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SORET EFFECT ON DOUBLE DIFFUSIVE CONVECTION IN A VISCOELASTIC FLUID SATURATED ANISOTROPIC SPARSELY PACKED POROUS LAYER

S. N. GAIKWAD*, ANILKUMAR PUJARI

Department of Mathematics,
Gulbarga University, Kalaburagi - 585 106, Karnataka, India.

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ABSTRACT

The Soret effect on the onset of double-diffusive convection in a viscoelastic fluid saturated anisotropic sparsely packed porous layer is studied using linear stability analyses. Linear theory is based on the normal mode technic.The modified Darcy law for viscoelastic fluid of the Oldroyd type model is employed for momentum equation. The onset criterion for stationary and oscillatory convection is derived analytically. The effect of Soret parameter, anisotropy parameters, Darcy-Prandlt number, and retardation retardation parameters on the stability of the system is investigated.

Keywords: *Double diffusive convection, viscoelastic fluid, sparsely packed porous layer, anisotropy, Soret parameter.*

NOMENCLATURE

a	Wave number
c	Specific heat
d	Height of the porous layer
D_1	Soret coefficient
Da	Darcy number, K_z/d^2
g	Gravitational acceleration, $(0,0,-g)$
i	Unit normal vector in x-direction
j	Unit normal vector in y-direction
K	Permeability tensor, $K_x^{-1}(ii+jj)+K_z^{-1}(kk)$
K	Permeability of the isotropic porous layer $(i.e, K_x = K_z + K)$
k	Unit normal vector in z-direction
Le	Lewis number, κ_T/κ_s
l, m	Horizontal wave number
P	Pressure
$\Pr_D$	Darcy-Prandtl number, $\gamma\varepsilon\nu d^2/\kappa_{TZ}K_z$
q	Velocity vector, (u,v,w)
Ra_T	Thermal Rayleigh number, $\beta_T g\Delta TdK/\nu\kappa_{TZ}$
Ra_s	Solute Rayleigh number, $\beta_s g\Delta SdK/\nu\kappa_{Tz}$
S	Solute concentration
S_r	Soret parameter, $D_1\beta_S/K_{TZ}\beta_T$
ΔS	Salinity difference between the walls
T	Temperature
ΔT	Temperature difference between the walls
t	Time
x, y, z	Space coordinates

GREEK SYMBOLS

β_T	Thermal expansion coefficient
β_S	Solute expansion coefficient
η	Thermal anisotropic parameter, κ_{Tx}/κ_{Tz}
ε	Porosity
γ	Ratio of specific heats, $(\rho c)_m/(\rho c)_f$
Θ	Dimensionless amplitude of temperature perturbation
θ	Dimensionless temperature
κ	Diffusivity
κ_S	Solute diffusivity
κ_T	Thermal diffusivity $\kappa_{Tx}(ii+jj)+\kappa_{Tz}(kk)$
$\overline{\lambda}_1$	Stress-relaxation time
$\overline{\lambda}_2$	Strain-retardation time
λ_1	Relaxation parameter, $(\kappa_T/\gamma d^2)\overline{\lambda}_1$
λ_2	Retardation parameter, $(\kappa_T/\gamma d^2)\overline{\lambda}_2$
μ	Dynamic viscosity
μ_e	Effective viscosity
ν	Kinematic viscosity, μ/ρ_0
ξ	Mechanical anisotropy parameter, K_x/K_Z
	Fluid density
σ	Growth rate

Corresponding Author: S. N. Gaikwad*,
Department of Mathematics,
Gulbarga University, Kalaburagi 585 106,
Karnataka, India.

ϕ	Normalized porosity, ε / γ
Φ	Dimensionless amplitude of concentration perturbation
ψ	Stream function

OTHER SYMBOLS

D	$\frac{d}{dz}$
∇_h^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$
∇^2	$\nabla_h^2+\frac{\partial^2}{\partial z^2}$
δ^2	π^2+a^2
δ_1^2	$\pi^2\xi^{-1}+a^2$
δ_2^2	$\pi^2+\eta a^2$

SUBSCRIPTS AND SUPERSCRIPTS

b	Basic state
c	Critical
f	Fluid phase
i	Imaginary wall value
l,u	Lower/upper wall value
o	Reference
r	Real part
s	Solid
m	Porous medium
*	Dimensionless quantity
'	Perturbed quantity
F	Finite amplitude
Osc	Oscillatory
St	Stationary

1. INTRODUCTION

In this paper, a study of the Soret effect on double diffusive convection in an anisotropic sparsely packed porous layer,saturated with viscoelastic fluid, heated from below and cooled from above is undertaken. The flow of viscoelastic fluid is of considerable importance due to its copious applications in different fields such as geothermal energy utilization, oil reservoir modeling, materials processing, petroleum, chemical and nuclear industries, bioengineering, building thermal insulation and nuclear waste disposals to mention a few. The nature of convective motion in a thin horizontal layer of viscoelastic fluid which is heated from below, in the classical Rayleigh-Benard convection geometry, has been the subject of discussion in the literature for nearly four decades, this was reviewed by Vest and Arpaci (1969), Sokolov and Tanner (1972), Rosenblat (1986),.Convection in a viscoelastic fluid saturated sparsely packed porous layer by Rudraiah *et al.* (1990).Martinez-Mardones and Perez-Garcia (1990,1992). Larson (1992), Khayat (1995). Viscoelastic fluids exhibit unique patterns of instabilities such as over stability that is not predicated or observed in Newtonian fluid. With the growing importance of non-Newtonian fluids in modern technology and also due to their natural occurrence, the investigations on such fluids are quite desirable. Although the problem of Rayleigh-Benard Convection (RBC) has been extensively investigated for Newtonian fluids, relatively little attention has been denoted to the thermal convection of viscoelastic fluids by Li and Khayat (2005) and references therein. The study of RBC in viscoelastic fluid may be important from a rheological point of view because the observation of the onset of convection provides potentially useful techniques to investigate the suitability of a constitutive model adopted for certain viscoelastic fluids.

Flow instability and turbulence are far less widespread in viscoelastic fluids than in Newtonian fluids because of the high viscosity of the polymeric fluids. It has long been a common belief that oscillatory convection is not possible in viscoelastic fluids in realistic experimental condition shown by Larson (1992). However, a series of experiments performed by Perkins et al. (1994) showed that dilute suspensions of long DNA molecules, in a buffer solution, behave as viscoelastic fluids, and also presented an oscillatory instability as first convective instability in that case. This possibility has been confirmed by Kolodner (1998) in his experiments on the elastic behavior of individual long strands of DNA in buffer solutions. This has pointed out the way towards obtaining a fluid in which oscillatory viscoelastic convection might be observed. He observed oscillatory convection in DNA suspensions in annular geometry. The onset of double diffusive convection in a viscoelastic fluid layer. Shown by Malashetty and Swamy (2010**)** .Therefore, in the present study, we intend to perform linear stability analyses of the onset of Soret effect on double diffusive convection in a viscoelastic fluid saturated anisotropic sparsely packed porous layer. Our objective is to study how the onset criterion for oscillatory convection is affected by the viscoelastic Darcy-Prandtl number, Lewis number, normalized porosity and other parameters. In the limiting cases, some previously published results can be recovered as the particular cases of our results.

2. MATHEMATICAL FORMULATION

Consider an infinite horizontal sparsely packed, viscoelastic fluid saturated anisotropic porous layer with fluid and confined between the plates $z=0$ and $z=d$, with vertically downward gravity force **g** acting on it. A uniform adverse temperature gradient $\Delta T = T_1 - T_u$ and a stabilizing concentration gradient $\Delta S = S_i - S_u$ $(T_1 > T_u$ and $S_1 > S_u)$ are maintained between the lower and upper surfaces. A Cartesian frame of reference is chosen with origin at the lower boundary and z-axis vertically upwards. The modified Darcy-Brinkman-Oldroyd model is employed for the momentum equation studied by Zhang *et al.* [19]. We however assume that Soret effect is weak and hence assume moderate values for the Soret coefficient. The transport of heat and solute is employed for the momentum equation by Philips [20].The governing equations under Boussinesq approximation are

$$\nabla.q = 0 \tag{1}$$

$$\left(1+\overline{\lambda}_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\partial t}+\nabla p-\rho g\right)=\left(1+\overline{\lambda}_2\frac{\partial}{\partial t}\right)\left(\mu_e\nabla^2 q-\mu\mathrm{K}.q\right) \tag{2}$$

$$\gamma\frac{\partial T}{\partial t}+(q.\nabla)T=\nabla.(\kappa_T.\nabla T) \tag{3}$$

$$\varepsilon\frac{\partial S}{\partial t}+(q.\nabla)S=(\kappa_s.\nabla^2 S)+D_1\nabla^2 T \tag{4}$$

$$\rho=\rho_0[1-\beta_T(T-T_0)+\beta_S(S-S_0)]. \tag{5}$$

where $q=(u,v,w)$ the velocity, p is pressure, $\overline{\lambda_1},\overline{\lambda_2}$ are the relaxation and retardation times respectively, g is the acceleration due to gravity, μ is the viscosity, ρ is the density, T and S are the temperature and concentration respectively and ε is the porosity of the porous medium. $\kappa_T=\kappa_{Tx}ii+\kappa_{Ty}jj+\kappa_{Tz}kk$ is the inverse of the permeability tensor and $\kappa_T=\kappa_{Tx}ii+\kappa_{Ty}+\kappa_{Tz}kk$ is the thermal diffusivity tensor, and D_1 is the Soret coefficient. $\gamma=(\rho c)_m/(\rho c_p)_f$, $(\rho c)_m=(1-\varepsilon)(\rho c_p)_f$, C_p is the specific heat of the fluid at constant pressure**,** **C** is the specific heat of the solid, the subscripts f,s and m denote fluid, solid and porous medium values respectively, and $\beta_T,\beta_S,\mu_f,\mu_e$ and κ_S are the thermal and solute expansion coefficients, fluid viscosity, effective viscosity, and solute diffusivity respectively. It is hereby stated that permeability is most strongly anisotropic than solute diffusivity. Therefore, we ignore the solute anisotropy.

It is assumed that the viscoelastic liquid has relaxed enough time; typically 1 s is enough for dilute polymeric suspensions. Thus the basic state is assumed to be quiescent and is given by

$$\mathbf{q}_b=(0,0,0),\ p=p_b(z),\ T=T_b(z),\ S=S_b(z),\ \rho=\rho_b(z)\,, \tag{6}$$

which satisfy the following equations

$$\frac{dp_b}{dz}=-\rho_b g,\quad \frac{d^2T_b}{dz^2}=0,\quad \frac{d^2S_b}{dz^2}=0,\ \rho_b=\rho_0\left(1-\alpha\left(T_b-T_0\right)+\beta\left(S_b-S_0\right)\right), \tag{7}$$

with boundary conditions

$$T=T_0+\Delta T \quad \text{and} \quad S=S_0+\Delta S \quad \text{at} \quad z=0, \tag{8}$$

$$T=T_0 \quad \text{and} \quad S=S_0 \quad \text{at} \quad z=d\,. \tag{9}$$

The steady state solutions are given as

$$T_b=T_0+\Delta T\left(1-\frac{z}{d}\right),\qquad S_b=S_0+\Delta S\left(1-\frac{z}{d}\right) \tag{10}$$

with $\Delta T=(T_l-T_u)$ and $\Delta S=(S_l-S_u)$, where $T_l>T_u$ and $S_l>S_u$.

From the reference motionless solution we will study the stability of the system. Let the basic state be perturbed by an infinitesimal perturbation, so that

$$q=q',\ T=T_b+T',S=S_b+S',\ P=p_b+p',\rho=\rho_b+\rho', \tag{11}$$

where the prime indicates that the quantities are infinitesimal perturbations. Substituting Eq. (11) into Eqs. (1)- (5) and using the basic state solutions, we obtaine the equations in the perturbations form

$$\nabla.q'=0 \tag{12}$$

$$\left(1+\overline{\lambda}_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial q'}{\partial t}+\nabla p'-\rho' g\right)=\left(1+\lambda_2'\frac{\partial}{\partial t}\right)\left(\mu_e\nabla^2 q'-\mu\mathrm{K}.q'\right) \tag{13}$$

$$\gamma\frac{\partial T'}{\partial t}+\left(q'.\nabla\right)T'-w\frac{\Lambda T}{d}=\nabla.\left(\kappa_T.\nabla T\right) \tag{14}$$

$$\varepsilon\frac{\partial S'}{\partial t}+\left(q'.\nabla\right)S'-w\frac{\Delta S}{d}=\left(\kappa_S.\nabla^2 S'\right)+D_1\nabla^2 T' \tag{15}$$

$$\rho'=-\rho_0(\beta_T T'+\beta_S S'). \tag{16}$$

By operating curl twice on equation (13) we eliminate p' from it and then render the resulting equation and Eqs. (14) and (15) dimensionless using the following transformations:

$$\left(x,y,z\right)=d\left(x^*,y^*,z^*\right),t=\left(\frac{\gamma d^2}{\kappa_{Tz}}\right)t^*,\left(u',v',w'\right)=\frac{\kappa_{Tz}}{d}\left(u^*,v^*,w^*\right),T'=\left(\Delta T\right)T^*,S'=\left(\Delta S\right)S^* \tag{17}$$

To obtain non-dimensional equations as (after dropping the asterisks for simplicity)

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{1}{\gamma\,\mathrm{Pr}_D}\frac{\partial}{\partial t}\nabla^2 w - Ra_T\nabla_h^2 T + Ra_S\nabla_h^2 S\right) = \left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(-\nabla_h^2 - \frac{1}{\xi}\frac{\partial^2}{\partial z^2} + Da\nabla^4\right)w, \tag{18}$$

$$\left(\frac{\partial}{\partial t} - \left(\eta\nabla_h^2 + \frac{\partial^2}{\partial z^2}\right)\right)T + \left(\vec{q}.\nabla\right)T - w = 0, \tag{19}$$

$$\left(\phi\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)S + \left(\vec{q}.\nabla\right)S - S_r\nabla^2 T - w = 0, \tag{20}$$

where $Ra_T = \beta_T g\Delta T d\mathrm{K}/\upsilon\kappa_{Tz}$ the thermal Rayleigh number, $Ra_S = \beta_S g\Delta S d\mathrm{K}_Z/\upsilon\kappa_{Tz}$ the solute Rayleigh number, $\lambda_1 = \left(\kappa_{Tz}/d^2\right)\overline{\lambda_2}$ retardation parameter, $S_r = D_1\beta_s/\kappa_{TZ}\beta_T$ the Soret parameter, $Da = \mu_e k_z/\mu_f d^2$ the Darcy number, $\mathrm{Pr}_D = \gamma\varepsilon\nu d^2/\mathrm{K}_z\kappa_{Tz}$ the Darcy-Prandtl number, $Le = \kappa_{Tx}/\kappa_{Tz}$ the Lewis number, $\xi = k_x/k_z$ the mechanical anisotropic parameter, $\eta = \kappa_{Tx}/\kappa Tz$ is the thermal anisotropic parameter, $\phi = \varepsilon/\gamma$ normalized porosity. Eqs. (18)-(20) are solved for stress free, isothermal and isosolutal and boundary conditions. Hence the boundary conditions for the perturbation variables are given by

$$w = T = S = 0, \text{ at } z = 0,1 \tag{21}$$

3. LINEAR STABILITY ANALYSIS

In this section we predict the thresholds of both stationary and oscillatory convection using linear theory. The Eigen value problem defined by Eqs. (18)-(20) subject to the boundary conditions (21) is solved using the time–dependent periodic disturbances in a horizontal plane, upon assuming that amplitudes are small enough and can be expressed as

$$\begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} = \begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix}\exp\left[i\left(lx+my\right)+\sigma t\right], \tag{22}$$

where l and m are the wave numbers in the horizontal plane and σ is the growth rate, Infinitesimal perturbations of the rest state may either damp or grow depending on the value of the parameter σ. Substituting Eq. (22) into Eqs. (18) - (20) we obtain

$$\left(1+\lambda_1\sigma\right)\left[\frac{\sigma}{\mathrm{Pr}_D}\left(D^2-a^2\right)W + a^2Ra_T\Theta - a^2Ra_S\Phi\right] = \left(1+\lambda_2\sigma\right)\left[Da\left(D^2-a^2\right)^2 + \left(a^2 - \frac{1}{\xi}D^2\right)\right]W, \tag{23}$$

$$\left[\sigma - \left(D^2 - a^2\eta\right)\right]\Theta - W = 0, \tag{24}$$

$$\left[\phi\sigma - \frac{1}{Le}\left(D^2-a^2\right)\right]\Phi - S_r\left(D^2-a^2\right) - W = 0, \tag{25}$$

where $D = d/dz$ and $a^2 = l^2 + m^2$. In case of stress-free boundary conditions, it possible to solve analytically the system of Eqs.(23) – (25) . This is a standard Eigen value-Eigen function problem. Here the Rayleigh number is taken as the Eigen value and it is expressed as a function of the other parameters which govern the stability of the system. The solution of Eqs. (23) – (25) satisfying the boundary conditions (21) are assumed in the form

$$\begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} = \begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix}\sin n\pi z, \left(n = 1,2,3......\right). \tag{26}$$

The most unstable mode corresponding to $n=1$ (fundamental mode). Therefore, by substituting Eqs. (26) with $n=1$ into Eqs. (23) – (25).we obtain a matrix equation of the form

$$\begin{pmatrix} M_{11} & -a^2Ra_T & a^2Ra_S \\ -1 & \sigma+\delta_2^2 & 0 \\ -1 & S_r\delta^2 & \phi\sigma + \frac{1}{Le}\delta^2 \end{pmatrix}\begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{27}$$

where $\delta^2 = \pi^2 + a^2$, $\delta_1^2 = \pi^2\xi^{-1} + a^2$ and $\delta_2^2 = \pi^2 + \eta a^2$.

The condition of non-trivial solutions of above system of homogeneous linear equations (27) yields the expression for the thermal Rayleigh number in the form

$$Ra_T = \left[\frac{\sigma\delta^2}{\mathrm{Pr}_D} + \left(\frac{1+\lambda_2\sigma}{1+\lambda_1\sigma}\right)\left(\delta_1^2 + Da\delta^4\right)\right]\left(\frac{\sigma-\delta_2^2}{a^2}\right) + Ra_S\left(\frac{\sigma+\delta_2^2 - S_r\delta^2}{\phi\sigma + Le^{-1}\delta^2}\right) \tag{28}$$

4. STATIONARY CONVECTION

For validity of the principal of exchange of stabilities (i.e., steady case), we have $\sigma = 0 \ \left(i.e., \sigma_r = \sigma_i = 0\right)$ at the margin of stability, then Eq. (28) gives the Rayleigh number at which marginally stable steady mode exists as

$$Ra_T = \left(\pi^2 + \eta a^2\right)\left(\frac{1}{a^2}\left[\left(\frac{\pi^2}{\xi} + a^2\right) + Da\left(\pi^2 + a^2\right)^2\right]\right) + \frac{LeRa_S\left(\delta_2^2 - S_r\delta^2\right)}{\delta^2}. \tag{29}$$

The minimum value of the Rayleigh number Ra_T^{st} occurs at the critical wave number $a = a_C^{st}$ where $a_C^{st} = \sqrt{x}$ satisfies a polynomial equation of degree four in x as

$$c_0 x^4 + c_1 x^3 + c_2 x^2 + c_3 x + c_4 = 0 \tag{30}$$

where

$$c_0 = Da\pi^2 + \eta + 6Da\pi^2\eta,$$

$$c_1 = 2Da\pi^4 + 2\pi^2\eta + 6Da\pi^2\eta,$$

$$c_2 = -Le\pi^2 Ra_S + \pi^4\eta + 2Da\pi^6\eta + Le\pi^2 Ra_S\eta - LeRa_S S_r - \frac{\pi^4}{\xi},$$

$$c_3 = -2Da\pi^8 - \frac{2\pi^6}{\xi},$$

$$c_4 = -Da\pi^{10} + 2Dax^5\eta - \frac{\pi^8}{\xi}.$$

In the absence of the Darcy number, i.e., $Da = 0$ equation (29) implies

$$Ra_T{}^{st} = \left(\pi^2 + \eta a^2\right)\left(\frac{1}{a^2}\left(\frac{\pi^2}{\xi} + a^2\right)\right) + \frac{Le\,Ra_S\left(\left(\pi^2 + \eta a^2\right) - S_r\left(\pi^2 + a^2\right)\right)}{\left(\pi^2 + a^2\right)}, \tag{31}$$

Equation (31) exactly coincides with the result of Malashetty *et.al* [21].

For an isotropic porous media, that is when $\xi = \eta = 1$, equation (29) gives

$$Ra_T{}^{st} = \frac{\left(\pi^2 + a^2\right)^2}{a^2}\left[1 + Da\left(\pi^2 + a^2\right)\right] + \frac{Le\ Ra_S(1 - S_r)}{\left(\pi^2 + a^2\right)}, \tag{32}$$

This is exactly the one given by Poulikakos [15] for double diffusive convection in a horizontal sparsely packed porous layer. For a single component fluid $Ra_S = 0$, the expression for stationary Rayleigh number given by equation (31) becomes

$$Ra_T{}^{st} = \frac{1}{a^2}\left(a^2 + \frac{\pi^2}{\xi}\right)\left(\pi^2 + \eta a^2\right), \tag{33}$$

which is the one obtained by Storesletten [19] for the case of a single component fluid. Further for an isotropic porous medium $\xi = \eta = 1$ the above equations (33) reduces to the classical results

$$Ra_T{}^{st} = \frac{1}{a^2}\left(a^2 + \pi^2\right)^2, \tag{34}$$

which has the critical value $Ra_C^{st} = 4\pi^2$ for $a_C^{st} = \pi^2$ obtained by Horton and Rogers [11] and Lapwood [10].

5. OSCILLATORY CONVECTION

We now set $\sigma = i\sigma$ in equation (28) and clear the complex quantities from the denominator to obtain

$$Ra_T = \Delta_1 + i\sigma_i\Delta_2 \tag{35}$$

where

$$\Delta_1 = \frac{\delta_1^2 + Da\,\delta^4}{a^2}\left(\frac{\delta_2^2\left(1+\lambda_1\lambda_2\sigma^2\right)}{1+\lambda_1^2\sigma^2} - \frac{\sigma^2\left(\lambda_2-\lambda_1\right)}{1+\lambda_1^2\sigma^2}\right) - \frac{\sigma^2\delta^2}{a^2\,\mathrm{Pr}_D} + Ra_s\left(\frac{\sigma^2\phi + Le^{-1}\delta^2\delta_2^2 - Le^{-1}S_r\delta^2}{\left(Le^{-1}\delta^2\right)^2 + \phi^2\sigma^2}\right), \quad (36)$$

$$\Delta_2 = \frac{\delta_1^2 + Da\,\delta^4}{a^2}\left(\frac{1+\lambda_1\lambda_2\sigma^2}{1+\lambda_1^2\sigma^2} + \frac{\delta_2^2\left(\lambda_2-\lambda_1\right)}{1+\lambda_1^2\sigma^2}\right) + \frac{\delta_2^2\delta^2}{a^2\,\mathrm{Pr}_D} + Ra_s\left(\frac{Le^{-1}\delta^2 - \phi\delta_2^2 + \phi S_r\delta^2}{\left(Le^{-1}\delta^2\right)^2 + \phi^2\sigma^2}\right). \quad (37)$$

Since Ra_T is a physical quantity, it must be real. Hence, from equation (35) it follows that either $\sigma_i = 0$ (steady onset) or $\Delta_2 = 0\ (\sigma_i \neq 0)$ (oscillatory onset). For oscillatory onset $\Delta_2 = 0\ (\sigma_i \neq 0)$ and this gives an expression for frequency of oscillations in the form (on dropping the subscript i)

$$a_0\left(\sigma^2\right)^2 + a_1\left(\sigma^2\right) + a_2 = 0 \quad (38)$$

Now equation (35) with $\Delta_2 = 0$ gives,

$$Ra_T^{osc} = \frac{\delta_1^2 + Da\,\delta^4}{a^2}\left(\frac{\delta_2^2\left(1+\lambda_1\lambda_2\sigma^2\right)}{1+\lambda_1^2\sigma^2} - \frac{\sigma^2\left(\lambda_2-\lambda_1\right)}{1+\lambda_1^2\sigma^2}\right) - \frac{\sigma^2\delta_2}{a^2\,\mathrm{Pr}_D} + Ra_s\left(\frac{\sigma^2\phi + Le^{-1}\delta^2(\delta_2^2 - S_r)}{\left(Le^{-1}\delta_2\right)^2 + \phi^2\sigma^2}\right), \quad (39)$$

we find the oscillatory neutral solutions from equation (39). It proceeds as follows: First determine the number of positive solutions of equation (38). If there are none, then no oscillatory instability is possible. If there are two, then the minimum (over a^2) of equation (39) with σ^2 given by equation (38) gives the oscillatory neutral Rayleigh number. Since equation (38) is quadratic in σ^2 , it can give rise to more than one positive value of σ^2 for fixed values of the parameters Ra_S, Da, λ_1, λ_2, Le, ϕ, Pr_D, ξ and η . However, our numerical solution of equation (38) for the range of parameters considered here gives only one positive value of σ^2 indicating that there exists only one oscillatory neutral solution. The analytical expression for oscillatory Rayleigh number given by equation (39) is minimized with respect to the wavenumber numerically, after substituting for $\sigma^2 (> 0)$ from equation (38), for various values of physical parameters in order to know their effects on the onset of oscillatory convection.

6. RESULTS AND DISCUSSION

The Soret effect on double diffusive convection in a viscoelastic fluid sparsely packed saturated anisotropic porous layer, which is heated and salted from below, is investigated analytically using the linear stability theory. In the linear stability theory the expressions for the stationary and oscillatory Rayleigh number are obtained analytically along with expression for frequency of oscillation. The variation of the critical oscillatory Rayleigh number with strain retardation parameter λ_2 for different parameters is shown in figures 1 - 8. Figure 1 displays the effect of relaxation parameter λ_1 on the critical Rayleigh number. We observe that an increase of relaxation parameter λ_1 , decreases the critical Rayleigh number indicating that relaxation parameter λ_1 destabilizes the system. Figure 2 shows the effect of the mechanical anisotropy parameter ξ on the critical Rayleigh number. We find that an increase of anisotropy parameter decreases the critical Rayleigh number indicating that the mechanical anisotropic parameter destabilizes the system. Figure 3 indicates the variation of the critical Rayleigh number with strain retardation parameter λ_2 for different values of thermal anisotropy parameter η . It is observed that an increase in the thermal anisotropy parameter η decreases the critical Rayleigh number. Thus the effect of thermal anisotropy parameter η is to advance the onset of convection. In Figure 4 it is observed that the critical Rayleigh number decreases with an increase in the Lewis number Le . Thus the effect of Lewis number Le is to advance the onset of convection. Figure 5 shows the effect of the solute Rayleigh number on the critical Rayleigh number; we find that an increase in the solute Rayleigh number Ra_S increases the critical Rayleigh number. Thus the effect of increasing solute Rayleigh number Ra_S is to stabilize the system. Figure 6 shows the effect of Darcy-Prandtl number Pr_D on the critical Rayleigh number for fixed values of the other parameters. We find that an increase in the value of Darcy-Prandtl number Pr_D decreases the region of stability indicating that the effect of Darcy-Prandtl number Pr_D is to destabilize the system. Figure 7 indicates the variation of the critical Rayleigh number with strain retardation parameter λ_2 for different values of the Darcy number Da . It is observed that the critical Rayleigh number Ra_{Tc} increases with an increase of Da indicating that the effect of Darcy number is to inhibit the onset of convection. From figure 8 we observe that the critical oscillatory Rayleigh number Ra_{Tc} decreases with an increase of normalized porosity ϕ. Thus the effect of ϕ is to destabilize the system. Figure 9 depicts the effect of Soret parameter Sr on oscillatory Rayleigh number with Ra_{Tc}. From this figure we observe that an increase in the values (positive) of the Soret parameter decreases the oscillatory Rayleigh number indicating that Soret parameter destabilizes the system for oscillatory mode. On the other hand, for increasing in the negative Soret parameter decreases the oscillatory Rayleigh number indicating that the effect of negative Soret parameter destabilizes the system for oscillatory convection.

7. CONCLUSIONS

The Soret effect on the onset of double diffusive convection in a viscoelastic fluid saturated anisotropic sparsely packed porous layer is investigated analytically using the linear stability theory. The usual normal mode technique is used to solve the linear problem. The following conclusions are drawn:

1. The Soret parameter stabilizing effect on oscillatory convection and destabilizing effect on stationary convection.
2. In the neutral stability curves, the effect of strain retardation parameter, thermal anisotropy parameter Lewis number, solute Rayleigh number and Darcy number stabilize the system whereas, stress relaxation parameter, mechanical anisotropy parameter, Darcy-Prandtl number and normalized porosity destabilize the system.
3. And for critical curves, the effect solute Rayleigh number and Darcy number stabilize the system whereas, stress relaxation parameter, mechanical anisotropy parameter, Lewis number, Darcy-Prandtl number and normalized porosity destabilize the system.

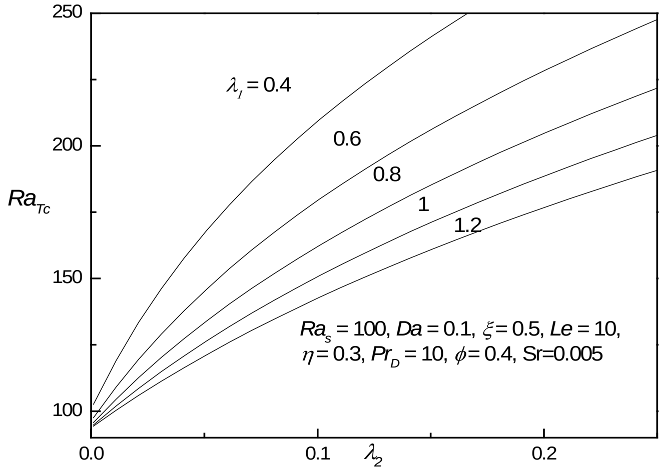

Fig.1. Variation of critical Rayleigh number with strain retardation parameter λ_2 for different values of stress relaxation parameter λ_1.

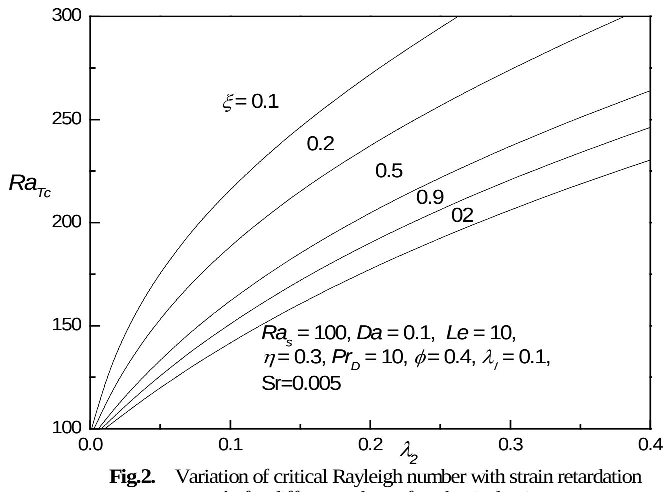

Fig.2. Variation of critical Rayleigh number with strain retardation parameter λ_2 for different values ofmechanical anisotropy parameter ξ.

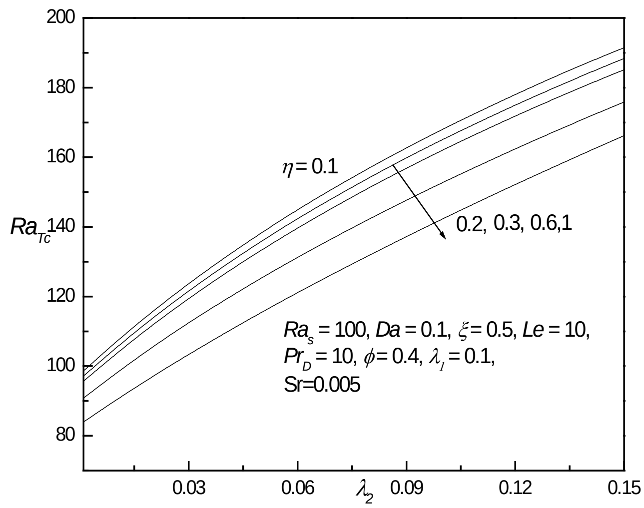

Fig.3. Variation of critical Rayleigh number with strain retardation parameter λ_2 for different values of thermal anisotopy η.

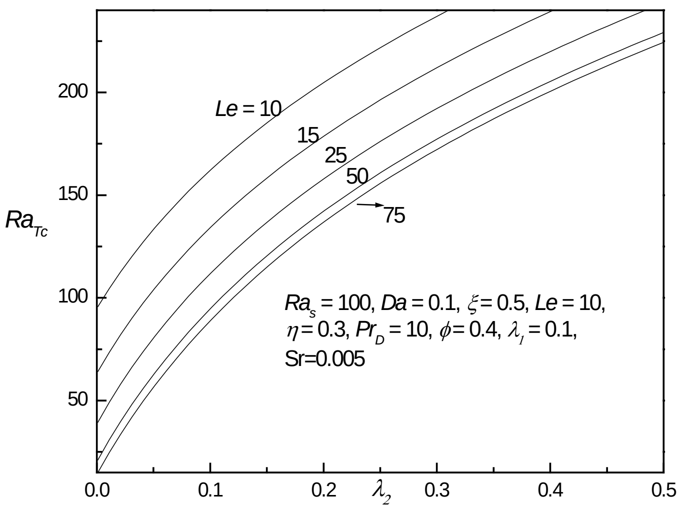

Fig.4 Variation of critical Rayleigh number with strain retardation parameter λ_2 for different values of Lewis number Le.

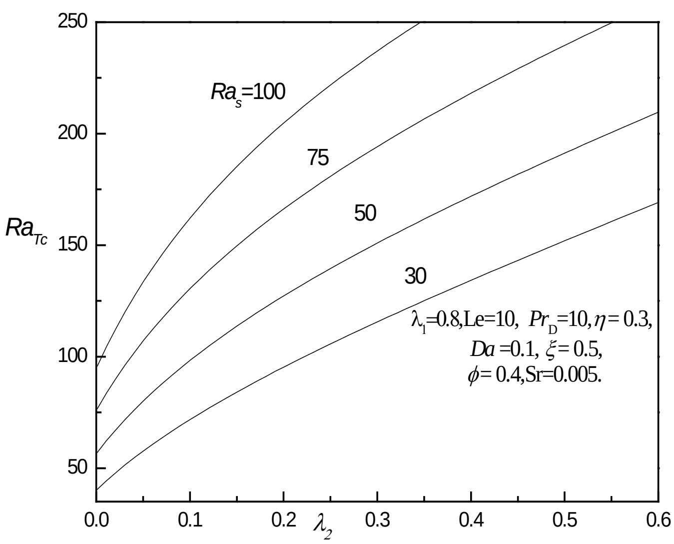

Fig.5 Variation of critical Rayleigh number with strain retardation parameter λ_2 for different values of Solute Rayleigh number Ra_s.

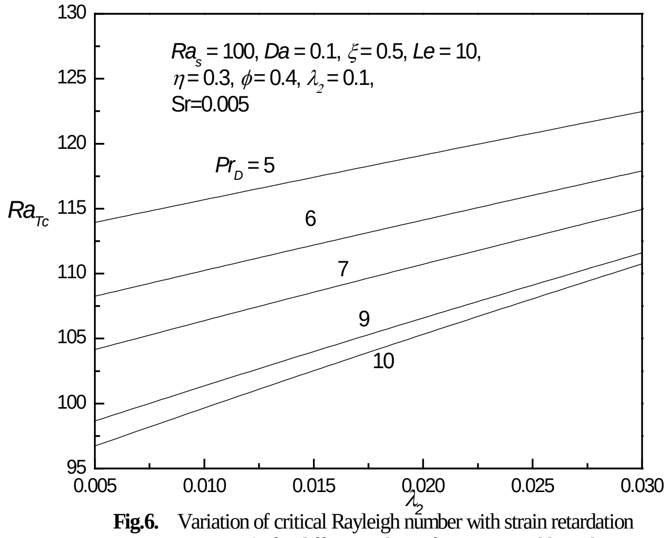

Fig.6. Variation of critical Rayleigh number with strain retardation parameter λ_2 for different values of Darcy- Prandtl number Pr_D.

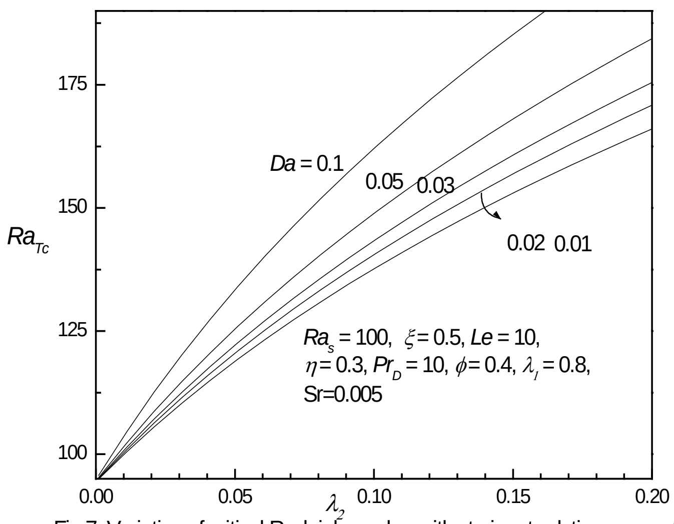

Fig.7. Variation of critical Rayleigh number with strain retardation parameter λ_2 for different values of Darcy number.

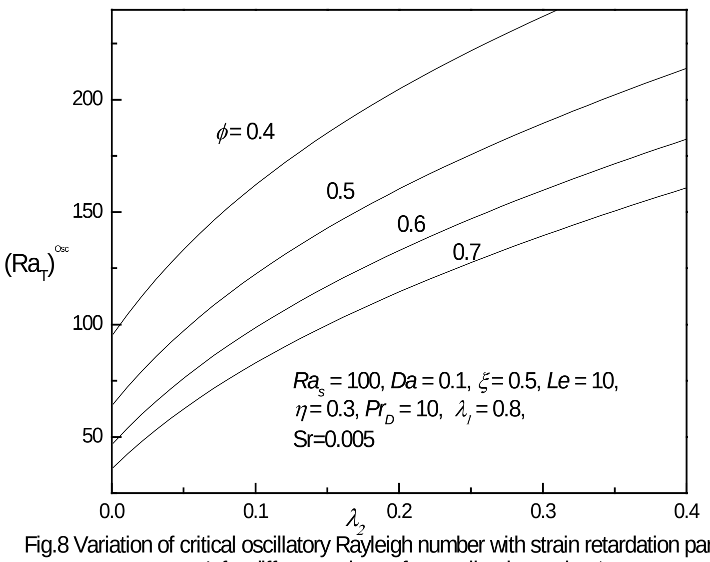

Fig.8 Variation of critical oscillatory Rayleigh number with strain retardation parameter λ_2 for different values of normalised porosity ϕ.

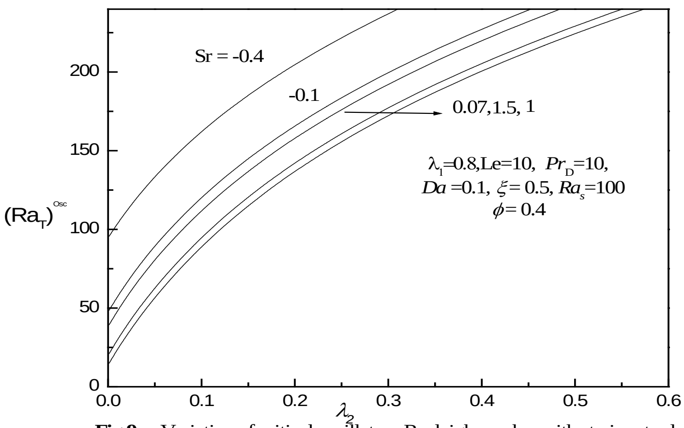

Fig.9. Variation of critical oscillatory Rayleigh number with strain retardation parameter λ_2 for different values of Soret parameter *Sr*.

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