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AN EPQ MODEL FOR NON-INSTANTANEOUS DETERIORATING PRODUCT IN WHICH PRODUCTION COST VARIES WITH TIME

Dr. C. SUGAPRIYA*
Assistant Professor, Department Of Mathematics, PG and Research,
Queen Mary's College, Chennai, Tamil Nadu, India.

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ABSTRACT

This paper discusses an Economic Production Quantity (EPQ) model with constant production rate and demand rate. The produced item undergoes non-instantaneous deterioration and the production cost varies with time. It is assumed that a single machine produces single product over an infinite planning horizon. The optimal production cycle time is derived under the conditions for continuous review model with deterministic demand and no shortage. The numerical solutions are also obtained for three Economic Production Quantity (EPQ) models for non-instantaneous deteriorating product.

1.1. INTRODUCTION

The Economic Production Quantity model (also known as the EPQ model) is an extension of the Economic Order Quantity model. The EPQ model was developed by E.W. Taft in 1918. The difference being that the EPQ model assumes orders are received incrementally during the production process. The function of this model is to balance the inventory holding cost and the average fixed ordering cost. The Economic order Quantity is also said to be the amount of orders at minimizes total variable cost required to order and hold inventory or size of an order at which the total of procurement cost and inventory carrying cost is at minimum.

To determine the Economic Production Quantity (EPQ) model assumes perfect product quality and perfect production processes. Deteriorating processes may affect production systems such as production stoppages and breakdowns are reducing the production rate due to production process inefficiency. Mohamed Ben-Daya *et al.* (2007) concentrated with EPQ model that incorporates the effect of shifts in production rate on lot sizing decisions due to speed losses. The cycle starts with a certain production rate and after a random time, the production rate shifts to a lower value.

Rau H *et al.* (2004) developed an inventory model for deteriorating items with a shortage occurring at the supplier involving a supply chain between the producer and buyer. The optimal number of deliveries is derived with the minimal joint total cost from the integrated viewpoint. This study compared with and without shortages and sensitivity analysis is given to explore the effect from a supplier shortage.

Ghosh S.K *et al.* (2006) developed an inventory model with time-dependent two-parameter Weibull demand rate, allowing shortages in the inventory. The shortages are completely backlogged. The production rate is assumed to be finite and proportional to the demand rate. The model is solved analytically to obtain the optimal solution of the problem.

Moncer Hariga (1997) presents two computationally efficient solution methods that determine the optimal replenishment schedules for exponentially deteriorating items and perishable products with fixed lifetime. For both models, the inventory items have general continuous time dependent demand. The models are formulated under the assumption of discrete opportunities for replenishment over a fixed planning horizon. The expected overall costs for an imperfect EPQ model with backlogging permitted is less than or equal to that of the one without backlogging. Yuan-Shyi Peter Chiu *et al.* (2006) included intangible backorder cost in mathematical analysis, an optimal lot-size policy that minimizes expected total costs as well as satisfied the maximal shortage level constraint for the EPQ model.

Tai-Yue Wang and Long-Hui Chen (2007) presented a production lot size model for deteriorating items with time-varying demand. The replenishment cycle and deterioration rates are allowed to vary over a finite planning horizon.

*Corresponding Author: Dr. C. Sugapriya**
Assistant Professor, Department Of Mathematics, PG and Research,
Queen Mary's College, Chennai, Tamil Nadu, India.

The Retailer's inventory system as a cost minimization problem to determine the retailer's optimal inventory cycle time. The retailer's will do order less quantity to take the permissible delays in payments more frequently and difference between unit selling price per item and unit purchasing price per item also larger dealt by Yung-Fu Huang and Kuang-Hua Hsu (2007).

The optimal production lot size is derived by Yuan-Shyi Peter Chiu and Singa Wang Chiu (2006) using the differential calculus on the production-inventory cost function. The EPQ model taking the random defective rate, imperfect rework process and the optimal lot size can be solved algebraically. Jui-Jung Liao (2007) derived a production model for the lot-size inventory system with finite production rate, permissible delay in payments, in which the restrictive assumption of a permissible delay is relaxed to that at the end of the credit period, the retailer will make a partial payment on total purchasing cost to the supplier and pay off the remaining balance by loan from the bank.

1.2. BASIC ASSUMPTIONS AND NOTATIONS

The following assumptions are used for the development of Economic Production Quantity models.

1. The demand rate for the product is known and finite.
2. Shortage is not allowed.
3. An infinite planning horizon is assumed.
4. Once a unit of the product is produced, it is available to meet the demand.
5. Once the product starts deterioration, the production is terminated and the Price discount is provided.
6. The deterioration follows an exponential distribution.
7. There is no replacement or repair for a deteriorated item.

The notations used for the development of Economic Production Quantity models are listed below:

p : production rate per unit time.
d : actual demand for the product per unit time.
A : set up cost.
θ : a constant deterioration rate (unit/unit time).
h : inventory carrying cost per unit time.
k=a+bt: production cost per unit time, where 'a' and 'b' are constants.
c : deterioration cost per unit item.
r : price discount per unit time.
T : optimal cycle time.
T_1 : production period.
T_2 : time during which there is no production of the product i.e, $T_2 = T - T_1$.
$I_1(t)$: inventory level of the product during the production period, i.e., $0 \le t \le T_1$.
$I_2(t)$: inventory level of the product during the period when there is no production i.e., $T_1 \le t \le T_2$.
I(M) : maximum inventory level of the product.
TVC (T): total cost per unit time.

1.3. Model Development

The inventory level is zero at t=0 time units. The production rate is p and the demand rate is d. The production and supply start simultaneously, the inventory builds up at the rate of p-d. It reaches the maximum level of I (M) at t $=T_1$ time units. Then the production is terminated at t $=T_1$ time units at which deterioration starts. From this point, the on-hand inventory diminishes to the extent of the supply plus the loss due to the deterioration. The production is resumed when all the units of the product are depleted at time T as shown in Figure 1.1. The deterioration depends on the existing amount of inventory present at any time.

This analysis has been represented by the differential equations:

$$\frac{dI_1(t)}{dt} = p - d \qquad \text{for } 0 \le t \le T_1 \tag{1.1}$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -d \qquad \text{for } 0 \le t \le T_2 \tag{1.2}$$

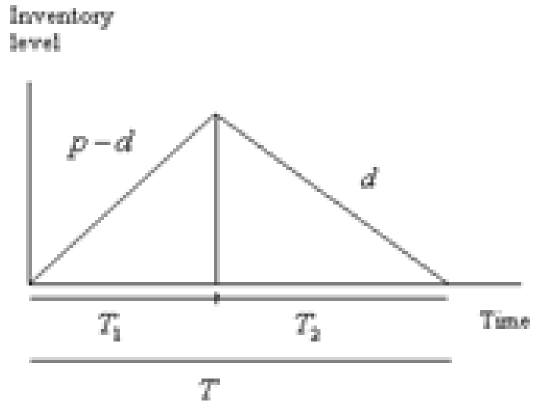

Figure-1.1: Representation of Inventory Model

The boundary conditions associated with these equations are: $I_1(0)=0$, $I_2(T)=0$

$$I_1(t)=(p-d)t \quad \text{for } 0\le t\le T_1 \tag{1.3}$$

$$I_2(t)=\frac{d}{\theta}\left[e^{\theta(T_2-t)}-1\right] \quad \text{for } 0\le t\le T_2 \tag{1.4}$$

The main objective here is to minimize the total cost per unit time. The average total cost per unit time depends on production cost, setup cost, holding cost and deterioration cost.

a) Production cost: The production cost per unit time is represented by the following equation:

$$\text{PC} = \frac{p}{T}\int_0^{T_1}(a+bt)dt \tag{1.5}$$

b) Setup cost:

$$\text{SC} = \frac{A}{T} \tag{1.6}$$

c) Holding cost (storage cost): The holding cost per unit time is given by

$$\text{HC} = \frac{h}{T}\left[\int_0^{T_1} I_1(t)dt + \int_0^{T_2} I_2(t)dt\right] \tag{1.7}$$

d) Deterioration cost: The number of units that deteriorate in a cycle is the difference between the maximum inventory and the number of units used to meet the demand during the period $[0, T_2]$. Hence, the deterioration cost per unit time is given as

$$\text{DC} = \frac{c}{T}\left[I_2(0) - \int_0^{T_2} d.dt\right] \tag{1.8}$$

Therefore, the average total cost per unit time is equal to sum of production cost, setup cost, holding cost and deterioration cost. Assuming $t\theta<1$, an approximate value is obtained by neglecting the terms of degree greater than or equal to 2 in $t\theta$, in the Tailor's expansion of the exponential function

TVC (T) = PC+SC+HC+DC

$$= \frac{p}{T}\left[aT_1+\frac{bT_1^2}{2}\right]+\frac{A}{T}+\frac{h}{T}\left[(p-d)\frac{T_1^2}{2}+\frac{dT_2^2}{2}\right]+\frac{cd\theta T_2^2}{2T} \tag{1.9}$$

If (1A.1) and (1A.2) are used to express T_1 and T_2 in terms of T neglecting the third and higher powers of θT terms the equation (1.10) becomes

$$\text{TVC(T)} = p\left[\frac{ad}{p}+\frac{bd^2}{2p^2}T\right]+\frac{A}{T}+h(p-d)\frac{d^2T}{2p^2}+\frac{hd(p-d)^2T}{2p^2}+\frac{cd\theta(p-d)^2T}{2P^2} \tag{1.10}$$

To minimize the total cost per unit time TVC (T), differentiate TVC (T) with respect to T and set the derivative equal to zero. The resultant production cycle time represented as

$$T = \sqrt{\frac{2p^2A}{pbd^2 + h(p-d)d^2 + hd(p-d)^2 + cd\theta(p-d)^2}} \tag{1.11}$$

$\dfrac{d^2TVC(T)}{dT^2} = \dfrac{2A}{T^3} > 0$, i.e., the second derivative is found to be positive. It is the basic requirement for T is to be minimum.

1.4. Numerical Example

Consider the EPQ problem in which the data are as given below: A= \$1000/set up, p =100 units/unit time, d = 40units/unit time, h = \$0.05 units/unit time, θ = 0.08, a = 30, b = 0.1, c = 0.15/unit time. The obtained results are: i) production cycle time (T) = 25.938 unit time, ii) average total cost TVC (T) = \$ 1277.1, iii) Production cost = \$ 1220.8, iv) Production run time = 10.375 unit time.

Based on this numerical values the total cost, production cycle time, production run time and production cost are computed for five different sets of values a and b with price discount. The production cost per unit time is k= a+bt where a and b are constants. The values of a and b of the product in a given set is increased from one set to another. The results are obtained for different set of constants a and b. The total cost is calculated and compared with production cycle time, production run time, price discount and production cost for different values a and b.

Table-1.1: Set of Time Independent Constants

a_1	a_2	a_3	a_4	a_5
30	35	40	45	50

Table-1.2: Set of Time DependentConstants

b_1	b_2	b_3	b_4	b_5
0.1	0.125	0.15	0.175	0.2

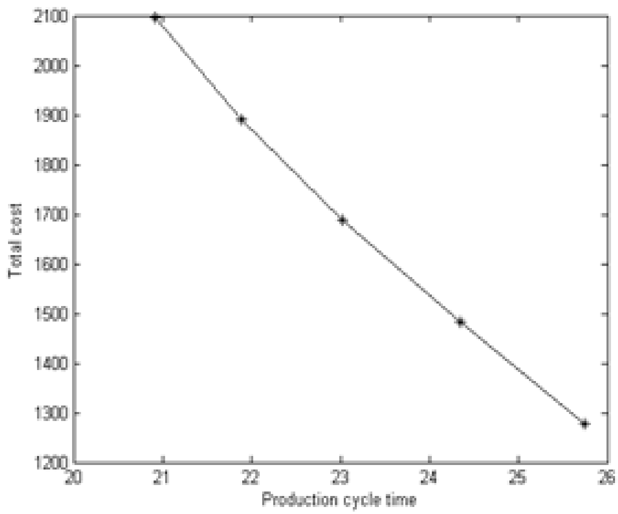

Figure-1.2: Production Cycle Time versus Total Cost for Model 1

Figure 1.2 Shows that, the total cost decreases for increased values of production cycle time. The Figure 1.3 shows that, the total cost varies with production run time. The analysis suggests that, the total cost decreases for increased values of production cycle time. Similarly, comparative graph was drawn between production cycle time and production cost as shown in Figure 1.4 and it is shown that the production cost decreases when production cycle time increases as expected.

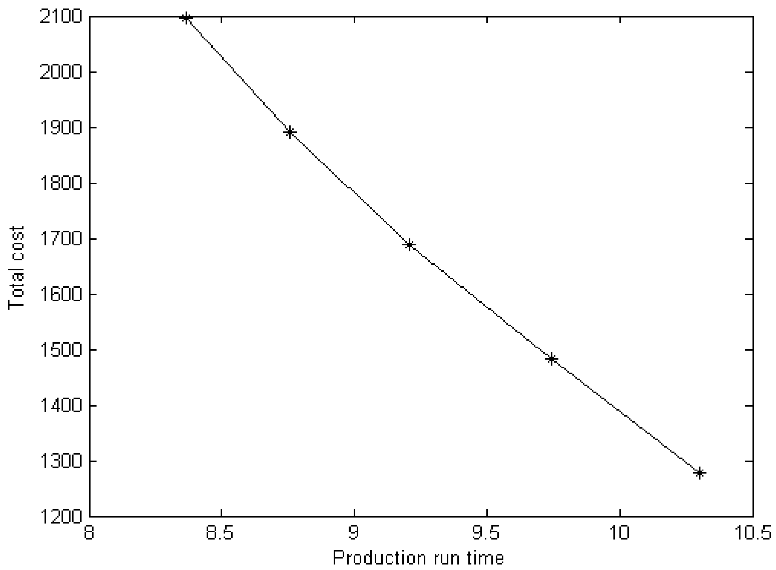

Figure-1.3: Production Run Time versus Total Cost

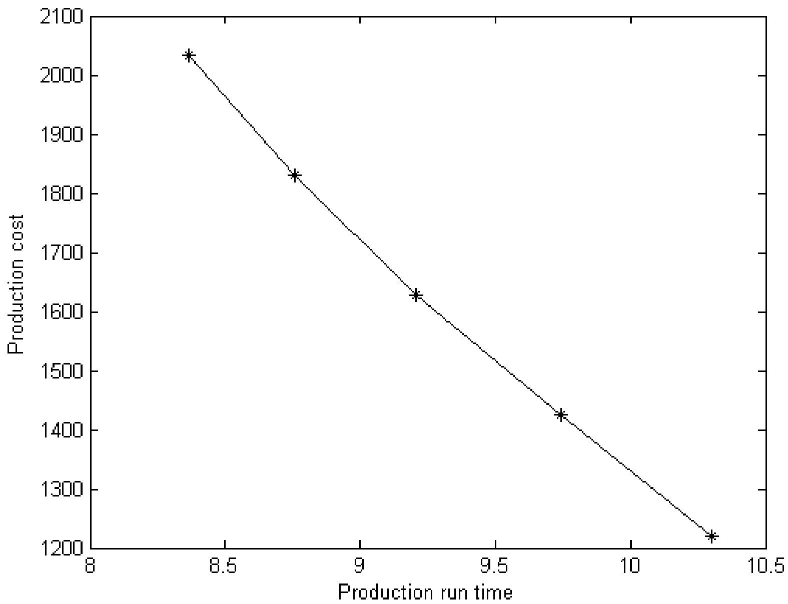

Figure-1.4: Production Run Time versus Production Cost

1.5. CONCLUSION

In this paper, EPQ model is analyzed for non- instantaneous deterioration. The total cost is minimized based on production cost, holding cost and deterioration cost. The EPQ model was discussed based on constant production rate and demand rate in which production cost varies with time. In this model is assumed that a single machine produces single product over an infinite planning horizon. The production cost per unit depends on the extend of time it consumes the machine time. The expressions obtained for total cost and production cycle time are useful to minimize the total cost. The optimal production cycle time is derived under conditions for continuous review model with deterministic demand and no shortage. The numerical solutions are also obtained with Economic Production Quantity (EPQ) model for non-instantaneous deteriorating product without lead time.

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Appendix 1A

To get the total cost per unit time TVC (T) in terms of the single variable T, the variables T_1 and T_2 are to be eliminated from the equation (1.10). At the moment when production is terminated. $I_1(T_1) = I_2(0)$

$$(p-d)\,T_1 = \frac{d}{\theta}\left(e^{\theta T_2} - 1\right)$$

Applying Taylor's expansion and approximation

$$(p-d)\,T_1 = d\left(T_2 + \frac{\theta T_2^{\,2}}{2}\right)$$

$$T_2 = \frac{(p-d)}{p}T \tag{1A.1}$$

$$T = T_1 + T_2 = \frac{p}{d}T_1 \tag{1A.2}$$

And $e^{\theta T_2} = 1 + \theta T_2 + \dfrac{(\theta T_2)^2}{2}$