

TENSOR PRODUCT OF A NEAR-FIELD SPACE AND SUB NEAR-FIELD SPACE OVER A NEAR-FIELD

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ABSTRACT

A tensor product of near-field space and sub near-field space over a near-field defined. But it turned out to be an abelian group. In order to avoid this special situation, this concept is generalized further by Dr N V Nagendram in this paper. Now there are two tensor products for the same pair of sub near-field spaces of a near-field space over a near-field. This situation fits better in the theory of near-field spaces and their sub near-field spaces over a near-field. It has been seen here that there may be two dual near-field spaces for a pair of sub near-field spaces over a near-field.

**Keywords:** sub near-field space, near-field space, semi simple near-field space, ordered Euclidean near-field spaces, vector lattices.

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SECTION 1: INTRODUCTION

We write maps on the right and hence use left near-field spaces and the traditional sub near-field spaces are right sub near-field spaces.

**Definition 1.1 (N-sub near-field space):** Let  $(N, +, \cdot)$  be a left near-field space. A sub near-field space  $(M, +)$  is called an N-sub near-field space i.e. traditional one if there is a near-field space homo-morphism  $\theta : N \rightarrow \text{Map}(M)$ . As usual, we write  $gn$  to mean  $g(n\theta)$  for  $g \in M$  and  $n \in N$ . In this case the group elements distribute over the near-field spaces.

**Definition 1.2 (Complementary N-near-field space):**  $M$  is called a complementary N-sub near-field space or N – co sub near-field space, for short, if there is a semi sub near-field space elements distribute over the sub near-field space elements and the action of  $N$  is usually written on the left of the elements of  $M$ .

**Definition 1.3 ((N, T) – bi sub near-field space):** Let  $N$  and  $T$  be two left near-field spaces. A sub near-field space  $M$  is called an  $(N, T)$  – bi sub near-field space if

- (a)  $M$  is an N-co sub near field space
- (b)  $M$  is an T-sub near-field space and (c)  $(ng)t = n(gt), \forall g \in M, n \in N, t \in T$ .

**Definition 1.4 (left strong N-sub near-field space):**  $M$  is called left strong N-sub near-field space if the action of  $N$  is defined on the left of  $M$  satisfying the following conditions  $\forall n, n' \in N$  and  $g, g' \in M$

- (a)  $(nn')g = n(n'g)$
- (b)  $n(g + g') = ng + ng'$  and (c)  $(n + n')g = ng + n'g$ .

**Note-1.5:** A right strong N-sub near-field space is defined similarly.  $(N, +)$  is an  $(N - N)$  – bi sub near-field space for the left as well as right near-field space  $N$  over a near-field. If  $N$  is distributive near-field space then  $(N, +)$  is a left as well as right strong N-sub near-field space. Many more examples of these structures are given in near-field space related topic.

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**Definition 1.6 (N-homomorphism):** Let  $M$  and  $K$  be two  $N$ -sub near-field spaces ( $N$ -co sub near-field space. A sub near-field space homomorphism  $\theta : M \rightarrow K$  is called an  $N$ -homomorphism if for any  $g \in M$  and  $n \in N$ ,  $(gn)\theta = (g\theta)n$ ,  $((rg)\theta = r(g\theta))$ .

**Note-1.7:** An  $(N - T) -$  homomorphism for  $(N - T)$ -bi sub near-field space are defined in a similar way.

## SECTION 2: TENSOR PRODUCT

Let  $N$  be left near-field space,  $A$  an  $N$ -sub near-field space and  $B$  an  $N$ -co sub near-field space. Let  $E$  be the free collection of sub near-field spaces on  $A \times B$ . Let  $P$  and  $Q$  be the normal sub near-field spaces of  $E$  generated by  $\{(a + a', b) - (a', b) - (a, b), (ar, b) - (a, rb) \mid a, a' \in A, b \in B, r \in N\}$  and  $\{(a, b + b') - (a, b') - (a, b), (ar, b) - (a, rb) \mid a \in A, b, b' \in B, r \in N\}$  respectively.

We call  $E/P$  the left tensor product of  $A$  and  $B$  and denote it by  $A_N \otimes B$  and call  $E/Q$  the right tensor product of  $A$  and  $B$  and denote it by  $A \otimes_N B$ . The coset  $(a, b) + P$  is denoted by  $a_l \otimes b$  and  $(a, b) + Q$  is denoted by  $a \otimes_r b$ . The coset  $P$  is denoted by  $0$  in both cases. Since  $E$  is generated by  $A \times B$ ,  $E/P = A_N \otimes B$  and  $E/Q = A \otimes_N B$  are generated by  $\{a_l \otimes b/a \in A, b \in B\}$  and  $\{a \otimes_r b/a \in A, b \in B\}$  respectively. An element of  $A_N \otimes B$  ( $A \otimes_N B$ ) is a finite sum of the form  $\sum \epsilon_i (a_i l \otimes b_i)$  ( $\sum \epsilon_i (a_i \otimes_r b_i)$ ), where each  $\epsilon_i = \pm 1$ .

The following result is a direct consequence of the definition of tensor product of near-field space and sub near-field space over a near-field.

**Theorem 2.1:**  $\forall a, a' \in A, b \in B, r \in N$  the following are satisfied in  $A_N \otimes B$ :

- (i)  $(a + a')_l \otimes b = a_l \otimes b + a'_l \otimes b$
- (ii)  $ar_l \otimes b = a_l \otimes rb$
- (iii)  $0_{A_l} \otimes b = 0$
- (iv)  $(-a)_l \otimes b = - (a_l \otimes b)$ .

**Theorem 2.2:**  $\forall a \in A, b, b' \in B, r \in N$  the following are satisfied in  $A \otimes_N B$ :

- (i)  $(a)_l \otimes (b + b') = a \otimes_r b + a \otimes_r b'$
- (ii)  $ar \otimes_r b = a \otimes_r rb$
- (iii)  $a \otimes_r bb = 0$
- (iv)  $a \otimes_r (-b) = - (a \otimes_r b)$ .

**Remark 2.3:** In general  $(a + a')r \neq ar + a'r$  in  $A$  as  $A$  is an  $N$ -sub near-field space. But we have

$$\begin{aligned} (a + a')r_l \otimes b &= (a + a')_l \otimes rb = a_l \otimes rb + a'_l \otimes rb \\ &= ar_l \otimes b + a'r_l \otimes b \\ &= (ar + a'r)_l \otimes b. \end{aligned}$$

This shows that in  $A_N \otimes B$  with  $b \neq 0$ .

**Remark 2.4:** It is possible that  $a \otimes_r b = 0$  in  $A \otimes_N B$ , with  $b \neq 0$ . Later on we will show these by examples and we generalize the definition of a middle linear map.

**Definition 2.5 (left N-middle linear map(LNMLM)):** Let  $N, A$  and  $B$  be as before and let  $C$  be any sub near-field space with a map  $f : A \times B \rightarrow C$ . then we call  $f$  a left  $N$ -middle linear map if  $(a + a', b)f = (a, b)f + (a', b)f$ ,  $(ar, b)f = (a, rb)f$ ,  $\forall a, a' \in A, b \in B, r \in N$ .

**Definition 2.6 (Right N-middle linear map(RNMLM)):** Let  $N, A$  and  $B$  be as before and let  $C$  be any sub near-field space with a map  $f : A \times B \rightarrow C$ . then we call  $f$  a right  $N$ -middle linear map if  $(a, b + b')f = (a, b)f + (a, b')f$ ,  $(ar, b)f = (a, rb)f$ ,  $\forall a, a' \in A, b, b' \in B, r \in N$ .

It is easy to see that  $\theta_l = j\pi_P : A \times B \rightarrow A_N \otimes B = E/P, (a, b) \mapsto a_l \otimes b$  and  $\theta_r = j\pi_Q : A \times B \rightarrow A \otimes_N B = E/Q, (a, b) \mapsto a \otimes_r b$  are

LNMLM and RNMLM respectively. Here  $j : A \times B \rightarrow E$  is the inclusion map and  $\pi_P : E \rightarrow E/P$  and  $\pi_Q : E \rightarrow E/Q$  are the natural homomorphisms. We call  $\theta_l$  ( $\theta_r$ ) the canonical LNMLM (RNMLM).

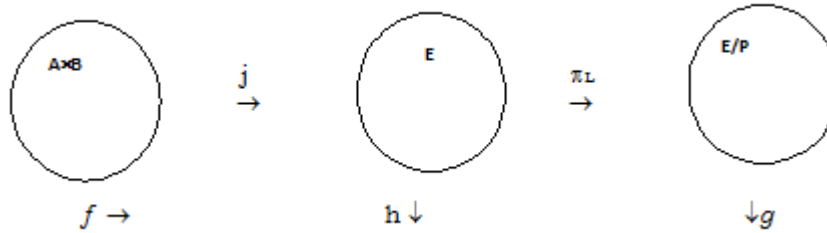
Now we prove the universal property of the tensor product of near-field space and sub near-field space over a near-field  $N$ .

**Theorem 2.7:** Let  $N$  be a left-near field space,  $A$  be an  $N$ -sub near-field space and  $B$  an  $N$ -co sub near-field space. Let  $C$  be a group with a function  $f: A \times B \rightarrow C$ , and  $\theta_l (\theta_r)$  be as above.

- (a). If  $f$  is a LNMLM, then there exists a unique sub near-field space homomorphism  $g: A_N \otimes B \rightarrow C$ , such that  $\theta_l g = f$
- (b). If  $f$  is a RNMLM, then there exists a unique sub near-field space homomorphism  $g: A \otimes_N B \rightarrow C$ , such that  $\theta_r g = f$ .

**Proof:** Given  $N$  be a left-near field space,  $A$  be an  $N$ -sub near-field space and  $B$  an  $N$ -co sub near-field space. Let  $C$  be a group with a function  $f: A \times B \rightarrow C$ .

**To prove (a):** Let us consider the following diagrammatic expression of near-field spaces over a near-field  $N$ .



where  $h$  is the unique homomorphism extending  $f$ , as  $E$  is the free sub near-field space on  $A \times B$ . Since  $f$  is an LNMLM,  $P \subseteq \text{Ker } h$ . This gives us a unique sub near-field space homomorphism  $g: E/P \rightarrow C$ , such that  $\pi_P g = h$ . It follows then  $\theta_l g = j\pi_L g = jh = f$ . Now for the uniqueness let  $g'$  be another homomorphism from  $E/P$  to  $C$  with  $\theta_l g' = f$ , then

$$(a_1 \otimes b)g' = (a, b)\theta_l g' = (a, b)f = (a, b)jh = (a, b)j\pi_L g = (a, b)\theta_l g = (a_1 \otimes b)g.$$

Therefore,  $g$  and  $g'$  agree on the generators of  $A_N \otimes B$  and hence are equal.

Hence proved (b). In similar manner (b) can be proved. This completes the proof of the theorem.

**Corollary 2.8:** Let  $A, A'$  be  $N$ -sub near-field spaces and  $B, B'$  be  $N$ -co sub near-field spaces over a near-field  $N$ ,  $f: A \rightarrow A'$  and  $g: B \rightarrow B'$  be  $N$ -homomorphisms of  $N$ -sub near-field spaces and  $N$ -co sub near-field spaces respectively. Then there are unique sub near-field space homomorphisms  $\phi: A_N \otimes B \rightarrow A'_N \otimes B'$  and  $\Psi: A \otimes_N B \rightarrow A' \otimes_N B'$  such that  $(a_1 \otimes b)\phi = af \otimes bg$  and  $(a \otimes b)\Psi = af \otimes bg$ .

**Note-2.9:** The homomorphism  $\phi$  and  $\Psi$  in the above corollary are denoted by  $f \otimes_l g$  and  $f \otimes_r g$  respectively.

If  $A, A', A''$  are  $N$ -sub near-field spaces of a near-field space over a near-field  $N$  and  $B, B', B''$  are  $N$ -sub near-field spaces of a near-field space over a near-field  $N$  with  $N$ -homomorphisms  $f: A \rightarrow A', f': A \rightarrow A'', g: B \rightarrow B'$  and  $g': B \rightarrow B''$ , then  $(f \otimes_l g)(f' \otimes_l g') = ff' \otimes_l gg'$  and  $(f \otimes_r g)(f' \otimes_r g') = ff' \otimes_r gg'$ . Moreover, if  $f$  and  $g$  are isomorphisms then  $f \otimes_l g$  and  $f \otimes_r g$  are isomorphisms.

**Theorem 2.10:** Let  $N$  and  $T$  be left near-field spaces over a near-field,  $A$  an  $(N - T)$  - bi sub near-field space and  $B$  an  $T$ -co sub near-field space over a near-field. The  $A_T \otimes B$  and  $A \otimes_T B$  are  $N$ -co sub near-field spaces over a near-field  $N$ .

**Proof:**  $\forall n \in N$ , define  $\alpha_n: A \times B \rightarrow A_T \otimes B$  by  $(a, b)\alpha_n = na \otimes b$ , for all  $(a, b) \in A \times B$ . We claim that  $\alpha_n$  is a LTMPM. For all  $a, a' \in A, b \in B$  and  $t \in T$  then we have the following relation as

$$(a + a', b)\alpha_n = n(a + a') \otimes b = (na + na') \otimes b = na \otimes b + na' \otimes b = (a, b)\alpha_n + (a', b)\alpha_n.$$

$$(at, b)\alpha_n = n(at) \otimes b = (na)t \otimes b = na \otimes sb = (a, sb)\alpha_n.$$

By theorem 2.9 there is a unique endomorphism  $\beta_n$  of  $A_T \otimes B$  such that  $\theta_l \beta_n = \alpha_n$ , where  $\theta_l$  is the canonical LTMPM:  $A \times B \rightarrow A_T \otimes B$ . The action of  $N$  on  $A_T \otimes B$  is now defined by  $nu = u\beta_n$  for  $n \in N$  and  $u \in A_T \otimes B$ . We claim that this action defines  $A_T \otimes B$  as an  $N$ -co sub near-field space over a near-field  $N$ . For all  $u, u' \in A_T \otimes B$  and  $n, n' \in N$  we have  $n(u + u') = (u + u')\beta_n = u\beta_n + u'\beta_n = nu + nu'$ . In order to prove that  $(nn')u = n(n'u)$ , it is enough to prove that  $\beta_{nn'} = \beta_n \beta_{n'}$  for all  $n, n' \in N$ . we look at their action on generators of  $A_T \otimes B$ .

$$(a \otimes b)\beta_{nn'} = (a, b)\theta_l \beta_{nn'} = (a, b)\alpha_{nn'} = (nn')a \otimes b = n(n'a) \otimes b = (n'a, b)\alpha_n = (n'a \otimes b)\beta_n = (a, b)\alpha_n \beta_n = (a \otimes b)\beta_n \beta_n.$$

The fact that  $A_T \otimes B$  is  $N$ -co sub near-field space over a near-field and similar manner one can prove that  $A \otimes_T B$  is an  $N$ -co sub near-field space over a near-field. This completes the proof of the theorem.

**Note-2.11:**  $N_N \otimes B$  and  $N \otimes_N B$  are co sub near-field spaces for any N-co sub near-field space B over a near-field.

**Note-2.12:** Let N be a left near-field space with 1, and B be any unital N-co sub near-field space. Then for  $n \in N$  and  $b \in B$ , we have  $n_l \otimes b = 1_l \otimes nb$  and  $n \otimes_n b = 1 \otimes_n nb$ . Therefore we have the following results:

- $N_N \otimes B$  is generated by  $\{ 1_l \otimes b \mid b \in B \}$
- $N \otimes_N B$  is generated by  $\{ 0 \otimes_n b, 1 \otimes_n b \mid b \in B \setminus \{0\} \}$ .
- $\theta_b : N \rightarrow N_N \otimes B$  defined by  $n\theta_b = n_l \otimes b$  is a sub near-field space homomorphism .
- $\theta_b : N \rightarrow N \otimes_N B$  defined by  $n\theta_b = n \otimes_n b$  is not a sub near-field space homomorphism in general.

**Theorem 2.13:** let N and T be left near-field spaces, A an N-sub near-field space and B an (N – T)-bi sub near-field space. Then  $A_N \otimes B$  is an T-sub near-field space with T acting on the right. If in addition B is a right strong T-sub near-field space then  $A \otimes_N B$  is also a T-co sub near-field space with T acting on the right.

**Proof:** For  $t \in T$  define  $\alpha_t : A \times B \rightarrow A_N \otimes B$  by  $(a, b) \alpha_t = a_l \otimes bt$ . It is easy to see that  $\alpha_t$  is a LNMLM.

This gives us a unique endomorphism  $\beta_t$  of  $A_N \otimes B$  such that  $\theta_l \beta_t = \alpha_t$ , where  $\theta_l : A \times B \rightarrow A_N \otimes B$  is the canonical LNMLM. For all  $(a_l \otimes b) \in A_N \otimes B$ , we have  $(a_l \otimes b) \beta_t = (a, b) \theta_l \beta_t = (a, b) \alpha_t = a_l \otimes (bt)$ .

Now we define an action of T on  $A_N \otimes B$  by  $ut = u\beta_t \forall u \in A_N \otimes B$ . Clearly,  $(u + u')t = ut + u't, \forall u, u' \in A_N \otimes B$ .

For the other condition we need to show that  $\beta_{tt'} = \beta_t \beta_{t'} \forall t, t' \in T$ . It is enough to look at their behaviour on the generators.

$(a_l \otimes b) \beta_{tt'} = a_l \otimes b (tt') = a_l \otimes (bt) t' = (a_l \otimes (bt)) \beta_{t'} = (a_l \otimes b) \beta_t \beta_{t'}$ . The second part can be proved in similar manner. This completes the proof of the theorem.

**Corollary 2.14:**  $A_N \otimes B$  is an N-co sub near-field space with N acting on the right. If N is distributive near-field space then  $A_N \otimes N$  and  $A \otimes_N N$  is an N- co sub near-field space with N acting on the right.

**Note-2.15:** Let N be a left near-field space with 1 and A be a unital N-sub near-field space. Then we have  $A_N \otimes N$  is generated by notation of a set  $\{a_1 \otimes 1, a_1 \otimes 0 \mid a \in A \setminus \{0\}\}$  and  $A \otimes_N N$  is generated by  $\{a \otimes_r 1 \mid a \in A\}$ .

**Note-2.16:** If A is any sub near-field space and B is an abelian sub near-field space then  $A_Z \otimes B$  and  $A \otimes_Z B$  are right Z-co sub near-field spaces and hence are abelian sub near-field spaces over a near-field N.

**Note-2.17:** If N is a near-field and A is an N-sub near-field space in the near-field spaces sense, that is  $(A, +)$  is not necessarily abelian, then  $A_N \otimes B$  and  $A \otimes_N B$  can be constructed, which may be different.

The structure of tensor products of near-field spaces can be explored further and these proofs here to show the importance of this concept.

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