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BAYESIAN SPECIAL TYPE DOUBLE SAMPLING PLAN WITH BETA PRIOR DISTRIBTUTION

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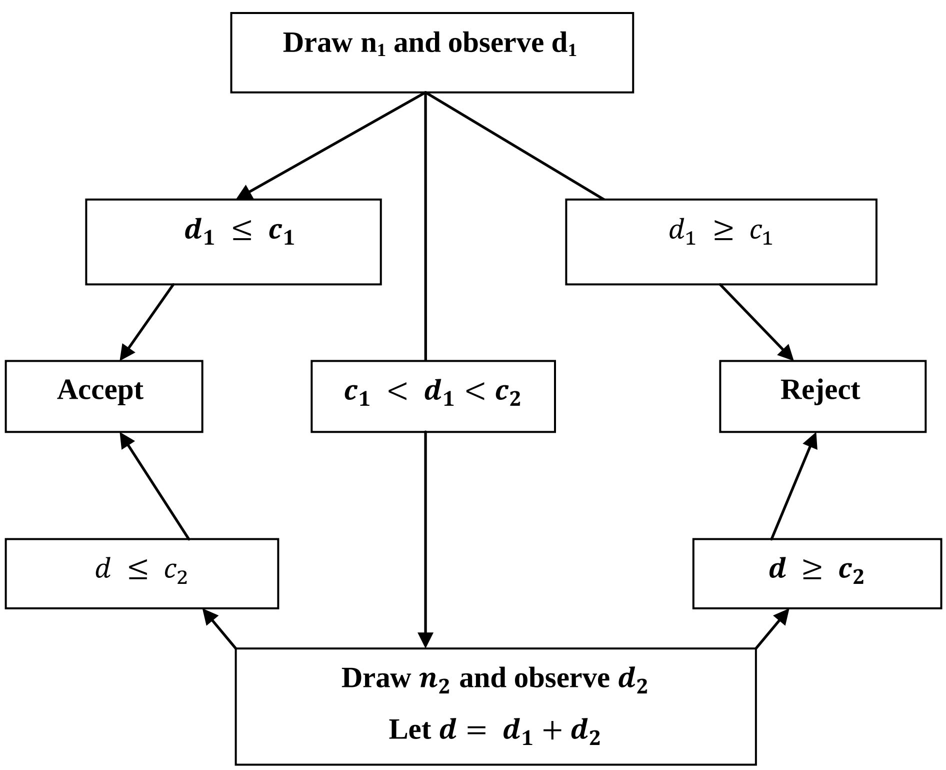
ABSTRACT

An iterative procedure of finding the parameters of a Special type Double sampling plan by attributes under Binomial distribution with Bea prior satisfying given conditions with respect to producer's and consumers' risks is presented. Tables are constructed for selecting the sampling parameters. The discriminating power of the Bayesian Special Type Double sampling is also discussed by determining the operating characteristic curve.

INTRODUCTION TO SPECIAL TYPE DOUBLE SAMPLING PLAN WITH BINOMIAL DISTRIBUTION

Single sampling plans are simple to use. Very often the producer is at a 'psychological' disadvantage if a single sampling plan is applied to the lots, since no second chance is given for the lots not accepted. In such situations, taking a second sample is preferable. The operating procedure of double sampling is given in following schematic diagram.

1.1 OPERATING PROCEDURE OF DOUBLE SAMPLING PLAN



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Rajagopal, *et al.* [6] developed an iterative procedure for finding the parameters of single sampling plan by attributes under Polya distribution satisfying the requirements to ensure protection to the producers and consumers and discussed the discriminating power of the plans through the associated OC curves with beta prior distribution

From the 1.1 figure, we can observe that, in usual double sampling, if there is not a clear accept/reject decision from the inspection of the first sample; a second sample is taken and the decision is made on the results of combined samples in which decision of acceptance can be made even before the inspections of second sample. But in special Type double sampling plan the decision of acceptance is made only after the second sample. Conflicting interest arise between producer and consumer in the use of single sampling plans either with c=1 or c= 0 for the product characteristics involving costly and destructive testing. Single sampling plan with c=0 favors the consumer and with c=1 favors the producer. To overcome the conflicting interests in single sampling and the disadvantages of double sampling Govindaraj [1] has developed a special type double sampling plan. In Special Type double sampling plan the decision of acceptance is made only after the second sample.

Govindaraj and Balamurali [2] studied the special type sampling plan under the assumption that the process fraction defective as a constant.

BAYESIAN SPECIAL TYPE DOUBLE SAMPLING PLAN

Suppose a process in a series of lots supplying is product. Due to random fluctuations, these lots will differ in quality even though the process is stable and in control. These fluctuations can be separated into within lot (sampling) variations of individual units and between lot (sampling and process) variations. Bayesian single sampling attributes plans are analyzed by Hald [3] for continuous prior distribution. The authors briefly reviewed an approach for choosing a prior distribution for a Baye's attributes (good/bad) acceptance sampling plan. A prior distribution is chosen from confidence levels corresponding to classical lower confidence bounds, where a Baye's plan is acceptable, the sample size cane be reduced.

If these two sources of variations are equal implying more process variation, the dispersion of the process about the process average is zero, and each lot can be considered as a random sample drawn from a process with a constant probability of producing a non-conforming unit. This is the premise behind conventional acceptance sampling.

Frequently,' between lot' variation is greater than the 'within lot' variation, indicating that process variation exists and that the probability of obtaining non-conforming unit varies continuously. Proper assessment of the sampling risks required evaluation of this process variation as well as of the sampling variation. Bayesian acceptance sampling considers both source of variation. Thus the distinction between the conventional and Bayesian approach is associated with utilization of prior process history or knowledge in selection of distribution to describe the random fluctuations involved in acceptance sampling. Lauer [4] considered Polya distribution as the sampling distribution of the number of non-conforming units and found that the OC curves of the sampling plans are different from that of the OC curves of non-Bayesian single sampling plan when lot-to-lot variation is presented.

Bayesian Special Type Double sampling has the psychological advantage of giving a second chance to inspect a second lot of items because to some people, especially to the producer, it may seem unfair to reject a lot in the basis of a single sample. Vijayaraghavan and Banumathi [7] proposed a procedure for designing special type double sampling plan by attributes for specified discrimination which ensure protection to the producer and consumer with small acceptance number using Bayesian methodology

BETA-BINOMIAL DISTRIBUTION

We make n independent Bernoulli trials (0 to1 trials) with probability parameter p. It is well known that the number of successes x has the binomial distribution. Considering the Probability parameter p unknown (but of course the sample-size parameter n is known), we have

$$x/p \sim bin\,(n;\,p)$$

This is given by,

$$p(x) = nc_x p^x (1-p)^{n-x}, \qquad n = 0,1,2,3,4 \ldots \ldots \ldots$$

We assume the prior distribution for p is Beta distribution,

i.e. $p \sim \beta(s,t)$ with density function, $w(p) = \frac{1}{\beta(s.t)} \int_0^1 p^{s-1}(1-p)^{t-1}$

The emerged distribution is called beta-binomial distribution and can be written as

$$x \sim betabin(n, s, t)$$

Operating Procedure of Special Type Double Sampling Plan (BSTDSP)

From the given lot select a random sample of size $n_1$and count the number of defectivesd_1. If $d_1 \geq 1$ reject the lot, if $d_1 = 0$ select a second sample of size n_2 and count the number of defectivesd_2. If $d_2 \leq 1$, accept the lot otherwise reject the lot.

OC FUNCTION OF BSPDSP

The OC function of the plan is given by

$$P_a = P_0 P_1$$

The probability of acceptance Bayesian Special Type Double sampling plan based on Binomial distribution is given by

$$P(x, n_2, p) = (1-p)^n + n_2 p(1-p)^{n-1}$$

In Special Type double sampling plan we consider $n = n_1 + n_2$

Based on the past history of inspection, it is observed that product quality $'p'$ follows the Beta distribution

$$w(p)dp = \frac{1}{\beta(s,t)} \int_0^1 p^{s-1} (1-p)^{t-1} dp$$

The emerged Probability of Acceptance is given by

$$\overline{P} = \int_0^1 P(x, n_2, p)\, w(p) dp \text{ can be written in terms of Beta distribution as}$$

$$\overline{P} = \frac{1}{\beta(s,t)} \int_0^1 (1-p)^n + n_2 p(1-p)^{n-1} \;\; p^{s-1}(1-p)^{t-1} dp$$

$$\overline{P} = \frac{1}{\beta(s,t)} \{\beta(s, n+t) + n_2 \beta(s+1, n+t-1)\} \tag{1.1}$$

Where $t = \frac{s - \mu s}{\mu}$ and $\mu = \frac{s}{s+t}$ = Product quality

CONSTRUCTION OF TABLES

For $s = 1$, the expression mentioned in (1.1) can be reduced into

$$\overline{P} = \frac{1-\mu}{n\mu+1-\mu} + \frac{n_2\mu(1-\mu)}{(n\mu+1-\mu)(n\mu+1-2\mu)} \tag{1.2}$$

For s = 2, then

$$\overline{P} = \frac{(2-2\mu)(2-\mu)}{(n\mu+2-\mu)(n\mu+2-2\mu)} + \frac{2n_2\mu(2-2\mu)(2-\mu)}{(n\mu+2-\mu)(n\mu+2-2\mu)(n\mu+2-3\mu)} \tag{1.3}$$

For s = 3,

$$\overline{P} = \frac{(3-3\mu)(3-2\mu)(3-\mu)}{(n\mu+3-\mu)(n\mu+3-2\mu)(n\mu+3-3\mu)} + \frac{3n_2\mu(3-3\mu)(3-2\mu)(3-\mu)}{(n\mu+3-\mu)(n\mu+3-2\mu)((n\mu+3-3\mu)(n\mu+3-4\mu)} \tag{1.4}$$

In general, for $s = r$, the expression mentioned in (4.1.1) can be reduced as

$$\overline{P} = \frac{(r-r\mu)[r-(r-1)\mu][r-(r-2)\mu][r-(r-3)\mu]......(r-\mu)}{[n\mu+r-r\mu][n\mu+r-(r-1)\mu][n\mu+r-(r-2)\mu][n\mu+r-(r-3)\mu]....[n\mu+r-\mu]}$$

$$+ \frac{n_2 r\mu(r-r\mu)[r-(r-1)\mu][r-(r-2)\mu][r-(r-3)\mu]......(r-\mu)}{[n\mu+r-(r+1)\mu][n\mu+r-r\mu][n\mu+r-(r-1)\mu][n\mu+r-(r-2)\mu][n\mu+r-(r-3)\mu]....[n\mu+r-\mu]} \tag{1.5}$$

Illustration: 1.1 From the Table 1.1, it is observed that for specified values $s = 2, n = 100$ n_2=50, μ =0.1, the Probability of Acceptance is given by 0.5911.

Illustration: 1.2 For specified values $s = 4, n = 100$ n_2=50, $\mu = 0.8$ the Probability of Acceptance is given by 0.0256 From the Illustration 1.1 and 1.2, it is observed that the probability acceptance is very lesser for the greater value of s and μ.

ANALYSIS OF OC CURVE

Figure 1.2 displays comparative OC curve for various values of μ with fixed values of n, n_2 and s, and demonstrated that Bayesian Special Type Double sampling plan decreases its probability of acceptance for smaller changes in μ and larger values of s. At the same time, the smaller values of s do not provide protection to the consumer against unsatisfactory quality levels. As the values of'sʼ increases, the corresponding OC curves exhibit the decreasing trend in the acceptance probabilities for each values of μ. It can be observed, further, from the figure the larger value of s provide better protection to the consumer with lesser risk of accepting the lot of unsatisfactory quality. It appears that if'sʼ increases, the OC curve will tend towards the curve corresponding to the largest values of s.

PERFORMANCE MEASURES OF BAYESIAN SPECIAL TYPE DOUBLE SAMPLING PLAN

Suresh and Latha [5] derived the OC function and Performance measures of Bayesian single sampling and chain sampling with Gamma distribution as prior distribution. She also obtained the performance measures of chain sampling and other plan parameters.

If the Probability of Acceptance obtained in the expressions (1.2), (1.3) and (1.4) can be equated to 0.5, the Indifference Quality level of the given plan will be obtained and displayed jn the Table 1.2

In the similar way, the expression mentioned in (1.2), (1.3) and (1.4) are equated to the given probability of acceptance, the AQL and LQL values of this plan can be obtained and displayed in the Table 1.3

Illustration: 1.3 From the Table 1.2, for specified values of s=1, $n = 100, n_2$=50, α=0.05 and β=0.10 the AQL = 0.0011 and LQL =0.1266. The corresponding point of control is given by IQL=0.0162

Illustration: 1.4 It is observed that, for specified values of s=3, $n = 100, n_2$=50, α=0.05 and β=0.10 the AQL =0.0011 and LQL =0.0505 and the point of control is given by IQL=0.0128.

SELECTION BAYESIAN TYPE DOUBLE SAMPLING PLAN FOR GIVEN AQL AND LQL

From Table 1.3, it is observed that for a specified μ_1, and μ_2, the operating ratio is constructed for varying values of s and for fixed n=100 and n_2=50. The operating ratio of μ_1, and μ_2 with beta prior distribution can be used as the measure of discrimination while designing the Bayesian Special Type Double sampling plan.

Illustration: 1.5 From the table 1.3, when s=2, for the given value of $n\mu_1$=0.0011 and $n\mu_2$ =0.0632, the operating ratio is calculated by 57.4545 and the plan parameters corresponding to this ratio is given as $s = 2$ and $n =$100, n_2=50

SELECTION OF PRODUCT QUALITY μ FOR THE GIVEN PROBABILITY OF ACCEPTANCE

The μ values are obtained by equating the expressions mentioned in 1.2 to 1.4 to the required probability of acceptance by using Method of approximation and values are displayed in Table 1.3

Illustration: 1.6 For specified s = 3, n = 100, n_2= 50 and $\overline{P} = .99$ the average product quality μ = 0.0003

For specified s = 4, n = 100, n_2= 50 and $\overline{P} = .10$ the product quality μ = 0.0453.

CONCLUSION

To overcome the conflicting interests in single sampling and the disadvantages of double sampling Special Type double sampling plan has developed. In Special Type Double Sampling plan the decision of acceptance is made only after the second sample. Bayesian sampling plan is a best technique to evaluate quality from the conformance view. This paper is related to Bayesian Special Type Double sampling plan and its various parameters with beta distribution as prior distribution. This plan will be more useful to the quality control practitioners to meet out the consumer requirements.

Table-1.1: Probability of Acceptance for various values s

s / μ	1	2	3	4
0	1.0000	1.0000	1.0000	1.0000
0.1	0.6231	0.5911	0.5788	0.5722
0.2	0.4400	0.3712	0.3421	0.3259
0.3	0.3374	0.2508	0.2140	0.1936
0.4	0.2722	0.1792	0.1410	0.1203
0.5	0.2273	0.1337	0.0970	0.0779
0.6	0.1945	0.1030	0.0691	0.0522
0.7	0.1695	0.0815	0.0507	0.0361
0.8	0.1498	0.0659	0.0382	0.0256
0.9	0.1339	0.0541	0.0293	0.0185
1.0	0	0	0	0

Table-1.2: μ Values of BSTDS for the given Probability of Acceptance with s and i.

$\overline{P}$ / s	0.99	0.95	0.90	0.50	0.10	0.05
1	0.0003	0.0011	0.0021	0.0162	0.1266	0.2430
2	0.0003	0.0011	0.0021	0.0135	0.0632	0.0999
3	0.0003	0.0011	0.0021	0.0128	0.0505	0.0741
4	0.0003	0.0011	0.0021	0.0125	0.0453	0.0639

Table-1.3: Certain parametric values of BSTDS Plan with Binomial Distribution

$\overline{P}$ / s	$n\mu_1$	$n\mu_2$	$n\mu_0$	μ_2 / μ_1
1	0.0011	0.1266	0.0162	115.0909
2	0.0011	0.0632	0.0135	57.4545
3	0.0011	0.0505	0.0128	45.9091
4	0.0011	0.0453	0.0125	41.182

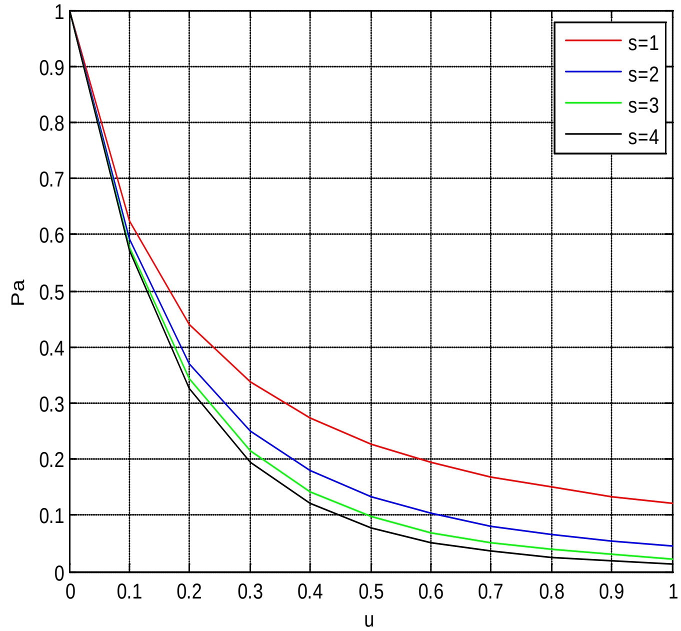

Figure-1.2: Comparative OC Curve for BSTDS with s=1, 2, 3, 4

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