vector $\tilde{v}_1$, $\tilde{v}_2$, …..,$\tilde{v}_k$.

i.e., $\alpha_1\tilde{v}_1$+$\alpha_2\tilde{v}_2$+…….+$\alpha_k\tilde{v}_k$=0 for scalars $\alpha_i \epsilon F_f$ then each $\alpha_i = 0$.

Definition 4.6: A set of fuzzy vectors $\{\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_k\}$ in a fuzzy vector space V_f is said to be linearly dependent provided the set is not linearly independent.

Definition 4.7: A set of fuzzy vectors $\{\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_k\}$ in a fuzzy vector space V_f is said to be a spanning set for $\tilde{S}$ provided that each fuzzy vector $\tilde{V}$ is expressible as a fuzzy linear combination of $\tilde{v}_1$, $\tilde{v}_2$, …..,$\tilde{v}_k$.

Definition 4.8: A set of fuzzy vectors $\{\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_k\}$ in a fuzzy vector space V_f is said to be a basis for $\widetilde{\mathcal{B}}$ provided that it is both a linearly independent set and a spanning set for $\tilde{S}$.

Definition 4.9: A fuzzy vector space V_f is said to be finite dimensional provided that there exists a finite set of fuzzy vectors in $\tilde{V}$ which is a basis for $\widetilde{\mathcal{B}}$. Otherwise $\tilde{V}$ is said to be infinite dimensional.

Definition 4.10: The dimension of a finite dimensional fuzzy vector space V_f is the number of elements in a basis of $\widetilde{\mathcal{B}}$.

Definition 4.11: Let $f: V \to W$ be a fuzzy linear map of fuzzy vector space V_f and W_f. The image of f is denoted by Imf is defined as I$mf = \{f(\tilde{v}); \tilde{v}\epsilon\tilde{V}\}$. The Kernal of f denoted Ker f is defined by Ker $f = \{\tilde{v}\epsilon\tilde{V}: f(\tilde{v}) = O\}$.

Definition 4.12: Let $f: V_f \to W_f$ be a fuzzy linear map of finite dimension fuzzy vector spaces V_f andW_f. We say that Imf and Ker f are subspaces of V_f and W_f respectively.

The dimension of $Im\, f$ is called the rank of f and is denoted $\rho_f(f)$.

The dimension of Ker f is called the nullity of f is denoted $n_f(f)$.

Definition 4.13: A square TFM A = $(\tilde{a}_{ij})$ is said to be singular TFM if $|A|$ = $\tilde{0}$.

Definition 4.14: A square TFM A = $(\tilde{a}_{ij})$ is said to be non singular TFM if $|A| \neq \tilde{0}$.

4.1 Elementary transformations:

We use the term elementary transformation of a TFM A= $(\tilde{a}_{ij})$ for the following transformations.

a. Interchange of two rows and columns.
b. Multiplication of a row (or a column) by an arbitrary triangular fuzzy number$\neq \tilde{0}$.
c. Addition of a multiple of one row (or column) by a triangular fuzzy number$\neq \tilde{0}$ to another row or column. Clearly these elementary transformation do not change the rank of TFM.

4.2 Algorithm for finding the rank of TFM to row reduced Echelon form.

Suppose we have TFM A = $\begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{2n} \\ \vdots & & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \tilde{a}_{mn} \end{pmatrix}$

Step-1: If $\tilde{R}(\tilde{a}_{11})$=0 then an interchange of rows and columns will change element in the position $(\tilde{a}_{11})$ so that $\tilde{R}(\tilde{a}_{11})$≠0.

Step-2: Convert the element $\tilde{a}_{11}$ to $\tilde{1}$ by multiplying the first row by 1/ $\tilde{a}_{11}$.

Step-3: Subtract from the i^{th} row, i>1, the first row multiplied by $\tilde{a}_{i1}(i$>1) will be replaced by $\tilde{0}$.

Step-4: Subtract the j^{th} column j>1, the first column multiplied by $\tilde{a}_{1j}$, whenever $\widetilde{R}$ $(\tilde{a}_{ij})$ ≠ 0,then the element $\tilde{a}_{ij}$(j >1) will be replaced by $\tilde{0}$.

At this stage we get a TFM of the form

$$A=\begin{pmatrix} \tilde{1} & \tilde{0} & \ldots & \tilde{0} \\ \tilde{0} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \tilde{0} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{pmatrix}$$

Step-5: Performing the same manipulation (step1 to step4) with the sub matrix that remains in the lower right corner, and so on. We finally alter a finite number of manipulation arrive at a diagonal-equivalent TFM with the same rank as the original TFM A.

Remark: Thus to find the rank of TFM it is necessary to convert the TFM by means of elementary transformation to diagonal equivalent TFM. Finally, we convert this diagonal equivalent TFM into matrix and count the number of units in the principal diagonal. The number of units gives the rank of the given TFM.

4.3. Properties and Theorems on Rank and Nullity of TFM.

Theorem: 4.3.1: The fuzzy linear map $f: v \to w$ is injective if and only if Ker f = 0.

Proof: We say that f(0)=0 where f is fuzzy linear. Thus if f is injective then Ker f = 0.

Conversely, suppose that Ker f=0, then $f(V_1) = f(V_2)$ implies by linearity that $f(V_1 - V_2) = 0$.

Hence $V_1 - V_2 = 0$. Since Ker f=0.

Therefore, f is injective.

Theorem: 4.3.2: Let $f: v \to w$ be a fuzzy linear transformation. Then dim V_f= rank T_f+nullity T_f

Proof: By the result

$$\frac{V_f}{Ker\ T_f} = T_f(V) => \dim(\frac{v_f}{KerT_f}) = \dim(T(V))$$

$$\dim\ V_f = \dim(\text{Ker}\ T_f) = \dim(T_f(V))$$

Therefore, dim V_f = nullity T_f= rank T_f

Therefore, dim V_f = nullity T_f= rank T_f

Theorem: 4.3.3: Let $f: \tilde{V}_f \to \widetilde{W}_f$ be a fuzzy linear map of finite dimensional fuzzy vector spaces $\tilde{V}_f$ and $\widetilde{W}_f$. Then $\rho_f(f) + n_f(f) = dim\ V$

Proof: Let nullity (f) = n and let $\{\tilde{u}_1, \tilde{u}_2 \ldots \tilde{u}_n\}$ be a basis of Ker f. Extend this to a basis for all of $\tilde{V}$.

i.e., $\{\tilde{u}_1, \tilde{u}_2, \ldots\ldots \tilde{u}_n, \tilde{v}_1, \tilde{v}_2,, \ldots\ldots \tilde{v}_n\}$ is a basis for $\widetilde{\mathcal{B}}$ for suitable elements $\tilde{v}_1, \tilde{v}_2, \ldots.. \tilde{v}_n$ of $\tilde{V}$.

We will show that $\{f(\tilde{v}_1), f(\tilde{v}_2), \ldots\ldots f(\tilde{v}_s)\}$ is a basis of I$m\ f$.

First to prove that this set is linearly independent suppose that $\alpha_1 f(\tilde{v}_1) + \alpha_1 f(\tilde{v}_2) + \cdots + \alpha_1 f(\tilde{v}_s) = 0$. Then the linearity of f implies that $f(\sum_{i=1}^{s} \tilde{v}_i)$ = 0. So that $\sum_{i=1}^{s} \alpha_i \tilde{v}_i$ belongs to Ker f.

Hence $\sum_{i=1}^{s} \alpha_i \tilde{v}_i$ is a linear combination of $\{\tilde{u}_1, \tilde{u}_2 \ldots \tilde{u}_n\}$ which implies that each α_i=0 because $\{\tilde{u}_1, \tilde{u}_2, \ldots\ldots \tilde{u}_n, \tilde{v}_1, \tilde{v}_2,, \ldots\ldots \tilde{v}_n\}$ is a basis for $\tilde{\mathcal{B}}$. Secondly an element of Imf is a fuzzy linear combination of the fuzzy vectors $\{f(\tilde{u}_1), f(\tilde{u}_2), \ldots\ldots f(\tilde{u}_n), f(\tilde{v}_1), f(\tilde{v}_2), \ldots\ldots f(\tilde{v}_s)\}$. The first n of these fuzzy vectors are zero as $\tilde{u}_1, \tilde{u}_2, \ldots\ldots \tilde{u}_n$ are in Ker f. So that $\{ f(\tilde{v}_1), f(\tilde{v}_2), \ldots\ldots f(\tilde{v}_s)\}$ spans Imf.

This proves that S=$\rho_f(f)$ and completes the proof.

Property 4.3.4: The followings are equivalent characterization of the rank of an mxn triangular fuzzy matrix of A.

i. $\rho_c(A)$ is the maximum number of linearly independent columns of A.
ii. $\rho_r(A)$ is the maximum number of linearly independent rows of A.
iii. $\rho_f(A)$ is the number of non zero equivalent rows in an each TFM to which A has been reduced by a finite sequence of elementary row operations.

Proof: The equivalent of (i) with the definition of $\rho_f(A)$as the elimination of $I_m f$ follows from our comments above.

Ker g is the solution set of the homogeneous system of equation $A\tilde{x}$=0 where $X = (\tilde{x}_i)$ in a columns fuzzy vector of n unknowns. This dimension of this solution set will be n-t where t is the number of non zero equivalent TFM rows in an echelon form for A. Now let $f: V \to W$ be a linear map of finite dimensional fuzzy vector spaces V_f and W_f. Then $\rho_f(f) + n_f(f) = \dim V_f$. Such that $\rho_f(g) = t$. This shows that (iii) is equivalent to our definition of fuzzy rank.

The nature of elementary row operations and the fact that they are all reversible implies that the subspace of F_f^n spanned by the rows of A coincide with the subspace of F_f^n Spanned by thr rows of an echelon form for A.

Thus (ii) and (iii) are equivalent.

Property 4.3.5:

i. If A is any mxn TFM with entries in F_f where
ii. If A and B are mxn matrices with in F then r(A+B)≤r(A)+r(B).
iii. If A is an mxn matrix and B is an nxp matrix each with entries in F then r(AB)≤min(r(A),r(B)) i.e r(AB)≤r(A) and r(AB)≤r(B).

Proof:

i. To prove (i) then by above property it is obivious.

Let g: $V_f^n \to W_f^m$ and h: $V_f^n \to W_f^m$ be defined by g(v)=Av_f. And h(v)=Bv_f for each column fuzzy vector VϵV_f^n then $Im(g+h)$ is contained in the subspace. $Img + Imh$ as defined in subspace spanned by a set of fuzzy vectors.
ρ_f(A+B)≤ $\rho_f(A) + \rho_f(B)$ which is obvious.

Let g: $V_f^n \to W_f^m$ be a given by g(v)=Bv_f for $v_f \epsilon f_f^R$ and let h: $V_f^m \to W_f^p$ be given by h(v)=Av_f for $v_f \epsilon V_f^m$. Then $\rho_f(A) = \rho_f(h)$ and $\rho_f(B) = \rho_f(g)$. Clearly $Im(hog)$ is contained in Imh which shows that $\rho_f(AB) \le \rho_f(A)$ clearly Ker g is contained in Ker(hog) and so $n_f(hog) \ge n_f(g)$. We have that V(g) = $n_f - n_f(g)$. So that $\rho_f(g) \ge n_f - n_f(hog)$. Hence by again $\rho_f(g) \ge \rho_f(hog)$. This proves that $\rho_f(AB) \le \rho_f(B)$.

5. NUMERICAL EXAMPLE

To find the Rank and Nullity of TFM is A

$$A=\begin{pmatrix} (-1,0,4) & (1,2,3) & (-2,8,9) \\ (1,2,3) & (1,3,5) & (2,4,6) \\ (1,3,5) & (-2,8,9) & (4,8,9) \end{pmatrix}$$

$$\sim \begin{pmatrix} (-1,0,4) & (1,2,3) & (-2,8,9) \\ (1,2,3) & (1,3,5) & (2,4,6) \\ (1,3,5) & (-2,8,9) & (4,8,9) \end{pmatrix}$$

$R_1 \to R_1$/(-1,0,4) where $\Re(-1,0,4) \neq 0$.

$$\sim \begin{pmatrix} (-1,0,4) & (1,2,3) & (-2,8,9) \\ (1,2,3) & (1,3,5) & (2,4,6) \\ (1,3,5) & (-2,8,9) & (4,8,9) \end{pmatrix}$$

$R_2 \to R_2 - (1,3,5)R_1$

$R_3 \to R_3 - (1,3,5)R_1$

$$\sim \begin{pmatrix} (-1,0,4) & (1,2,3) & (-2,8,9) \\ (-2,0,2) & (-5,-1,3) & (-13,-6,1) \\ (-4,0,4) & (-12,2,7) & (-21,-7,4) \end{pmatrix}$$

$R_2 \to R_2$/(-5,-1,3) where $\Re(-5,-1,3) \neq 0$.

$$\sim \begin{pmatrix} (-1,0,4) & (1,2,3) & (-2,8,9) \\ (-2,0,2) & (-3,1,5) & (-1,6,13) \\ (-4,0,4) & (-12,2,7) & (-21,-7,4) \end{pmatrix}$$

$R_3 \to R_3 + R_2$

$$\sim \begin{pmatrix} (-1,0,4) & (1,2,3) & (-2,8,9) \\ (-2,0,2) & (-3,1,5) & (-1,6,13) \\ (-8,0,8) & (-15,3,12) & (-22,-1,17) \end{pmatrix}$$

$C_2 \to C_2 - (1,2,3)C_1$
$C_3 \to C_3 - (-2,8,9)C_1$

$$\sim \begin{pmatrix} (-1,0,4) & (-2,0,2) & (-11,0,11) \\ (-2,0,2) & (-3,1,5) & (-1,6,13) \\ (-8,0,8) & (-15,3,12) & (-22,-1,17) \end{pmatrix}$$

$C_3 \to C_3 - (-1,6,13)C_2$

$$\sim \begin{pmatrix} (-1,0,4) & (-2,0,2) & (-11,0,11) \\ (-2,0,2) & (-3,1,5) & (-31,0,31) \\ (-8,0,8) & (-15,3,12) & (-22,-1,17) \end{pmatrix}$$

$$\sim \begin{pmatrix} (-1,0,4) & (-2,0,2) & (-11,0,11) \\ (-2,0,2) & (-3,1,5) & (-31,0,31) \\ (-8,0,8) & \left({}^{15}/_{2}, {}^{-3}/_{2}, -6\right) & \left(11, {}^{1}/_{2}, {}^{-17}/_{2}\right) \end{pmatrix}$$

$R_3 \to R_3/(-11,0,11)$

$$c = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The basis of $\mathbb{C}$(A)= $\begin{pmatrix} (-1,0,4) & (1,2,3) & (-2,8,9) \\ (1,2,3) & (1,3,5) & (2,4,6) \\ (1,3,5) & (-2,8,9) & (4,8,9) \end{pmatrix}$

The basis in $\Re$(A)= $\begin{pmatrix} (-1,0,4) & (-2,0,2) & (-11,0,11) \\ (-2,0,2) & (-3,1,5) & (-31,0,31) \\ (-8,0,8) & \left({}^{15}/_{2}, {}^{-3}/_{2}, -6\right) & \left(11, {}^{1}/_{2}, {}^{-17}/_{2}\right) \end{pmatrix}$

$$\rho_{r(A)} = \rho_{c(A)} = 3$$

Hence, the Rank of TFM $\rho_f(A)$=3

6. CONCLUSION

In this work, we discussed the notion of Rank and Nullity of Triangular Fuzzy Matrix. Also we have been justify some of the relevant properties with the aid of this notion. In this work can be extended in the consistency of linear system in non-homogeneous equations using the Rank of Triangular Fuzzy Matrix in future.

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