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RADIATION EFFECTS ON UNSTEADY MHD FLOW OVER A VERTICAL PLATE WITH RAMPED WALL TEMPERATURE IN THE PRESENCE OF CHEMICAL REACTION

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ABSTRACT

The effects of radiation on unsteady MHD free convection flow of an incompressible viscous, electrically conducting fluid near an infinite vertical plate with ramped wall temperature in the presence of chemical reaction has been studied. The motion of the plate is a rectilinear translation with an arbitrary time dependent velocity. The radiative flux is described using the differential approximation for radiation. The partial differential equations governing the flow are transformed into a non-dimensional form and are solved by Ritz FEM. The results obtained are discussed for the influence of magnetic field that is fixed relative to the fluid and plate. It has been found that when the thermal and mass Grashof numbers are increase, the thermal and concentration buoyancy effects are enhanced and the fluid velocity increases. Furthermore, when the magnetic field, heat absorption parameter and chemical reaction parameter increase the fluid velocity decrease. These results are in good agreement with earlier results.

Key words: *Radiation, MHD, free convection, ramped wall temperature, chemical reaction.*

1. INTRODUCTION

The problem of magneto-hydrodynamics (MHD) boundary layer flow of an electrically conducting fluid with heat and mass transfer plays an important role in different areas of science and technology like chemical engineering, mechanical engineering, biological science, petroleum engineering, biomechanics, irrigation engineering and aerospace technology. The class of exact solutions of the unsteady magneto-hydrodynamic free convection flows studied by Tokis (1985). Gokhale and Samman (2003) presented the effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi-infinite vertical plate with constant heat flux. Aldose and Al-Nimr (2005) presented the effect of local acceleration term on the MHD transient free convection flow over a vertical plate. The effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection presented by Kandasamy *et. al* (2005). Chaudary and Jain (2007) studied combined heat and mass transfer effects MHD free convection flow past an oscillating plate embedded in a porous medium. Heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous flat plate in the presence of radiative heat transfer studied by Mbeledogu and Ogulu (2007). Sharma *et. al* (2007) presented the Hall effect on MHD mixed convective flow of a viscous incompressible fluid pasta vertical porous plate immersed in a porous medium with heat sourec/sink. Thermal radiation effects on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity presented by Mohamoud (2009). Finite element analysis of heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer presented by Reddy and Rao (2010). Shanaker et. al (2010) presented the radiation and mass transfer effects on unsteady MHD free convective fluid flow embedded in a porous medium with heat generation/absorption. Seth *et. al* (2011) studied MHD natural convection flow with radiative heat transfer past an impulsively moving plate with ramped wall temperature. Fluctuating heat and mass

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transfer on unsteady MHD free convection flow of radiating and reacting fluid past a vertical porous plate in slip-flow regime studied by Singh and Kumar (2011). Ghara *et. al* (2012) presented the effect of radiation on MHD free convection flow past an impulsively moving vertical plate with ramped wall temperature. Ravi kumar *et. al* (2012) presented heat and mass transfer effects on MHD flow of viscous fluid through non-homogenous porous medium in presence of temperature dependent heat source. Seshaiah *et. al* (2013) presented the induced magnetic field effects on free convective flow of radiative, dissipative fluid past a porous plate with temperature gradient heat source. Ahmed and Dutta (2013) studied transient mass transfer flow past an impulsively started infinite vertical plate with ramped plate velocity and ramped wall temperature. Narahari and Debnath (2013) studied unsteady magneto-hydrodynamic free convection flow past an accelerated vertical plate with constant heat flux and heat generation/absorption. Unsteady magneto-hydrodynamic free convection flow of a second grade fluid in a porous medium with ramped wall temperature studied by Samiulhaq *et. al* (2014). Seth *et. al* (2015) presented Soret and Hall effects on unsteady MHD free convective flow of radiating and chemically reactive fluid past a moving vertical plate with ramped wall temperature in rotating system. Mass transfer effects on unsteady MHD free convective flow of an incompressible viscous dissipative fluid an infinite vertical porous plate presented by Reddy (2016).

Hence, based on the above investigations and applications, the objective of the present paper is to analyze the radiation effects on unsteady MHD free convection flow of an incompressible, electrically conducting fluid near an infinite vertical plate with ramped wall temperature in the presence of chemical reaction. The motion of the plate is a rectilinear translation with an arbitrary time dependent velocity. The Ritz FEM has been adopted to solve the system of partial differential equations which is more economical from computational point of view. The fluid is electrically conducting and regarding the applied magnetic field two cases are considered, namely, when the magnetic lines of force are fixed to the fluid and plate. The differences between fluid velocities in the two cases are studied and some properties are highlighted.

2. MATHEMATICAL ANALYSIS

Consider unsteady MHD free convection flow of an incompressible, electrically conducting fluid near an infinite vertical plate with ramped wall temperature and constant concentration. The motion of the plate is a rectilinear translation with an arbitrary time dependent velocity. We introduce a coordinate system with $x'-$axis along the plate in the vertical upward direction, and $y'-$ axis normal plate. A uniform transverse magnetic field of strength B_0 is applied. Initially, at time $t=0$ the plate and the fluid are at rest with the same temperature T_∞ the species concentration in the fluid C_∞. After time $t=0^+$ the plate moves with the velocity $U_0 f(t)$ in its own plane along the $x'-$axis. Here, U_0 is a constant velocity and $f(\cdot)$ is a dimensionless piecewise continuous function whose values $f(0)=0.$ Heat is supplied to the plate as a time-ramped function in the presence of heat source and chemical reaction. The species concentration at the plate is constant $C_w.$ we further assume that:

1) The magnetic Reynolds number is small so that the induced magnetic field is negligible in comparison to the applied magnetic field.
2) Viscous dissipation and Joule heating terms are neglected in the energy equation (usually in free convection flows the velocity has small values).
3) No external electric field is applied and the effect of polarization of ionized fluid negligible, therefore, electric field is assumed to be zero.
4) There exists a first order chemical reaction between the fluid and species concentration. The level of the species concentration is very low, so that the heat generated by chemical reaction can be neglected.

Since the plate is infinite extended in x and z directions, therefore all the physical quantities are functions of the spatial coordinate y and t only. Then, under the Boussinesq's approximation, the flow governed by the following system of equations:

$$\frac{\partial u(y,t)}{\partial t}=g\beta_T\left(T(y,t)-T_\infty\right)+g\beta_C\left(C(y,t)-C_\infty\right)+\nu\frac{\partial^2 u(y,t)}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u(y,t) \tag{1}$$

$$\rho C_p\frac{\partial T(y,t)}{\partial t}=k\frac{\partial^2 T(y,t)}{\partial y^2}-\frac{\partial q_r}{\partial y}-Q\left(T(y,t)-T_\infty\right) \tag{2}$$

$$\frac{\partial C(y,t)}{\partial t}=D_m\frac{\partial^2 C(y,t)}{\partial y^2}-k_r\left(C(y,t)-C_\infty\right) \tag{3}$$

Where u, T and C are velocity, temperature and species concentration of the fluid, respectively, ν is kinematic viscosity of the fluid, g is the acceleration due to gravity, ρ is the fluid density, C_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid, D_m is the chemical molecular diffusivity, β_T is the volumetric coefficient of thermal expansion, β_C is the volumetric coefficient of concentration expansion, Q is the heat generation or absorption coefficient, k_r is the chemical reaction parameter, B_0 is the uniform magnetic field, t is the time.

Equation (1) is valid, when the magnetic lines of force are fixed relative to the fluid. If the magnetic field is fixed to the plate, the momentum equation (1) is replaced by [1, 17]

$$\frac{\partial u(y,t)}{\partial t} = g\beta_T\left(T(y,t)-T_\infty\right)+g\beta_C\left(C(y,t)-C_\infty\right)+\nu\frac{\partial^2 u(y,t)}{\partial y^2}-\frac{\sigma B_0^2}{\rho}\left(u(y,t)-U_0 f(t)\right) \tag{4}$$

Equations (1) and (4) combined as:

$$\frac{\partial u(y,t)}{\partial t} = g\beta_T\left(T(y,t)-T_\infty\right)+g\beta_C\left(C(y,t)-C_\infty\right)+\nu\frac{\partial^2 u(y,t)}{\partial y^2}-\frac{\sigma B_0^2}{\rho}\left(u(y,t)-\varepsilon U_0 f(t)\right) \tag{5}$$

where

$\varepsilon = 0,$ if B_0 is fixed relative to the fluid

$= 1,$ if B_0 is fixed relative to the plate.

The corresponding initial and boundary conditions are:

$$u(y,0)=0, T(y,0)=T_\infty, C(y,0)=C_\infty,\quad y\ge 0$$

$$u(0,t)=U_0 f(t), T(0,t)=\begin{cases} T_\infty+(T_w-T_\infty)\dfrac{t}{t_0}, 0<t\le t_0 \\ T_w, t>t_0 \end{cases}, C(0,t)=C_w$$

$$u(y,t)<\infty, T(y,t)\to T_\infty, C(y,t)\to C_\infty \quad as\ y\to\infty \tag{6}$$

By using the Rosseland approximation, the radiative flux vector q_r can be written as:

$$q_r = \frac{4\sigma^*}{3k^*}\frac{\partial T^4(y,t)}{\partial y} \tag{7}$$

It is assumed that the temperature differences within the flow are sufficiently small so that $T^4(y,t)$ can be expanded in a Taylor series about the free stream temperature T_∞, so that after neglecting the higher order terms

$$T^4(y,t)\approx 4T_\infty^3 T(y,t)-3T_\infty^4 \tag{8}$$

The energy equation after substitution of equations (7) and (8) can be written as:

$$\frac{\partial T(y,t)}{\partial t} = \frac{k}{\rho c_p}\frac{\partial^2 T(y,t)}{\partial y^2}+\frac{16\sigma^* T_\infty^3}{3\rho c_p k^\bullet}\frac{\partial^2 T(y,t)}{\partial y^2}-Q\left(T(y,t)-T_\infty\right) \tag{9}$$

The following non-dimensional quantities introduced to transform equations (3), (5), (6) and (9) into dimensionless form:

$$u^*=\frac{u}{U_0}, y^*=\frac{y}{\sqrt{\nu t_0}}, t^*=\frac{t}{t_0}, T^*=\frac{T-T_\infty}{T_w-T_\infty}, C^*=\frac{C-C_\infty}{C_w-C_\infty}, S_c=\frac{\nu}{D_m}, P_r=\frac{\mu C_p}{k}, M=\sqrt{\nu t_0}B_0\sqrt{\frac{\sigma}{\mu}},$$

$$k_r^*=k_r t_0, K_1=\frac{16\sigma^* T_\infty^3}{3k^\bullet k}, Q^*=\frac{Q t_0}{\rho C_p}, G_r=\frac{g\beta_T t_0(T_w-T_\infty)}{U_0}, G_m=\frac{g\beta_C t_0\left(C_w-C_\infty\right)}{U_0}.$$

Dropping out the star notions, the non-dimensional forms of governing equations are:

$$\frac{\partial u(y,t)}{\partial t}=\frac{\partial^2 u(y,t)}{\partial y^2}+G_rT(y,t)+G_mC(y,t)-M^2\left(u(y,t)-\varepsilon f(t)\right) \tag{10}$$

$$\frac{\partial T(y,t)}{\partial t}=\left(\frac{1+K_1}{P_r}\right)\frac{\partial^2 T(y,t)}{\partial y^2}-QT(y,t) \tag{11}$$

$$\frac{\partial C(y,t)}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C(y,t)}{\partial y^2}-k_rC(y,t) \tag{12}$$

where G_r is the thermal Grashof number, G_m is the mass Grashof number, M is the magnetic parameter, K_1 is the radiation parameter P_r is the Prandtl number, Q is the heat absorption parameter, S_c is the Schmidt number, k_r is the chemical reaction parameter.

The corresponding boundary conditions in non-dimensional form are:

$$u(y,0)=0, T(y,0)=T_\infty, C(y,0)=C_\infty,\ \ y\geq 0$$

$$u(0,t)=f(t), T(0,t)=\begin{cases}t, 0\leq t\leq 1\\ 1, t>1\end{cases}=tH(t)-(t-1)H(t-1), C(0,t)=1$$

$$u(y,t)<\infty, T(y,t)\to 0, C(y,t)\to 0\ \ as\ y\to\infty \tag{13}$$

where

$H(t)=\begin{cases}0, t\leq 0\\ 1, t>0\end{cases}$ is the Heaviside unit step function.

3. METHOD OF SOLUTION

The systems of partial differential equations (10) - (12) are solved numerically using the Ritz FEM, subject to the boundary conditions given in equation (13). The Ritz FEM has been employed extensively by the authors in many challenging heat transfer, biomechanics and metallurgical transport phenomena problems over the past few years. The method entails the following steps.

1. Division of the whole domain into smaller elements of finite dimensions called “finite elements”.
2. Generation of the element equations using variational formulations.
3. Assembly of element equations as obtained in step 2.
4. Imposition of boundary conditions to the equations obtained in step 3.
5. Solution of the assembled algebraic equations.

The assembled equations can be solved by any of the numerical technique viz. Gauss-Seidal iteration method. Due to simple and efficient use in computations, linear shape functions are used in the present problem. To prove convergence and stability of the Ritz FEM, the computations are carried out by making a small changes time t and $y-$directions. For these slightly changed values, no significant change was observed in the values of velocity u, temperature θ and concentration C. Hence, the Ritz finite element method is convergent and stable.

4. NUMERICAL RESULTS AND DISCUSSION

The effect of radiation on unsteady MHD free convection flow of an incompressible, electrically conducting fluid near an infinite vertical plate with ramped wall temperature and constant concentration is studied. The radiative heat flux is described using the differential approximation. The motion of the plate is a rectilinear translation with an arbitrary time dependent velocity. The Ritz FEM has been adopted to solve dimensionless governing equations of the flow. The influence of the transverse magnetic field that is fixed relative to the fluid and plate are studied. It is important to note that, depending on how the magnetic lines of force are fixed relative to the fluid or plate, flow parameters are significantly different. Here, the translational motion with constant velocity is considered. Numerical results for the velocity have been computed for the material parameters and then presented in figures.

The velocity profiles versus the spatial variable y for constant plate velocity $\left(f(t)=H(t)\right)$ are presented in figures 1-6 respectively. The figures, corresponding to the velocity $u(y,t)$ are plotted when the magnetic field is being fixed to the fluid $(\varepsilon=0)$ and to the moving plate $(\varepsilon=1)$.

Figure 1 depicts the effect of the magnetic field M on the velocity field. It is seen that fluid velocity decreases as the magnetic field increases. We note that, under the influence of magnetic field on an electrically conducting fluid, a resistive force arises (so called the Lorentz force). This force has tendency to slow down the fluid motion in the boundary layer. The effects of the thermal Grashof number G_r and mass Grashof number G_m on the velocity field are presented in the figures 2 and 3, respectively. It is seen that the fluid velocity increases with increasing values of both thermal Grashof number G_r and mass Grashof number G_m. It is further observed that the fluid velocity has a maximum value in the vicinity of the plate and tends to the finite value for larger values of the spatial coordinate y. Figure 4 depict the effect of radiation parameter K_1 on the velocity field. It can be seen that an increase in the radiation parameter leads to increase in the fluid velocity. The effects of the heat absorption parameter Q on the velocity field are presented in the figure 5. It is seen that the fluid velocity decreases with increasing values of heat absorption parameter. The influence of the dimensionless chemical reaction parameter k_r on the velocity field is presented in figure 6. It is observed that the fluid velocity decreases with increasing values of chemical reaction parameter. Also, it is noted that if the magnetic field is fixed to the fluid $(\varepsilon=0)$ the values of the fluid velocity are lower than in the case of the magnetic field is fixed to the plate $(\varepsilon=1)$.

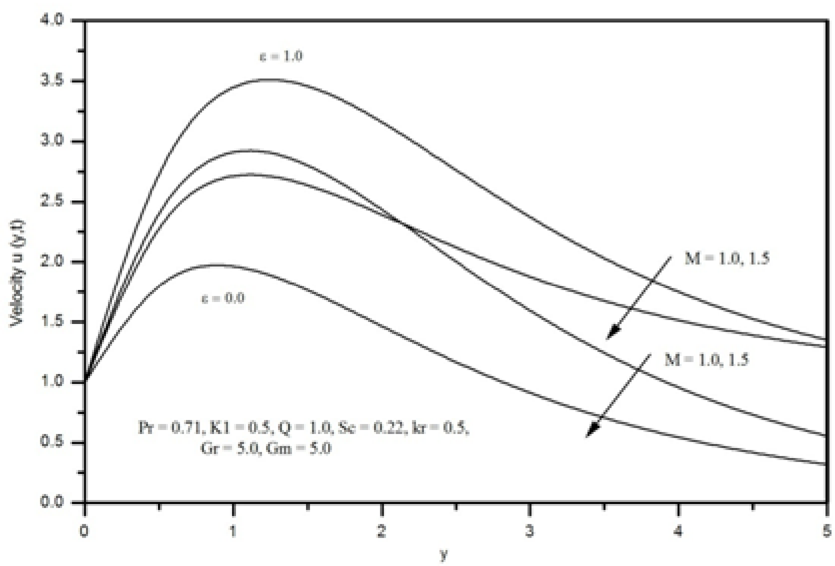

Figure-1: Effect of magnetic parameter M on the velocity profiles

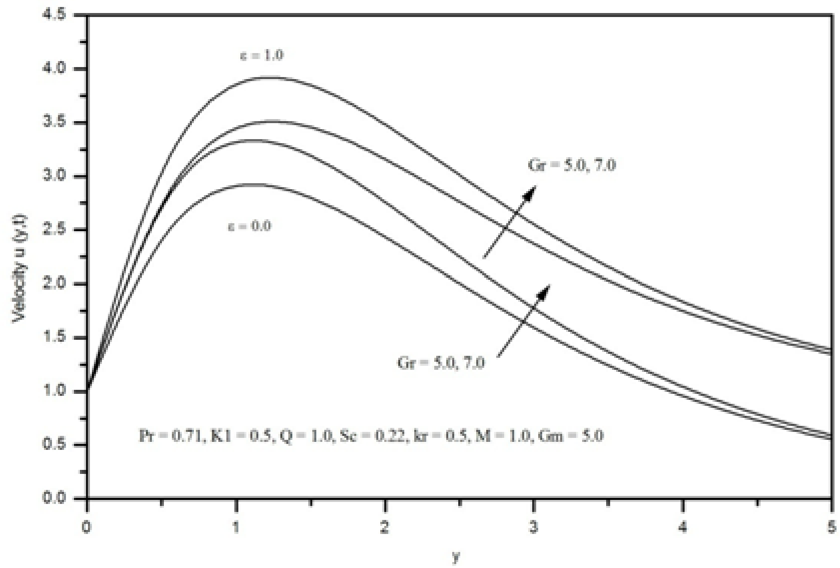

Figure-2: Effect of thermal Grashof number G_r on the velocity profiles

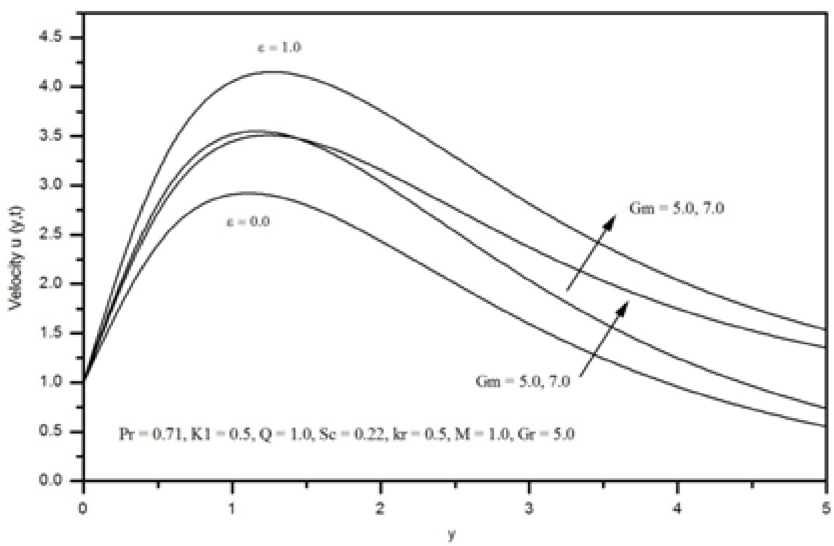

Figure-3: Effect of mass Grashof number G_m on the velocity profiles

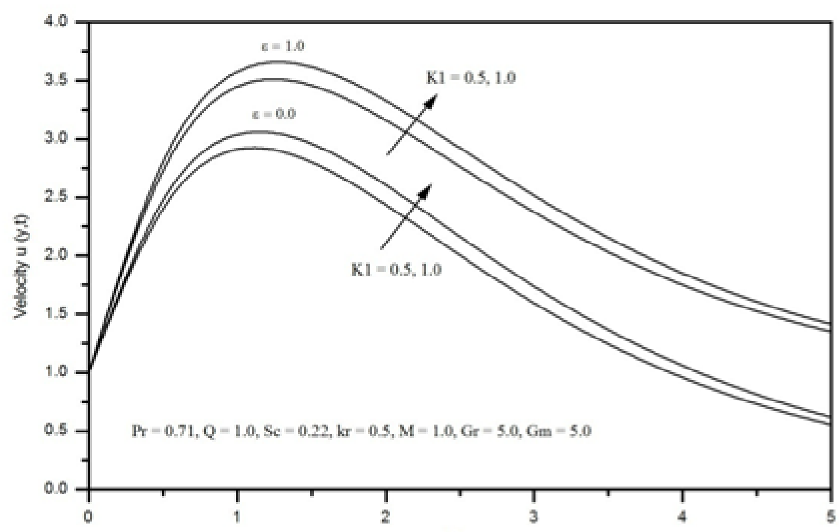

Figure-4: Effect of radiation parameter K_1 on the velocity profiles

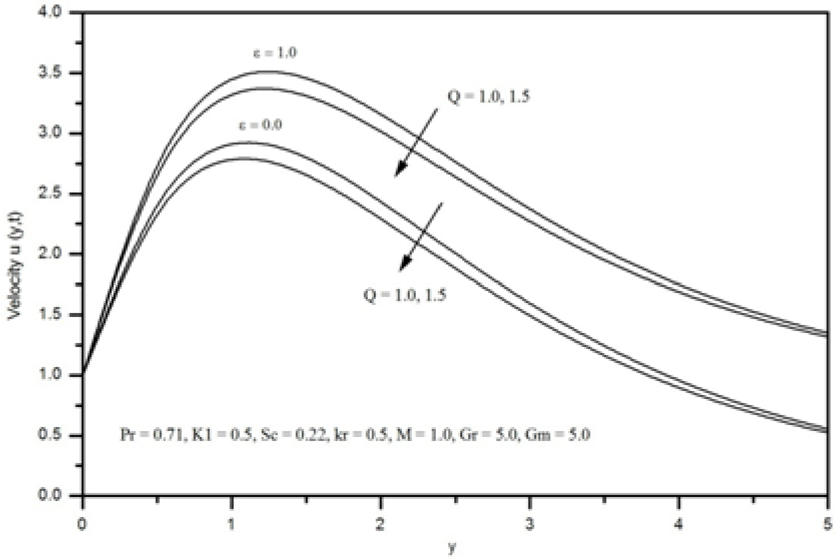

Figure-5: Effect of heat absorption parameter Q on the velocity profiles

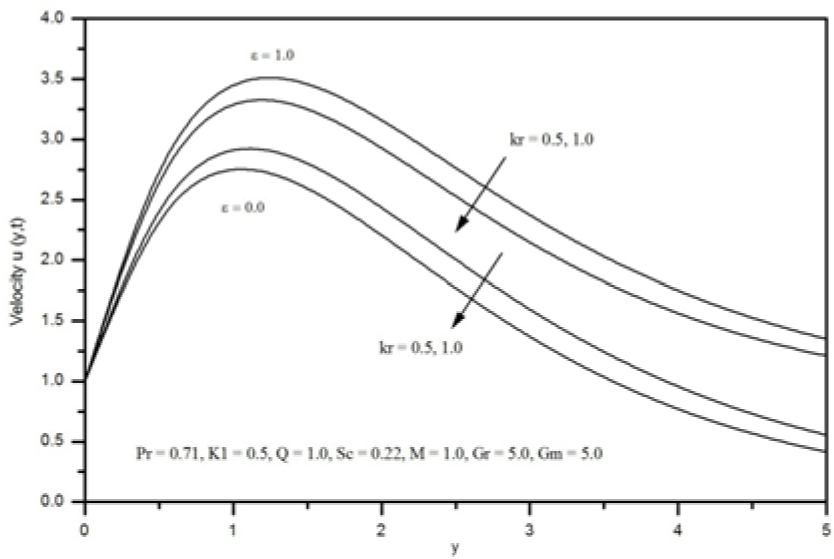

Figure-6: Effect of chemical reaction parameter k_r on the velocity profiles

CONCLUSIONS

The effects of radiation on unsteady MHD free convection flow of an incompressible, electrically conducting fluid near an infinite vertical plate with ramped wall temperature and constant concentration in the presence of chemical reaction has been studied. The motion of the plate is a rectilinear translation with an arbitrary time dependent velocity, the plate temperature changes as a time-ramped function and the concentration on the plate is constant. The problem is governed by system of coupled partial differential equations and are solved by Ritz FEM. The results obtained are discussed for the transverse magnetic field that is fixed relative to the fluid $(\varepsilon = 0)$ and plate $(\varepsilon = 1)$. The fluid velocity differs significantly, when the magnetic field is fixed relative to the moving plate from the fluid velocity corresponding to the case of magnetic field is fixed relative to the fluid. An increase in the magnetic parameter decreases the fluid velocity. i.e., stronger magnetic field leads to slower flows. The fluid velocity increases with increasing values of the thermal Grashof number and mass Grashof number and decreases with an increase in heat absorption parameter and chemical reaction parameter. The magnetic field is fixed to the fluid $(\varepsilon = 0)$, the values of the fluid velocity are lower than in the case of the magnetic field is fixed to the plate $(\varepsilon = 1)$.

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