International Journal of Mathematical Archive-8(6), 2017, 102-111
Available online through www.ijma.info ISSN 2229 – 5046

A TYPE OF FUZZY TOPOLOGY ON A GROUP

G. VASANTHAKANNAN*1, K. INDIRANI2

1Assistant Professor,
Department of Mathematics,
RVS college of Arts and Science, Coimbatore, India.

2Associate Professor and Head of Department,
Nirmala College for Women, Coimbatore, India.

Received On: 09-06-17; Revised & Accepted On: 27-06-17)

ABSTRACT

In this paper, we will introduce fuzzy prime topology on a group and also will investigate the concepts of fuzzy contra continuous, fuzzy contra open, fuzzy contra closed, fuzzy supra contra continuous, fuzzy supra contra open, fuzzy supra closed, via fuzzy topology and fuzzy supra topology on a group.

Keywords: *fuzzy topology, fuzzy continuous, fuzzy supra topology, Group theory.*

1. INTRODUCTION

Fuzzy set theory [13] was started in the year 1965 after the publication on the seminar work on fuzzy sets by Lotfi Asker Zadeh's (Professor of electrical engineering at the University of California in Berkeley). The basis of the fuzzification of mathematical concepts are based on the generalization of these concepts from characteristics function to membership function. Fuzzy sets theory is much wider than the classical set theory. Fuzzy topology in such a branch, combing order structure will topological structure. This branch has emerged from the background processing fuzziness and locate theory, proposed from the angle of pure mathematics by Ehresmann.

Change [3], Wang [12], Lowen [9] and others applied some basic concepts from the general topology to fuzzy sets and develop a fuzzy topological spaces. Foster [5] combined the structure of a fuzzy topological spaces with that of fuzzy group (introduced bt Rosenfeld [11]) to formulate the elements of the theory of fuzzy topological groups. Mash hour et aai[10] introduced the concepts os supra topological spaces. In 1987, M.E Abd, E.L – Monsef and A.E Ramadan [1] introduced the fuzzy supra topological spaces. N. Kuroki has studied fuzzy ideas and bi ideas in a semi group [8]. Many classes of semi groups were studied and discussed further by using fuzzy ideas. For example, Kuroki characterized intra-regular semi groups by fuzzy prime ideas.

2. PRELIMINARIES

Definition 2.1 [2]: Let G be a nonempty set. A mapping $\mu: G \rightarrow [0,1]$ is called a fuzzy set of G.

Definition 2.2 [2]: Let G be a nonempty set. A fuzzy topology on G is a family τ of fuzzy sets in G which satisfices the following conditions

1. For any $c \in [0,1]$, the fuzzy set $\mu_c(x) = c \; \forall \; x \in G$,is in τ.
2. If $A, B \in \tau$ then $(A \cap B)$, where $(A \cap B)\,(x) = \min\{A(x), B(x0\}$
3. If $A_j \in \tau$ for $j \in J$ then $\cup_{j\in J} A_j \in \tau$ where $\cup_{j\in J} \; A_j(x) = \sup_{j\in J}\{A_j\,(x)\}$.

Definition 2.3 [2]: Let G and H be two nonempty set, $f : G \rightarrow H$ a mapping and μ a fuzzy sets in$H$. The pre image of μ under f by $f^{-1}(\mu)\;(or\;\mu^f)$ a fuzzy set of G defined by $f^{-1}(\mu)(x) = \mu^f(x) = \mu\left(f(x)\right)$ for all $x \in G$.

Corresponding Author: G. Vasanthakannan*,
1Assistant Professor, Department of Mathematics,
RVS college of Arts and Science, Coimbatore, India.

Definition 2.4 [2]: Let μ be a fuzzy set in a $G$and, $f : G \to H$ a mapping. The mapping, $f\ (\mu): H \to [0,1]$ defined by $f(\mu)(y) = \sup_{x\in f^{-1}(y)}\{\mu\ (x)\}\ if\ f^{-1}(y) \neq \emptyset$ and $f(\mu)$(y) = 0

If $f^{-1}(y) \neq \emptyset$ is called the image of μ under f, where$f^{-1}\ (y) = \{x \in G : f(x) = y\}$.

Definition 2.5 [2]: Let $(G,\tau),(H,\sigma)$ be two fuzzy topological spaces. A mapping f of (G,τ), into (H,σ) is fuzzy continuous if for each open fuzzy set $V \in \sigma$ the inverse image $f^{-1}(V)$ is inτ. If f is said to be fuzzy open if for each $U\ \in \tau. f$ is said to be open if for each $U\ \in \tau, f(U) \in \sigma$.

Definition 2.6 [6]: If $(G,.) and\ (H,.)$ are any two groups then the function $f : G \to H$ is called a group homomorphism if $f\ (xy) = f(x)f(y) for\ all\ x,y\ \in G$.

Definition 2.7 [7]: A fuzzy set (fuzzy sets) μ of a group G is called a fuzzy prime ideal if for all $x,y\ \in G,\ \mu(xy) = \mu(x)\ \ or\ \mu(x,y) =\ \mu(y)$

Definition 2.8 [2]: Let G and H be two nonempty sets, $f : G \to H$ be a mapping, and A be a fuzzy set in G. Then A is said to be f invariant if for each $x,y\ \in G$ such that$f(x) = f(y)$, we have $A(x) = A(y)$.

Definition 2.9 [4]: A function f from a fuzzy topological space (G,τ) to fuzzy topological space (H,σ) is called fuzzy contra continuous if $f^{-1}(\lambda)$ is fuzzy closed in (G,τ) for every fuzzy open set λ of (H,σ).

3. ON FUZZY PRIME TOPOLOGY AND FUZZY TOPOLOGY ON A GROUP

Let $(G,.)$ be a group and τ is a family of fuzzy sets in $(G,.)$ satisfying the conditions in Definition 2.2, then τ is a fuzzy topology on group G and (G,τ) is a fuzzy topological spaces.

Example 3.1: Let $(G,.)$ be a group and let $\tau\!: \{\ \mu\!: G \to [0,1]\!: \mu(xy) =\ \mu(x);\ \forall x,u \in G\}$.

To check whether if the family τ satisfies the conditions of a fuzzy topology. First we have to prove that the fuzzy sets $\mu_c\ \in \tau$ where $\mu_c(x) = c\ for\ c \in [0,1]$.

Now, let $x,y\ \in G$.

Then $\mu_c(xy) = c = \mu_c(x)\ \ then\ \mu_c \in \tau$

Next, let, $\lambda,\mu \in \tau$

$$\begin{aligned} \text{Then}\ \ (\lambda \cap \mu)(xy) &=\ min\{\lambda(xy),\mu(xy)\} \\ &=\ min\{\lambda(x),\mu(x)\} \\ &= (\lambda \bigcap \mu)\ (x) \end{aligned}$$

Then $(\lambda \cap \mu)\ \in \tau$. Finally, let $\mu_i \in \tau, \tau\ i \in I$

$$\begin{aligned} \text{Then}\ \textstyle\bigcup_{i\in I}\mu_i(xy) &=\ \sup_{i\in I}\{\mu_i(xy)\} \\ &= \sup_{i\in I}\{\mu_i(xy)\} \\ &= \bigcup_{i\in I}\mu_i(x). \end{aligned}$$

Thus $\bigcup_{i\in I}\mu_i\ \in \tau$ and so τ is a fuzzy topology on G.

Definition 3.2: The family τin example 3.1 is called the fuzzy prime topology in G.

Theorem 3.3: Let $(G,\tau),(H,\sigma)$ be two fuzzy topological spaces, τ is the fuzzy prime topology in G and $f\!: (G,\tau) \to (H,\sigma)$be an epimorphism of group$G$. If f is fuzzy continuous, then any fuzzy subset η in H such that $\eta \in \sigma$ is a fuzzy prime topology.

Since η is a fuzzy subset of H, we are omitting the regularity method. Now, let $y_1,y_2 \in H.$ since f is onto there exist $x_1,x_2 \in G$ such that $f(x_1) =\ y_1\ and\ \ f(x_1) = f(y_2)$ such that

$$\begin{aligned} \eta(y_1y_2) &=\ \eta((f(y_1))\eta(f(y_2)) \\ &=\ \eta(f(x_1x_2)) \\ &= \eta^f\,(x_1\,x_2) \\ &=\ f^{-1}(\eta)(x_1\,x_2) \end{aligned}$$

Since f is fuzzy continuous and $\eta\ \in \sigma, f^{-1}(\eta)(x_1) =\ \eta(y_1)$.

Lemma 3.4: Let X and Y be two nonempty sets, $f: X \to Y$ mapping and A be an F-invariant fuzzy set in X. Then $f^{-1}\big(f(A)\big) = A$.

Corollary 3.5: Let $\tau\ and\ \sigma$ be the fuzzy prime topology on G and H respectively. If f is epimorphism and every fuzzy set in G is f is fuzzy open.

Proof: Suppose that f is onto homomorphism and every fuzzy set G is f invariant and let $\cup \in \tau$ and $y_{1,} y_2 \in H$, then exist $x_1, x_2 \in G$ such that $f(x_1) = y_1\ and\ f(x_2) = y_2$ so that

$$\begin{aligned} f(U)(y_1 y_2) &= f(U)(f(x_1) f(x_2)) \\ &= f(U)(f(x_1)(x_2)) \\ &= f^{-1}(f(U))(x_1 x_2) \end{aligned}$$

Since U is$f$ invariant, by lemma 3.4. Hence

$$\begin{aligned} f(U)(y_1 y_2) &= U\ (x_1 x_2) \\ &= U(x_1) \\ &= f(U)(f((x_1)) \\ &= f(U)(y_1) \end{aligned}$$

Definition 3.6: Let $(G, .)$ be a group. Let (G, τ) be a fuzzy topological spaces and A fuzzy set in G. Then the induced fuzzy topology on G, denoted by $\tau_A = \{\ A\ \cap S : S \in \tau\}$. the pair (A, τ_A) is called the fuzzy subspaces of (G, τ).

Example 3.7: Consider the set $G = \{\ 1, -1, i, -i\}$ with following caley table (Table).Then $(G, .)$ is a group. Let the fuzzy subset $\mu_i: G \to [0,1], i = 1, 2, 3, 4, 5, 6$ be given by

$$\mu_1(x) = \begin{cases} .7 & i\ fx = 1 \\ .5 & i\, fx =\ -1, i \\ .4 & i\ fx = -i \end{cases}$$

$$\mu_2(x) = \begin{cases} .6 & i\ fx = 1 \\ .3 & i\, fx =\ -1, i \\ .2 & i\ fx = -i \end{cases}$$

$$\mu_3(x) = \begin{cases} .6 & i\ fx = 1 \\ .2 & i\, fx =\ -1, i \\ .1 & i\ fx = -i \end{cases}$$

$$\mu_4(x) = \begin{cases} .0 & i\ fx = 1 \\ .2 & i\, fx =\ -1, i \\ .1 & i\ fx = -i \end{cases}$$

$$\mu_5(x) = \begin{cases} .7 & i\ fx = 1 \\ .4 & i\, fx =\ -1, i \\ .2 & i\ fx = -i \end{cases}$$

$$\mu_6(x) = \begin{cases} .6 & i\ fx = 1 \\ .4 & i\, fx =\ -1, i \\ .1 & i\ fx = -i \end{cases}$$

Then the collection $\tau = \{\ \emptyset\ . G, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5\}$ is a fuzzy topology on G. Hence (G, τ) is a fuzzy topological spaces. Let $A =\ \mu_{6.}$ Then $\tau_A = \{\ \emptyset\ , \mu_2, \mu_3, \mu_4, \mu_5\}$ and $A = (\mu_{6.} \tau_A)$ is a fuzzy topological subspaces of (G, τ)

Definition 3.8: Let $(G, .)$, $(H, .)$ be two groups. A function f from fuzzy topological spaces (G, τ) to fuzzy topological space (H, σ) is called fuzzy contra continuous if $f^{-1}(\eta)$ is fuzzy closed in (G, τ) for every fuzzy open η in (H, σ).

Example 3.9: Consider the set $G = \{1, -1, i, -i\}$ with the following caley table (Table 1).Then $(G, .)$ is a group. Let the fuzzy subsets $\mu_i: G \to [0,1], i = 1,2,3,4,5,6$ be given by

$$\mu_1(x) = \begin{cases} .7 & i\ fx = 1 \\ .5 & i\, fx =\ -1, i \\ .4 & i\ fx = -i \end{cases}$$

$$\mu_2(x) = \begin{cases} .6 & i\ fx = 1 \\ .3 & i\, fx =\ -1, i \\ .2 & i\ fx = -i \end{cases}$$

$$\mu_3(x) = \begin{cases} .6 & i\ fx = 1 \\ .2 & i\, fx =\ -1, i \\ .1 & i\ fx = -i \end{cases}$$

$$\mu_4(x) = \begin{cases} .0 & i\ fx = 1 \\ .2 & i\,fx = \ -1, i \\ .1 & i\ fx = -i \end{cases}$$

$$\mu_5(x) = \begin{cases} .7 & i\ fx = 1 \\ .4 & i\,fx = \ -1, i \\ .2 & i\ fx = -i \end{cases}$$

$$\mu_6(x) = \begin{cases} .6 & i\ fx = 1 \\ .4 & i\,fx = \ -1, i \\ .1 & i\ fx = -i \end{cases}$$

Table-1: Table caption

.	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	1	-1
-i	-i	i	1	-1

Table-2: Table caption

.	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

Then the collection $\tau = \{\emptyset\,.\,G, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5\}$ is a fuzzy topology on G. Hence (G, τ) is a fuzzy topological space. The member of τ are fuzzy open sets in(G, τ).

Now the complement of $\tau = \{\emptyset, \mu_1{}', \mu_2{}', \mu_3{}', \mu_4{}', \mu_5{}'\}$ are fuzzy closed set in (G, τ)., where

$$\mu_1{}'(x) = \begin{cases} .3 & i\ fx = 1 \\ .6 & i\,fx = \ -1, i \\ .5 & i\ fx = -i \end{cases}$$

$$\mu_2{}'(x) = \begin{cases} .4 & i\ fx = 1 \\ .7 & i\,fx = \ -1, i \\ .8 & i\ fx = -i \end{cases}$$

$$\mu_3{}'(x) = \begin{cases} .4 & i\ fx = 1 \\ .8 & i\,fx = \ -1, i \\ .9 & i\ fx = -i \end{cases}$$

$$\mu_4{}'(x) = \begin{cases} .1 & i\ fx = 1 \\ .8 & i\,fx = \ -1, i \\ .9 & i\ fx = -i \end{cases}$$

$$\mu_5{}'(x) = \begin{cases} .3 & i\ fx = 1 \\ .6 & i\,fx = \ -1, i \\ .8 & i\ fx = -i \end{cases}$$

$$\mu_6{}'(x) = \begin{cases} .4 & i\ fx = 1 \\ .6 & i\,fx = \ -1, i \\ .9 & i\ fx = -i \end{cases}$$

Consider the set $H = \{1, \omega, \omega^2\}$ with the following caley table (Table 2)

Let the fuzzy subset $\eta_i : H \ \rightarrow, i = 1,2,3,4$ be given by

$$\eta_1(y) = \begin{cases} .3 & i\,fy = \ 1 \\ .8 & i\,fy = \ \omega \\ .6 & i\,fy = \ \omega^2 \end{cases}$$

$$\eta_2(y) = \begin{cases} .4 & i\,fy = \ 1 \\ .9 & i\,fy = \ \omega \\ .8 & i\,fy = \ \omega^2 \end{cases}$$

$$\eta_3(y) = \begin{cases} .3 & i\,fy = \ 1 \\ .6 & i\,fy = \ \omega \\ .5 & i\,fy = \ \omega^2 \end{cases}$$

$$\eta_4(y) = \begin{cases} .4 & i\,fy = \ 1 \\ .8 & i\,fy = \ \omega \\ .7 & i\,fy = \ \omega^2 \end{cases}$$

Then the collection $\sigma = \{\emptyset, H, \eta_1, \eta_2, \eta_3, \eta_4\}$ is a fuzzy topology on H. Hence (H, σ) is a fuzzy topological space. Let $f: G \to H$ be a function given by$f(1) = 1, f(-1) = \omega^2, f(i) = \omega^2, f(-i) = .$ then $f^{-1}(\eta)(x) = \eta(f(x))$ for all $x \in G$ for any$\eta \in H$.

$f^{-1}(\eta_1)(x) = \mu_5,' \quad f^{-1}(\eta_2)(x) = \mu_3,' \quad f^{-1}(\eta_3)(x) = \mu_1,'$

$f^{-1}(\eta_4)(x) = \mu_2,'$ Hence $f^{-1}(\eta)$ is fuzzy closed in (G, τ) for every fuzzy open set η in (G, τ)

Definition 3.10: Let$(G, .), (H, .)$ be two groups. Let $(G, \tau), (H, \sigma)$ be fuzzy topological spaces. A mapping $f: (G, \tau)$ into (H, σ) is fuzzy contra open iff the mapping of each τ open fuzzy set is σ closed fuzzy set.

Example 3.11: Consider the groups$(G, .), (H, .)$ as in the example 3.9. Let the fuzzy subsets $\mu_i: G \to, [0,1], i = 1, 2,3,4$ be given by

$$\mu_1(x) = \begin{cases} .7 & i\ fx = 1 \\ .5 & i\,fx = -1, i \\ .4 & i\ fx = -i \end{cases}$$

$$\mu_2(x) = \begin{cases} .6 & i\ fx = 1 \\ .3 & i\,fx = -1, i \\ .2 & i\ fx = -i \end{cases}$$

$$\mu_3(x) = \begin{cases} .6 & i\ fx = 1 \\ .2 & i\,fx = -1, i \\ .1 & i\ fx = -i \end{cases}$$

$$\mu_4(x) = \begin{cases} .0 & i\ fx = 1 \\ .2 & i\,fx = -1, i \\ .1 & i\ fx = -i \end{cases}$$

Then the collection $\tau = \{\emptyset, G, \mu_1, \mu_2, \mu_3, \mu_4\}$ is a fuzzy topology on G. Hence (G, τ) is a fuzzy topological space.

Consider the group as in the example 3.9.Let the fuzzy subsets $\eta_i: H \to, [0,1], \; i = 1, 2,3,4$ be given by

$$\eta_1(y) = \begin{cases} .3 & i\,fy = 1 \\ .8 & i\,fy = \omega \\ .6 & i\,fy = \omega^2 \end{cases}$$

$$\eta_2(y) = \begin{cases} .4 & i\,fy = 1 \\ .9 & i\,fy = \omega \\ .8 & i\,fy = \omega^2 \end{cases}$$

$$\eta_3(y) = \begin{cases} .3 & i\,fy = 1 \\ .6 & i\,fy = \omega \\ .5 & i\,fy = \omega^2 \end{cases}$$

$$\eta_4(y) = \begin{cases} .4 & i\,fy = 1 \\ .8 & i\,fy = \omega \\ .7 & i\,fy = \omega^2 \end{cases}$$

$$\eta_5(y) = \begin{cases} .1 & i\,fy = 1 \\ .9 & i\,fy = \omega \\ .8 & i\,fy = \omega^2 \end{cases}$$

Then the collection $\sigma = \{\emptyset, H, \eta_1, \eta_2, \eta_3, \eta_4, \eta_5\}$ is a fuzzy topology on H. Hence (H, σ) is a fuzzy topological space. The member in σ are called as fuzzy open set in (H, σ) Now the complement of $\sigma = \{\emptyset, H, \eta'_1, \eta'_2, \eta'_3, \eta'_4, \eta'_5\}$ are fuzzy closed in (H, σ), where

$$\eta'_1(y) = \begin{cases} .7 & i\,fy = 1 \\ .2 & i\,fy = \omega \\ .4 & i\,fy = \omega^2 \end{cases}$$

$$\eta'_2(y) = \begin{cases} .6 & i\,fy = 1 \\ .1 & i\,fy = \omega \\ .2 & i\,fy = \omega^2 \end{cases}$$

$$\eta'_3(y) = \begin{cases} .7 & i\,fy = 1 \\ .4 & i\,fy = \omega \\ .5 & i\,fy = \omega^2 \end{cases}$$

$$\eta'_4(y) = \begin{cases} .6 & i\,fy = 1 \\ .2 & i\,fy = \omega \\ .3 & i\,fy = \omega^2 \end{cases}$$

$$\eta'_5(y) = \begin{cases} .0 & if\, y = 1 \\ .1 & if\, y = \omega \\ .2 & if\, y = \omega^2 \end{cases}$$

Let $f: G \to H$ be a function given by $f(1) = 1, f(-1) = \omega^2, f(i) = \omega^2\ f(-i) = \omega$. Then
$f^{-1}(1) = 1, f^{-1}(\omega^2) = -1,\ f^{-1}(\omega^2) = i, f^{-1}(\omega) = -i$.

Let $f(\mu_1) = \eta'_3,\ f(\mu_2) = \eta'_4,\ f(\mu_3) = \eta'_2,\ f(\mu_4) = \eta'_5$. Hence the image of each τopen fuzzy set isσ closed fuzzy set.

Definition 3.12: Let$(G,.)$, $(H,.)$ be two groups. Let $(G,\tau), (H,\sigma)$ be fuzzy topological spaces. A mapping $f: (G,\tau)$ into (H,σ) is fuzzy contra closed iff the image of each τ closed fuzzy set is σ open fuzzy set.

Example 3.13: Consider the groups$(G,.)$ as in the example 3.9. Let the fuzzy subsets $\mu_i: G \to, [0,1], i = 1, 2,3,4$ be given by

$$\mu_1(x) = \begin{cases} .7 & if\, x = 1 \\ .5 & if\, x = -1, i \\ .4 & if\, x = -i \end{cases}$$

$$\mu_2(x) = \begin{cases} .6 & if\, x = 1 \\ .3 & if\, x = -1, i \\ .2 & if\, x = -i \end{cases}$$

$$\mu_3(x) = \begin{cases} .6 & if\, x = 1 \\ .2 & if\, x = -1, i \\ .1 & if\, x = -i \end{cases}$$

$$\mu_4(x) = \begin{cases} .0 & if\, x = 1 \\ .2 & if\, x = -1, i \\ .1 & if\, x = -i \end{cases}$$

Then the collection $\tau = \{ \emptyset . G, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5\}$ is a fuzzy topology on G. Hence (G,τ) is a fuzzy topological spaces.

The member of τ are fuzzy open set in (G,τ). Now the complement of $\tau = \{ \emptyset, G, \mu_1', \mu_2', \mu_3', \mu_4'\}$ are fuzzy closed set in(G,τ), where

$$\mu_1'(x) = \begin{cases} .3 & if\, x = 1 \\ .6 & if\, x = -1, i \\ .5 & if\, x = -i \end{cases}$$

$$\mu_2'(x) = \begin{cases} .4 & if\, x = 1 \\ .7 & if\, x = -1, i \\ .8 & if\, x = -i \end{cases}$$

$$\mu_3'(x) = \begin{cases} .4 & if\, x = 1 \\ .8 & if\, x = -1, i \\ .9 & if\, x = -i \end{cases}$$

$$\mu_4'(x) = \begin{cases} .1 & if\, x = 1 \\ .8 & if\, x = -1, i \\ .9 & if\, x = -i \end{cases}$$

Consider the group as in the example 3.9. Let the fuzzy subsets $\eta_i: H \to, [0,1],\ \ i = 1, 2,3,4$ be given by

$$\eta_1(y) = \begin{cases} .3 & if\, y = 1 \\ .8 & if\, y = \omega \\ .6 & if\, y = \omega^2 \end{cases}$$

$$\eta_2(y) = \begin{cases} .4 & if\, y = 1 \\ .9 & if\, y = \omega \\ .8 & if\, y = \omega^2 \end{cases}$$

$$\eta_3(y) = \begin{cases} .3 & if\, y = 1 \\ .6 & if\, y = \omega \\ .5 & if\, y = \omega^2 \end{cases}$$

$$\eta_4(y) = \begin{cases} .4 & if\, y = 1 \\ .8 & if\, y = \omega \\ .7 & if\, y = \omega^2 \end{cases}$$

$$\eta_5(y) = \begin{cases} .1 & if y = 1 \\ .9 & if y = \omega \\ .8 & if y = \omega^2 \end{cases}$$

Then the collection $\sigma = \{ \emptyset, H, \eta_1, \eta_2, \eta_3, \eta_{4,}\eta_5\}$ is a fuzzy topology on H. Hence (H, σ) is a fuzzy topological space. Let $f: G \to H$ be a function given by$f(1) = 1, f(-1) = \omega^2, f(i) = \omega^2, f(-i) = .$ then $f^{-1}(1) = 1, f^{-1}(\omega^2) = -1$, $f^{-1}(\omega^2) = i\,, f^{-1}(\omega) = -i$.

$f(\mu_1{}') = \eta_3, \quad f(\mu_2{}') = \eta_4, f(\mu_3{}') = \eta_2, f(\mu_4{}') = \eta_5.$ Hence the image of each τ closed fuzzy set is σ open fuzzy set.

4. FUZZY SUPRA TOPOLOGICAL SPACE ON A GROUP

Definition 4.1: let $(G,.)$ be a group. A fuzzy supra topology on G is a family of fuzzy sets in G which satisfies the following conditions

1. $\emptyset, G \in \tau$
2. If $A_i \in \tau$ for $i \in I\ then\ \bigcup_{i \in I} A_i \in \tau$ where I is an index set. The pair (G, τ) is called a fuzzy supra topological space and the member of τ are called fuzzy supra open set.

Definition 4.2: Let$(G,.)$, $(H,.)$ be two groups. A function f from fuzzy supra topological space (G, τ) to fuzzy supra topological spaces (H, σ) is called fuzzy supra contra continuous if $f^{-1}(\eta)$ is fuzzy supra closed in (G, τ) foe every fuzzy supra open set η in (H, σ)

Example 4.3: Consider the group$(G,.)$ as in example 3.9. Let the fuzzy subset $\mu_i: G \to, [0,1], i = 1,2,3,4,5,6$ be given by

$$\mu_1(x) = \begin{cases} .6 & i\ fx = 1 \\ .4 & i fx = -1, i \\ .3 & i\ fx = -i \end{cases}$$

$$\mu_2(x) = \begin{cases} .7 & i\ fx = 1 \\ .6 & i fx = -1, i \\ .9 & i\ fx = -i \end{cases}$$

$$\mu_3(x) = \begin{cases} .5 & i\ fx = 1 \\ .4 & i fx = -1, i \\ .6 & i\ fx = -i \end{cases}$$

$$\mu_4(x) = \begin{cases} .3 & i\ fx = 1 \\ .5 & i fx = -1, i \\ .4 & i\ fx = -i \end{cases}$$

$$\mu_5(x) = \begin{cases} .4 & i\ fx = 1 \\ .5 & i fx = -1, i \\ .3 & i\ fx = -i \end{cases}$$

$$\mu_6(x) = \begin{cases} .1 & i\ fx = 1 \\ .2 & i fx = -1, i \\ .3 & i\ fx = -i \end{cases}$$

Then the collection $\tau = \{ \emptyset\ .\ G, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6\}$ is a fuzzy topology on G. Hence (G, τ) is a fuzzy supra topological spaces.

The member of τ are fuzzy supra open set in (G, τ). Now the complement of $\tau = \{ \emptyset, G, \mu_1{}', \mu_2{}', \mu_3{}', \mu_4{}'\}$ are fuzzy closed set in(G, τ), where

$$\mu_1{}'(x) = \begin{cases} .4 & i\ fx = 1 \\ .6 & i fx = -1, i \\ .7 & i\ fx = -i \end{cases}$$

$$\mu_2{}'(x) = \begin{cases} .3 & i\ fx = 1 \\ .4 & i fx = -1, i \\ .1 & i\ fx = -i \end{cases}$$

$$\mu_3{}'(x) = \begin{cases} .5 & i\ fx = 1 \\ .6 & i fx = -1, i \\ .4 & i\ fx = -i \end{cases}$$

$$\mu_4'(x) = \begin{cases} .7 & i\ fx = 1 \\ .5 & i\,fx = -1, i \\ .6 & i\ fx = -i \end{cases}$$

$$\mu_5'(x) = \begin{cases} .6 & i\ fx = 1 \\ .5 & i\,fx = -1, i \\ .7 & i\ fx = -i \end{cases}$$

$$\mu_6'(x) = \begin{cases} .9 & i\ fx = 1 \\ .8 & i\,fx = -1, i \\ .7 & i\ fx = -i \end{cases}$$

Consider the group $(H, .)$ as in example 3.9. Let the fuzzy subsets $\eta_i: H \to, [0,1]$, $i = 1, 2,3,4$ be given by

$$\eta_1(y) = \begin{cases} .4 & i\,fy = 1 \\ .3 & i\,fy = \omega \\ .1 & i\,fy = \omega^2 \end{cases}$$

$$\eta_2(y) = \begin{cases} .6 & i\,fy = 1 \\ .5 & i\,fy = \omega \\ .4 & i\,fy = \omega^2 \end{cases}$$

$$\eta_3(y) = \begin{cases} .5 & i\,fy = 1 \\ .6 & i\,fy = \omega \\ .7 & i\,fy = \omega^2 \end{cases}$$

$$\eta_4(y) = \begin{cases} .8 & i\,fy = 1 \\ .9 & i\,fy = \omega \\ .7 & i\,fy = \omega^2 \end{cases}$$

Then the collection $\sigma = \{\emptyset, H, \eta_1, \eta_2, \eta_3, \eta_{4,}\}$ is a fuzzy supra topology on H. Hence (H, σ) is a fuzzy supra topological space. Let $f: G \to H$ be a function given $f(1) = \omega$, $f(-1) = 1$, $f(i) = 1, f(-i) = \omega^2$. then $f^{-1}(\eta)(x) = \eta\big(f(x)\big) for\ all\ x \in G$ for any fuzzy subset $\eta \in H$.

$f^{-1}(\eta_1) = \mu_2'$, $f^{-1}(\eta_2) = \mu_3'$, $f^{-1}(\eta_3) = \mu_5'$, $f^{-1}(\eta_4) = \mu_6'$. Hence $f^{-1}(\eta)$ is fuzzy supra closed in (G, τ) for every fuzzy supra open set η in (H, σ)

Definition 4.4: Let$(G, .)$, $(H, .)$ be two groups. Let (G, τ), (H, σ) be fuzzy supra topological spaces. A mapping $f: (G, \tau)$ into (H, σ) is fuzzy supra contra open iff the image of each τ open fuzzy supra set is σ closed fuzzy supra set.

Example 4.5: Consider the group$(G, .)$ as in the example 3.9. Let the fuzzy subset $\mu_i: G \to, [0,1], i = 1, 2,3,4,5,6$ be given by

$$\mu_1(x) = \begin{cases} .6 & i\ fx = 1 \\ .4 & i\,fx = -1, i \\ .3 & i\ fx = -i \end{cases}$$

$$\mu_2(x) = \begin{cases} .7 & i\ fx = 1 \\ .6 & i\,fx = -1, i \\ .9 & i\ fx = -i \end{cases}$$

$$\mu_3(x) = \begin{cases} .5 & i\ fx = 1 \\ .4 & i\,fx = -1, i \\ .6 & i\ fx = -i \end{cases}$$

$$\mu_4(x) = \begin{cases} .3 & i\ fx = 1 \\ .5 & i\,fx = -1, i \\ .4 & i\ fx = -i \end{cases}$$

Then the collection $\tau = \{\emptyset\,.\,G, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5\}$ is a fuzzy supra topology on G. Hence (G, τ) is a fuzzy supra topological spaces.

Consider the group$(H, .)$ as in the example 3.9. Let the fuzzy subset $\eta_i: H \to, [0,1], i = 1, 2,3,4,5,$ be given by

$$\eta_1(y) = \begin{cases} .6 & i\,fy = 1 \\ .5 & i\,fy = \omega \\ .4 & i\,fy = \omega^2 \end{cases}$$

$$\eta_2(y) = \begin{cases} .5 & i\,fy = 1 \\ .7 & i\,fy = \omega \\ .6 & i\,fy = \omega^2 \end{cases}$$

$$\eta_3(y) = \begin{cases} .4 & if y = 1 \\ .3 & if y = \omega \\ .1 & if y = \omega^2 \end{cases}$$

$$\eta_4(y) = \begin{cases} .8 & if y = 1 \\ .7 & if y = \omega \\ .9 & if y = \omega^2 \end{cases}$$

$$\eta_5(y) = \begin{cases} .6 & if y = 1 \\ .4 & if y = \omega \\ .7 & if y = \omega^2 \end{cases}$$

Then the collection $\sigma = \{ \emptyset, H, \eta_1, \eta_2, \eta_3, \eta_{4,}\eta_5\}$ is a fuzzy supra topological on H. Hence (H, σ) is a fuzzy supra topological space. The member in σ are called as fuzzy supra open set in (H, σ) Now the complement of $\sigma = \{ \emptyset, H, \eta'_1, \eta'_2, \eta'_3, \eta'_4, \eta'_5\}$ are fuzzy supra closed set in (H, σ), where

$$\eta'_1(y) = \begin{cases} .4 & if y = 1 \\ .5 & if y = \omega \\ .6 & if y = \omega^2 \end{cases}$$

$$\eta'_2(y) = \begin{cases} .5 & if y = 1 \\ .3 & if y = \omega \\ .4 & if y = \omega^2 \end{cases}$$

$$\eta'_3(y) = \begin{cases} .6 & if y = 1 \\ .7 & if y = \omega \\ .9 & if y = \omega^2 \end{cases}$$

$$\eta'_4(y) = \begin{cases} .2 & if y = 1 \\ .3 & if y = \omega \\ .1 & if y = \omega^2 \end{cases}$$

$$\eta'_5(y) = \begin{cases} .4 & if y = 1 \\ .6 & if y = \omega \\ .3 & if y = \omega^2 \end{cases}$$

Consider the group $(H, .)$ as in example 3.9. Let the fuzzy subsets $\eta_i: H \to, [0,1]$, $i = 1, 2,3,4$ be given by

$$\eta_1(y) = \begin{cases} .6 & if y = 1 \\ .8 & if y = \omega \\ .7 & if y = \omega^2 \end{cases}$$

$$\eta_2(y) = \begin{cases} .5 & if y = 1 \\ .7 & if y = \omega \\ .6 & if y = \omega^2 \end{cases}$$

$$\eta_3(y) = \begin{cases} .6 & if y = 1 \\ .4 & if y = \omega \\ .5 & if y = \omega^2 \end{cases}$$

$$\eta_4(y) = \begin{cases} .4 & if y = 1 \\ .3 & if y = \omega \\ .1 & if y = \omega^2 \end{cases}$$

$$\eta_5(y) = \begin{cases} .1 & if y = 1 \\ .2 & if y = \omega \\ .3 & if y = \omega^2 \end{cases}$$

Then the collection $\sigma = \{ \emptyset, H, \eta_1, \eta_2, \eta_3, \eta_{4,}\eta_5\}$ is a fuzzy supra topological on H. Hence (H, σ) is a fuzzy supra topological space.

REFERENCES

1. M.E.Abd,EL-Monesf and A.E.Ramadan, Fuzzy supra topological spaces, Indian Journal of Pure and Applied Mathematics, 18(4), 1987, (322-329).
2. BAO-TING LERNER, On Fuzzy (Right- Topology) semi-groups, Journal of Mathematics Analysis and Applications, 134,1988, (306-311).
3. C.L.Chang, Fuzzy Topological spaces, Journal of Mathematics Analysis and Application, 24, 1968, (182-190).
4. E.Etienne, Erdal, Ekici and Kerre, On Fuzzy contra-continuous, Advanced in Fuzzy Mathematics, 1(1), 2006, (35-44).

5. D.H.FOSTER, Fuzzy Topology Groups, J.Math.Anal.App, 67, 1979, (549-564).
6. K.Jeyaraman and A.Sheik Abdullah, The Homomorphism and Anti Homomorphism of Level Sub groups of Fuzzy sub groups, International Mathematical Forum,5(46), 2010, (2293-2298).
7. O.Kazanci and B.Davvaz, On the Structure of rough prime (primary) ideals and rough fuzzy prime (primary) ideals in commutative rings, Information Sciences, 5(46), 2010, (35-44).
8. N.Kuroki, Fuzzy semi-prime ideals in semi groups, Fuzzy sets and systems, 8, 1982, (71-79).
9. R.Lowen, Fuzzy Topology Spaces and Compactness, J.Math.Anal.App. 56, 1976, (621-633).
10. A.S.Mashour, A.A.Allam and F.S>Mahmound and F.H.Khedr, Supratopological Spaces, Indian Journal of Pure and Applied Mathematics, 14, 1984, (592-610).
11. A.Rosenfield, Fuzzy groups, J. Math. Anal. Appl, 35, 1971 (512-521).
12. C.K Wang Fuzzy topology Product and quotient thermos, J. Math. Anal. Appl, 45, 1984, (512-517).
13. L.A. Zadeh, Fuzzy sets, Inform and control, 8, 1965, (338-353).

Source of support: Nil, Conflict of interest: None Declared.