International Journal of Mathematical Archive-8(6), 2017, 167-172
Available online through www.ijma.info ISSN 2229 – 5046

RADIATION EFFECT ON MHD FREE CONVECTIVE FLOW PAST A VERTICAL POROUS PLATE

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Received On: 27-03-17; Revised & Accepted On: 26-06-17)

ABSTRACT

An unsteady MHD free convective flow past a vertical porous plate in the presence of radiation has been considered. The fluid is considered to be a gray, absorbing emitting but non – scattering medium and the Rossel and approximation is used to describe the radiative heat flux in the energy equation. The dimensionless governing coupled, nonlinear boundary layer partial differential equations are solved by an efficient finite element method. A parametric study of the physical parameters involved in the problem is considered and a representative set of numerical results is illustrated graphically.

Keywords: *Radiation, free convective, vertical plate, MHD, skin-friction.*

1. INTRODUCTION

Free convection flows past different types of vertical bodies are studied because of their wide applications and hence, it has attracted the attention of numerous investigators and scientists. Literature on unsteady MHD convection heat transfer with or without Hall currents are very extensive due to its technical importance in the scientific community (Elbashbeshy, 1988; Rohsenow, 1998). The radiation effects on MHD flow and heat transfer problems have become more important industrially. At high operating temperature, radiation effect can be quite significant (Sahoo, 2003). Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites, and space vehicles are examples of such engineering areas. Heat and mass transfer of a moving vertical plate with suction was studied by Erickson *et al*, (1966) and (Gupta & Gupta, 1977) with different conditions. Convective heat transfer in an electrically conducting fluid at stretching surface was studied by (Vajravelu & Hadjinicolaou, 1997). Unsteady free convection flow past a vertical porous plate was investigated by (Helmy, 1988). Acharya *et al.* (2000) have studied free convection and mass transfer flow through a porous medium bounded by vertical infinite surface with constant suction and heat flux (Chen & Char, 1998). But in those studies, they considered the flow to be steady (Danberg & Fansber1976). Coming back to unsteady case, (Young, 2000), investigated unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. (Chamka, 2004), considered the case taking mass transfer with heat absorption (Sakiadis, 1961). Muthukumaraswamy & Kumar, (2004) investigated heat and mass transfer effect on moving vertical plate in the presence of Thermal Radiation (Carnahan, 1969; Takhar *et al.*, 1996; Singh *et al.*, 2003).

In this paper, we have discussed free convection MHD flow of a viscous incompressible fluid past an infinite vertical porous plate with heat and mass transfer in the presence of radiation. The governing system of partial differential equations and then solved numerically using finite difference method.

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2. MATHEMATICAL FORMULATION

Consider the effect of thermal radiation on an unsteady free convective mass flow of a viscous incompressible electrically conducting fluid flow past an accelerating vertical infinite porous flat plate. Choose the Cartesian coordinate system such that x-axis is taken to be along the vertical plate and the y- axis normal to the plate. Since the plate is considered infinite in x- direction, hence all physical quantities will be independent of x-direction. The wall is maintained at constant temperature T_w and concentration C_w higher than the ambient temperature T_∞ and concentration C_∞ respectively. A uniform magnetic field B_0 of magnitude is applied normal to the plate. The transverse applied magnetic field and magnetic Reynold's number are assumed to be very small, so that the induced magnetic field is negligible. The homogeneous chemical reaction is of first order with rate constant between the diffusing species of the fluid is neglected. It is assumed that there is no applied voltage which implies the absence of an electric field. It is assumed that the plate is accelerating with a velocity $u=U_0$ in its own plane for $t\geq 0$. The fluid has constant kinematic viscosity and constant thermal conductivity and the Boussinesq's approximation have been adopted for the flow. The magneto-hydrodynamic unsteady free convective boundary layer equations under the Boussinesq's approximations are:

$$\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=g\beta\left(T'-T'_\infty\right)-\nu\frac{u'}{k'}+g\beta^*\left(C'-C'_\infty\right)+\nu\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma B_o^2}{\rho}u' \tag{2}$$

$$\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}=\frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'} \tag{3}$$

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2} \tag{4}$$

The boundary conditions of the problem are:

$$\left.\begin{array}{l} t'\leq 0:\left\{\ u'=0,\ v'=0,\ T'=0,\ C'=0\ \ for\ all\ y'\right. \\ t'\geq 0:\left\{\begin{array}{l} u'=U_O,\ v'=-v'_o,\ T'=T'_w,\ C'=C'_w\ \ at\ y'=0 \\ u'\to 0,\ T'\to T'_\infty,\ C'\to C'_\infty\ \ as\ y'\to\infty \end{array}\right\} \end{array}\right\} \tag{5}$$

The radiative heat flux term is simplified by making use of the Rossel and approximation [28] as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'} \tag{6}$$

Here σ^* is Stefan- Boltzmann constant and k^* is the mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficient small such that T^4 may be expressed as a linear function of temperature.

$$T'^4\cong 4T_\infty'^3T'-3T_\infty'^4 \tag{7}$$

Using (6) and (7) in the last term of Equation (3) we obtain

$$\frac{\partial q_r}{\partial y'}=-\frac{16\sigma^*}{3k^*}\frac{\partial T_\infty'^3}{\partial y'^2} \tag{8}$$

In order to write the governing Equations and the boundary conditions in dimensional following non-dimensional quantities are introduced.

$$\left.\begin{array}{l} y=\frac{y'v'_o}{\nu},\ t=\frac{t'v_o'^2}{4\nu},\ \omega=\frac{4\nu\omega'}{v_o'^2},\ u=\frac{u'}{U_o},\ M=\left(\frac{\sigma B_o^2}{\rho}\right)\frac{\nu}{v_o'^2}, \\ K=\frac{k'\,v_o'^2}{\nu^2},Sc=\frac{\nu}{D},\ \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty},\ C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\ \Pr=\frac{\nu}{k}, \\ Gr=\frac{\nu g\beta\left(T'_w-T'_\infty\right)}{U_o v_o'^3},\ Gc=\frac{g\beta^*\nu\left(C'_w-C'_\infty\right)}{U_o v_o'^3},\ \ R=\frac{k\,k^*}{4\sigma T_h'^3} \end{array}\right\} \tag{9}$$

Where ν the kinematics viscosity, *k* is the thermal diffusivity, β and β^* are the volumetric coefficient of expansion for heat and mass transfer respectively, ρ is the fluid density, σ is the electrical conductivity of the fluid, g is the acceleration due to gravity, T is the temperature, U0 is the free stream velocity, μ is the coefficient of viscosity, C_p is the specific heat at constant, q_r is the radiative heat flux, M is the magnetic field, R is the radiation parameter, Gr is the Grashof number, Gc is the solutal Grashof number, Pr is the Prandtl number, Sc is the Schmidt number T_∞ is the temperature of the fluid far away from the plate, C is the concentration, C_∞ is the concentration of the fluid far away from the plate, K is the permeability parameter.

Substituting (9) in equations (2), (3) and (4) under boundary conditions (5), we get

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+(Gr)\theta+(Gc)C-\left(M+\frac{1}{K}\right)u \qquad (10)$$

$$\frac{\partial \theta}{\partial t}-\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\left(1+\frac{4}{3R}\right)\frac{\partial^2 \theta}{\partial y^2} \qquad (11)$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \qquad (12)$$

The corresponding boundary conditions are

$$\left.\begin{array}{l} u=1,\ \theta=1,\ C=1\ at\ y=0 \\ u\to 0,\ \theta\to 0,\ C\to 0\ \ as\ y\to\infty \end{array}\right\} \qquad (13)$$

3. SOLUTION OF THE PROBLEM

The above set of coupled non-linear partial differential equations subject to the boundary conditions is solved by Finite Element Technique. The fundamental steps comprising the method are as follows.

- Step 1: Discretization of the domain into elements.
- Step 2: Derivation of the element equations.
- Step 3: Assembly of element equations.
- Step 4: Imposition of Boundary Conditions.
- Step 5: Solution of the assembled equations.

The problem of radiative heat transfer to unsteady magnetohydromagnetic flow involving heat and mass transfer is addressed in this study. Numerical solutions have been carried out for the non-dimensional Temperature Concentration *C*, velocity (u) keeping the other parameters of the problem fixed. Numerical calculations of these results are presented graphically in the figures from (1) to(11). These results show the effect of different parameters on the velocity, temperature distribution and concentration profiles at the wall. To find out the solution of this problem, we have placed an infinite vertical plate in a finite length in the flow. Hence, we solve the entire problem in a finite boundary.

However, in the graphs, the *y* values vary from 0 to 4 and the velocity, temperature, and concentration tend to zero as *y* tends to 4. This is true for any value of *y*. Thus, we have considered finite length. In the present study we adopted the following default parameter values of finite element method. *Gr*=1.0, *Gc* = 1.0, *M* =1.0, *K* =1.0, Pr = 0.71, *N* =1.0, *Sc* =0.22. All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

For various values of the magnetic parameter *M*, the velocity profiles are plotted in Fig 1. It can be seen that as *M* increases, the velocity decreases. This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the flow. The effect of the permeability parameter *K* on the velocity field is shown in Fig 2. As depicted in this figure, the effects of increasing the values of porous permeability parameter is to increase the value of the velocity component in the boundary layer due to the fact that drag is reduced by increasing the values of the porous permeability on the fluid flow which results in increased velocity. The trend shows that the velocity is accelerated with increasing porosity parameter. Figs. 3 and 4 show the influence of the thermal buoyancy force parameter Gr and solutal buoyancy force parameter Gc, respectively. As seen from this figure that maximum peak value attains for Gr = 3 and Gc=4.5 and minimum peak value is observed in the absence of buoyancy force. This is due to the fact that buoyancy force enhances fluid velocity and increase the boundary layer thickness with increase in the value of Gr or Gc.In figure 5 illustrate that the effect of velocity profiles for different values of Prandtl number (Pr). It is observed that the velocity field decrease with an increasing the Prandtl number (Pr). Figure 6 presents the behavior of the fluid velocity when the radiation parameter is increased. Figure (7) illustrates that the velocity distribution for different values Schmidt number (Sc). It is noticed that the velocity profiles decreases with an increasing Schmidt number. Figure 8 presents the effect of Prandtl number Pr on velocity profiles. As Prandtl number increases the velocity profiles decreases. The effect of radiation parameter (R) on the temperature field is illustrated in Fig.9. It is obvious that the radiation parameter increases an increasing of the fluid temperature. The concentration profiles for different values of Schmidt number is presented figure 10. From the figure, it is seen that the concentration decreases with an increases in Schmidt number.

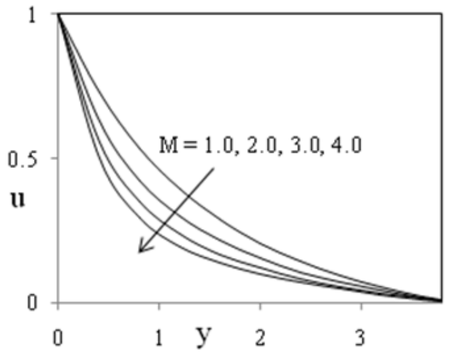

Fig.1: Velocity profiles for different values of magnetic parameter (M)

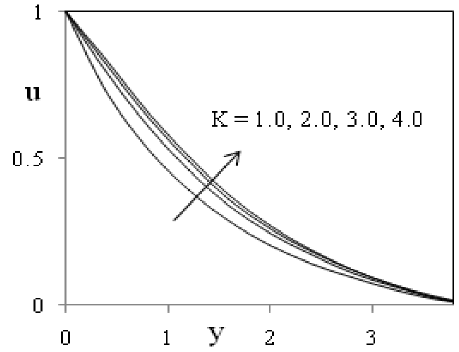

Fig.2: Velocity profiles for different values of permeability parameter (K).

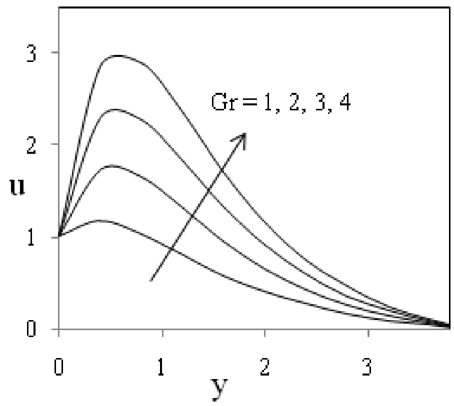

Fig.3: Velocity profiles for different values of Grashof number (Gr).

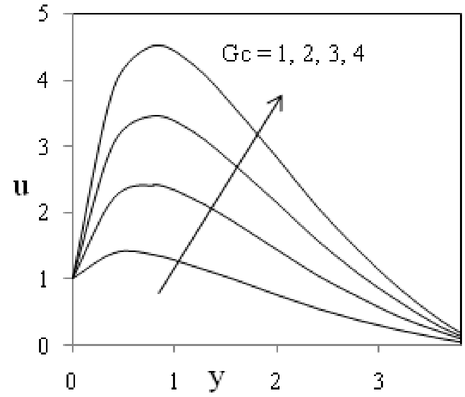

Fig.4: Velocity profiles for different values of modified Grashof number (Gc).

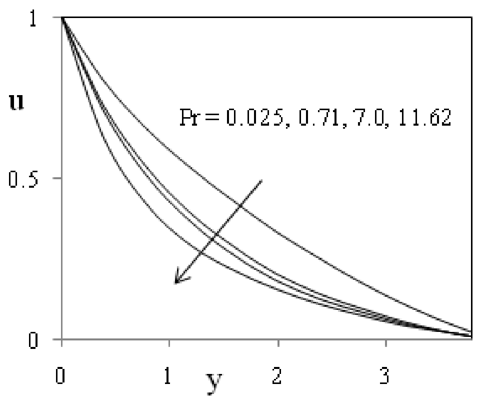

Fig.5: Velocity profiles for different values of Prandtl number (Pr).

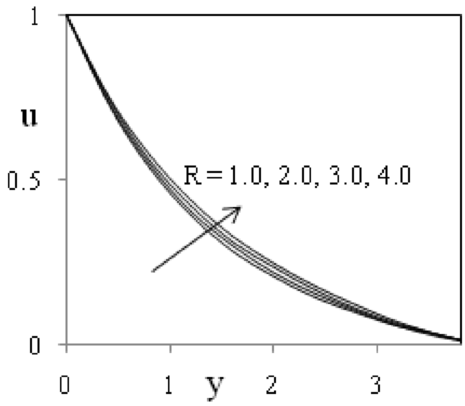

Fig.6: Velocity profiles for different values of radiation parameter (R).

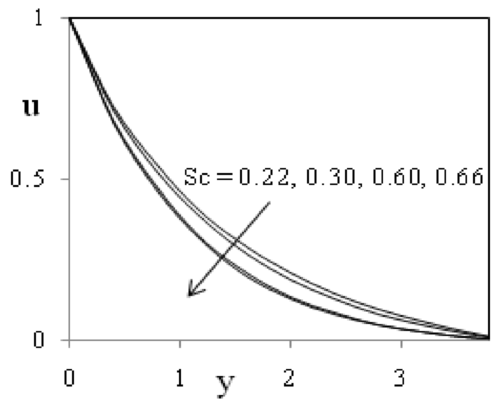

Fig.7: Velocity profiles for different values of Schmidt number (Sc).

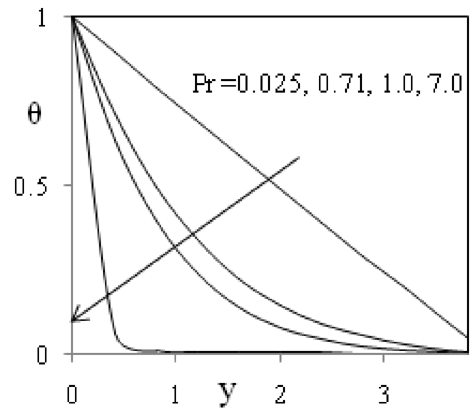

Fig.8: Temperature profiles for different values of Prandtl number (Pr).

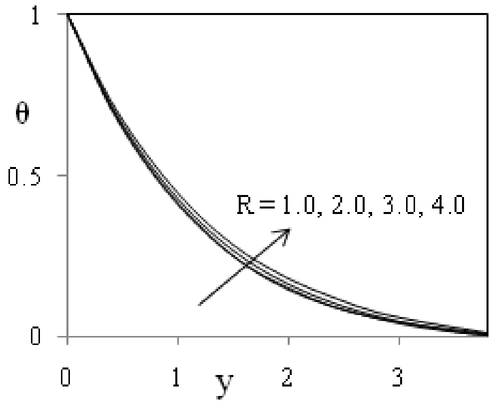

Fig.9: Temperature profiles for different values of radiation parameter (R).

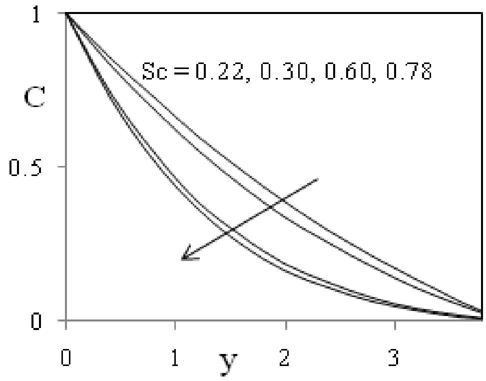

Fig.10: Concentration profiles for different values of Schmidt number (Pr).

Table – 1: Skin – friction results (τ) for the values of $Gr, Gc, \Pr, Sc, M, K, R$

M	K	R	Gr	Gc	Pr	Sc	τ
1.0	1.0	1.0	1.0	1.0	0.71	0.22	3.2954
1.0	1.0	1.0	2.0	1.0	0.71	0.22	4.4918
1.0	1.0	1.0	1.0	2.0	0.71	0.22	5.3946
1.0	1.0	1.0	1.0	1.0	7.00	0.22	2.1650
1.0	1.0	1.0	1.0	1.0	0.71	0.30	3.1168
2.0	1.0	1.0	1.0	1.0	0.71	0.22	2.6004
1.0	2.0	1.0	1.0	1.0	0.71	0.22	3.7905
1.0	1.0	2.0	1.0	1.0	0.71	0.22	3.4094

Table – 2: Rate of heat transfer (Nu) values for different values of radiation (R) and Prandtl number (Pr)

R	Pr	Nu
1.0	0.71	4.4972
1.0	7.00	1.0897
2.0	0.71	4.6087

Table – 3: Rate of mass transfer (Sh) values for different values of Schmidt number (Sc) and Soret number (So)

Sc	So	Sh
0.22	1.0	6.9193
0.30	1.0	6.5249
0.22	2.0	7.0338

CONCLUSIONS

We summarize below the following results of physical interest on the velocity, temperature and concentration distributions of the flow field and also on the skin – friction, rate of heat and mass transfer at the wall.

- An increasing the magnetic parameter or Prandtl number or Schmidt number retards the velocity of the flow field at all points.
- The effects of increasing Grashof number or Modified Grashof number or Permeability parameter or Eckert number or Permeability parameter or thermal radiation parameter are to accelerate velocity of the flow field at all points.
- An increasing the Prandtl number decreases temperature of the flow field at all points and increases with increasing of radiation parameter.
- The Schmidt numberdecreases the concentration of the flow field at all points.
- A growing Hartmann number or Prandtl number or Schmidt number decreases the skin – friction while increasing Grashof number or Modified Grashof number or Permeability parameter or radiation parameter increases the skin – friction.
- The rate of heat transfer is decreasing with increasing of Prandtl number and increases with increasing of Thermal radiation parameter.
- The rate of mass transfer is decreasing with increasing of Schmidt number.

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Source of support: Nil, Conflict of interest: None Declared.