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DETERMINATION OF ENTROPY FUNCTIONAL FOR DHS DISTRIBUTIONS

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ABSTRACT

This paper presents a study of the entropy functional for some continuous distributions. This study is concerned to determine the entropy functional for some various families of deformed hyperbolic secant distributions "DHS distributions". Finally, we discuss some results based on the introduced parameters or suggested parametric functions.

Keywords: *Deformed hyperbolic distributions, Entropy, Hyperbolic secant distribution.*

2010 Mathematics Subject Classification: *37A35, 94A17, 37B40, 60E05, 60E10, 62E17, 62E20.*

1. INTRODUCTION

Nowadays, the deformation technique has been applied for the continuous hyperbolic secant distribution "HS-Distribution" [1-7]. Some families of the continuous deformed hyperbolic secant distributions have been constructed and studied. Each DHS distribution has been obtained by introducing positive scalar deformation parameters as factors of the exponential growth and decay parts of the hyperbolic secant function "HS function" in the HS distribution. The probability density function (pdf) of the HS distribution is given as

$$f_{\text{HS}}(x) = \frac{1}{2}\operatorname{sech}(\pi x / 2); \qquad x \in R, \tag{1}$$

see [1-2]. Based on [3, 5], some classes of deformed hyperbolic secant distributions have been presented by introducing two parametric functions instead of the parameters. Some important properties of these classes of distributions have been discussed and some measures are derived in [3-7].

The main aim of this paper is to study and determine the entropy functional for each class of the deformed hyperbolic secant distributions.

Recently, the entropy functional has been suggested for some different probability distributions [8-10]. The entropy is known as a measure of uncertainly and dispersion and it is defined for the continuous random variable X as

$$\text{H}(f) = -\int_{-\infty}^{\infty} f(x) \log\big(f(x)\big)\, dx \tag{2}$$

where $f(x)$ is the pdf, see [11, 12].

This paper is laid out as follows: Section 2 presents a family of deformed hyperbolic secant distributions and its properties. Section 3 is devoted to the determined entropy functional of the hyperbolic secant distribution based on using of the deformation technique. Section 4 is considered for power comparison for some different values of the introduced parameters or parametric functions. Finally, the results of the paper have been concluded in Section 5.

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2. FAMILY OF DEFORMED HYPERBOLIC SECANT DISTRIBUTIONS

In order to obtain the deformed hyperbolic secant distributions "DHS distributions" by means of the deformation for the hyperbolic functions [4, 6,7], some properties of the deformed hyperbolic functions (DHF) will first be recalled, that is

$$\cosh_{pq} x = \frac{p e^{x} + q e^{-x}}{2} \quad \text{and} \quad \operatorname{sech}_{pq} x = \frac{1}{\cosh_{pq} x},$$

where $x \in R$ and the deformation parameters $p, q > 0$.

Based on [4], the pdf of a *pq*-DHS distribution as:

$$f_{pq-\mathrm{DHS}}(x; p, q) = \frac{\sqrt{p\,q}}{2} \operatorname{sech}_{pq}(\pi x/2). \tag{3}$$

According to [6, 7], two special cases of this distribution are given as follows:

- at $p = 1,\ q > 0$, this gives a *q*-DHS distribution with pdf

$$f_{q-\mathrm{DHS}}(x; q) = \frac{\sqrt{q}}{2} \operatorname{sech}_{q}(\pi x/2). \tag{4}$$

- at $q = 1,\ p > 0$, this gives a *p*-DHS distribution with pdf

$$f_{p-\mathrm{DHS}}(x; p) = \frac{\sqrt{p}}{2} \operatorname{sech}_{p}(\pi x/2). \tag{5}$$

Similarly, according to [3, 5], if $p(w)$ and $q(w)$ are two arbitrary real positive deformation parametric differentiable functions of w, $w \in R$, we consider the following $p(w)q(w)$-deformed hyperbolic functions:

$$\cosh_{p(w)q(w)} \varphi = \frac{p(w)e^{\varphi} + q(w)e^{-\varphi}}{2} \qquad \text{and} \qquad \operatorname{sech}_{p(w)q(w)} \varphi = \frac{1}{\cosh_{p(w)q(w)} \varphi},$$

with a real differentiable function $\varphi = \varphi(x, w)$ of x and w, and this function is linear in x with positive partial derivative with respect to x, i.e. $\varphi = C(w)x + D(w)$, $C(w) \in (0, \infty)$ as a derivative of φ with respect to x, and $D(w) \in R$. Moreover, a *p(w)q(w)*-DHS distribution has the following pdf:

$$f_{p(w)q(w)-\mathrm{DHS}}(\varphi; p(w), q(w)) = \frac{C(w)\sqrt{p(w)q(w)}}{2} \operatorname{sech}_{p(w)q(w)}\left(\frac{\pi\varphi}{2}\right), \quad x, w \in R \tag{6}$$

Based on [5], two special cases of this distribution are also given as follows:

- a *q(w)*-DHS distribution with pdf

$$f_{q(w)-\mathrm{DHS}}(\varphi; q(w)) = \frac{C(w)\sqrt{q(w)}}{2} \operatorname{sech}_{q(w)}\left(\frac{\pi\varphi}{2}\right); \quad x, w \in R, \tag{7}$$

- a *p(w)*-DHS distribution with pdf

$$f_{p(w)-\mathrm{DHS}}(\varphi; p(w)) = \frac{C(w)\sqrt{p(w)}}{2} \operatorname{sech}_{p(w)}\left(\frac{\pi\varphi}{2}\right); \quad x, w \in R. \tag{8}$$

In [3-7], some properties, measures and functions (expectation, variance, score function, unmodallity, measures of tendency, moments generating function, characteristic function, cumulants generating function, skewness, excess kurtosis and maximum likelihood estimators) of the families of DHS distribution are presented and discussed.

In the next section we present a study of the entropy functional for some various families of DHS distributions.

3. ENTROPY FUNCTIONAL FOR SOME DHS DISTRIBUTIONS

According to [8-12] the differentiable entropy $\mathrm{H}(f)$ is defined as a measure for a continuous random variable X by (2).

In this section, the suggested entropy functional $\mathrm{H}(f\ )$ will be derived for the deformed hyperbolic secant variables ($X_{p\,q-\mathrm{DHS}}$, $X_{p-\mathrm{DHS}}$, $X_{q-\mathrm{DHS}}$, $X_{p(w)\,q(w)-\mathrm{DHS}}$, $X_{p(w)-\mathrm{DHS}}$ and $X_{q(w)-\mathrm{DHS}}$) which based on the constructed DHS distributions [3-7].

Theorem 1: Let $X_{p\,q-\mathrm{DHS}}$ and $X_{p(w)\,q(w)-\mathrm{DHS}}$ be the *pq*-DHS variable and the *p(w)q(w)*-DHS variable with pdf's

$$f_{pq-\mathrm{DHS}}(x) = f_{pq-\mathrm{DHS}}(x\,;p\,,q) = \frac{\sqrt{p\,q}}{2}\,\mathrm{sech}_{pq}(\pi\ x\ /2)$$

and

$$f_{p(w)q(w)-\mathrm{DHS}}(\varphi) = f_{p(w)q(w)-\mathrm{DHS}}(\varphi;\,p(w),q(w)) = \frac{C(w)\sqrt{p(w)\,q(w)}}{2}\ \mathrm{sech}_{p(w)q(w)}\left(\frac{\pi\,\varphi}{2}\right),$$

respectively. Then,

(i) the entropy functional for the *pq*-DHS variable is

$$\mathrm{H}(f_{pq-\mathrm{DHS}}) = -\frac{1}{2}\log(p\,q) + \sqrt{p\,q}\ \ I_{pq-\mathrm{DHS}}, \tag{9}$$

where $$\mathrm{I}_{pq-\mathrm{DHS}} = \int_{-\infty}^{\infty} \frac{\log(p\,e^{\pi x/2} + q\,e^{-\pi x/2})}{p\,e^{\pi x/2} + q\,e^{-\pi x/2}}\,d\,x\ .$$

(ii) the entropy functional for the *p(w)q(w)*-DHS variable is

$$\mathrm{H}(f_{p(w)q(w)-\mathrm{DHS}}) = -[\log C(w) + \frac{1}{2}\log\ (\ p(w)\,q(w))] + C(w)\sqrt{p(w)\,q(w)}\,\mathrm{I}_{p(w)q(w)-\mathrm{DHS}} \tag{10}$$

where $$\mathrm{I}_{p(w)q(w)-\mathrm{DHS}} = \int_{-\infty}^{\infty} \frac{\log(\ p(w)\ e^{\pi\,\varphi/2} + q(w)\ e^{-\pi\,\varphi/2})}{p(w)\ e^{\pi\,\varphi/2} + q(w)\ e^{-\pi\,\varphi/2}}\ d\,x\,.$$

Proof: By using the form (2) of $\mathrm{H}(f\ )$, the entropy functional for the mentioned deformed hyperbolic secant variables in the two cases (i) and (ii) can be derived as follows:

(i) Similarly for the *pq*-DHS variable, we find that

$$\begin{aligned}
\mathrm{H}(\ f_{pq-\mathrm{DHS}}) &= -\int_{-\infty}^{\infty} f_{pq-\mathrm{DHS}}(x)\ \ \log\ (\ f_{pq-\mathrm{DHS}}(x)\ )\ d\,x \\
&= -\int_{-\infty}^{\infty} \frac{\sqrt{p\,q}}{2}\,\mathrm{sech}_{pq}(\pi\ x/2)\ \log\left(\frac{\sqrt{p\,q}}{2}\,\mathrm{sech}_{pq}(\pi\ x/2)\right) d\,x \\
&= -\int_{-\infty}^{\infty} \frac{\sqrt{p\,q}}{p\,e^{\pi x/2} + q\,e^{-\pi x/2}}\ \left\{\frac{1}{2}\log(p\ q) - \log(p\,e^{\pi x/2} + q\,e^{-\pi x/2})\right\} d\,x \\
&= -\frac{1}{2}\log(p\ q)\int_{-\infty}^{\infty} \frac{\sqrt{p\,q}}{p\,e^{\pi x/2} + q\,e^{-\pi x/2}}\,d\,x + \sqrt{p\,q}\int_{-\infty}^{\infty} \frac{\log(p\,e^{\pi x/2} + q\,e^{-\pi x/2})}{p\,e^{\pi x/2} + q\,e^{-\pi x/2}}\,d\,x
\end{aligned}$$

From the definition of the pdf of the *pq*-DHS variable, we find that

$$\int_{-\infty}^{\infty} \frac{\sqrt{p\,q}}{p\,e^{\pi x/2} + q\,e^{-\pi x/2}}\,d\,x = 1$$

and we can also obtain the required formula (9) of the entropy functional $\mathrm{H}(f_{pq-\mathrm{DHS}})$ for the *pq*-DHS distribution.

(ii) Without loss of generality, we consider $D(w\ ) = 0$. In this case, for some different values of w and for each fixed pair of the parametric functions $p(w\ )$ and $q(w\ )$ we can also derive the required formula of the entropy functional for the *p(w)q(w)*-DHS distribution as follows:

$$\mathrm{H}(f_{p(w)q(w)-\mathrm{DHS}}) = -\int_{-\infty}^{\infty} f_{p(w)q(w)-\mathrm{DHS}}(\varphi) \quad \log\ (f_{p(w)q(w)-\mathrm{DHS}}(\varphi))\ d\ x$$

$$= -\int_{-\infty}^{\infty} \frac{C(w)\sqrt{p(w)\,q(w)}}{2}\ \mathrm{sech}_{\,p(w)q(w)}(\pi\ \varphi / 2) \times$$

$$\times \log\left(\frac{C(w)\sqrt{p(w)\,q(w)}}{2}\ \mathrm{sech}_{\,p(w)q(w)}(\pi\ \varphi / 2) \right) d\ x$$

$$= -\int_{-\infty}^{\infty} \frac{C(w)\sqrt{p(w)\,q(w)}}{p(w)\,e^{\pi\,\varphi/2} + q(w)\,e^{-\pi\,\varphi/2}}$$

$$\times \{\log\ C(w) + \frac{1}{2}\log\ (p(w)\,q(w)) - \log\,(p(w)\,e^{\pi\,\varphi/2} + q(w)\,e^{-\pi\,\varphi/2})\}d\ x$$

$$= -\{\log C(w) + \frac{1}{2}\log\ (p(w)\,q(w))\} \int_{-\infty}^{\infty} \frac{C(w)\sqrt{p(w)\,q(w)}}{p(w)\,e^{\pi\,\varphi/2} + q(w)\,e^{-\pi\,\varphi/2}}\ dx$$

$$+ C(w)\sqrt{\ p(w)\,q(w)}\ \int_{-\infty}^{\infty} \frac{\log(\,p(w)\,e^{\pi\,\varphi/2} + q(w)\,e^{-\pi\,\varphi/2})}{p(w)\,e^{\pi\,\varphi/2} + q(w)\,e^{-\pi\,\varphi/2}} d\ x$$

From the definition of the pdf of the *p(w)q(w)*-DHS variable, we find that

$$\int_{-\infty}^{\infty} \frac{C\,(w\,)\sqrt{p(w\,)\,q\,(w\,)}}{p(w\,)e^{\pi\,\varphi/2} + q\,(w\,)e^{-\pi\,\varphi/2}}\ dx\ = 1$$

and we can also obtain the required formula (10) of the entropy functional $\mathrm{H}(f_{\,p(w\,)q\,(w\,)-\mathrm{DHS}})$ for the *p(w)q(w)*-DHS distribution.

Theorem 2: Let $X_{\,p-\mathrm{DHS}}$, $X_{\,q-\mathrm{DHS}}$, $X_{\,p(w\,)-\mathrm{DHS}}$ and $X_{\,q(w\,)-\mathrm{DHS}}$ be the *p*-DHS variable, the *q*-DHS variable, the *p(w)*-DHS variable and the *q(w)*-DHS variable with pdf's

$$f_{\,p-\mathrm{DHS}}(x\,) = f_{\,p-\mathrm{DHS}}(x\,;p\,) = \frac{\sqrt{p}}{2}\,\mathrm{sech}_{p}\,(\pi\ x\ / 2)\,,$$

$$f_{\,q-\mathrm{DHS}}(x\,) = f_{\,q-\mathrm{DHS}}(x\,;q\,) = \frac{\sqrt{q}}{2}\,\mathrm{sech}_{q}\,(\pi\ x\ / 2)$$

$$f_{\,p(w)-\mathrm{DHS}}(\varphi) = \ f_{\,p(w)-\mathrm{DHS}}(\varphi;\,p(w)) = \frac{C(w)\sqrt{p(w)}}{2}\ \mathrm{sech}_{\,p(w)}\left(\frac{\pi\ \varphi}{2}\right)$$

and

$$f_{q(w)-\mathrm{DHS}}(\varphi) = \ f_{q(w)-\mathrm{DHS}}(\varphi;\,q(w)) = \frac{C(w)\sqrt{\ q(w)}}{2}\ \mathrm{sech}_{\,q(w)}\left(\frac{\pi\ \varphi}{2}\right),$$

respectively. Then,

(i) The entropy functional for the *p*-DHS variable is

$$\mathrm{H}(f_{\,p-\mathrm{DHS}}) = -\frac{1}{2}\log\,(p) + \sqrt{\,p}\ \ \mathrm{I}_{\,p-\mathrm{DHS}}\ \ , \qquad (11)$$

where $$\mathrm{I}_{p-\mathrm{DHS}} = \int_{-\infty}^{\infty} \frac{\log\,(p\,e^{\pi\,x\,/2} + e^{-\pi\,x\,/2})}{p\,e^{\pi\,x\,/2} + e^{-\pi\,x\,/2}}\ d\ x\ .$$

(ii) The entropy functional for the *q*-DHS variable is

$$\mathrm{H}(f_{\,q-\mathrm{DHS}}) = -\frac{1}{2}\log\,(q) + \sqrt{\,q}\ \ \mathrm{I}_{\,q-\mathrm{DHS}}\ \ , \qquad (12)$$

where $$\mathrm{I}_{q-\mathrm{DHS}} = \int_{-\infty}^{\infty} \frac{\log\,(e^{\pi\,x\,/2} + q\,e^{-\pi\,x\,/2})}{e^{\pi\,x\,/2} + q\,e^{-\pi\,x\,/2}}\ d\ x\ .$$

(iii) The entropy functional for the *p(w)*-DHS variable is

$$\mathrm{H}(f_{p(w)-\mathrm{DHS}}) = -[\log C(w) + \frac{1}{2}\log p(w)] + C(w)\sqrt{p(w)}\ \mathrm{I}_{p(w)-\mathrm{DHS}}, \tag{13}$$

where $$\mathrm{I}_{p(w)-\mathrm{DHS}} = \int_{-\infty}^{\infty} \frac{\log(p(w)e^{\pi\varphi/2} + e^{-\pi\varphi/2})}{p(w)e^{\pi\varphi/2} + e^{-\pi\varphi/2}}\, dx\ .$$

(iv) The entropy functional for the *q(w)*-DHS variable is

$$\mathrm{H}(f_{q(w)-\mathrm{DHS}}) = -[\log C(w) + \frac{1}{2}\log q(w)] + C(w)\sqrt{q(w)}\ \mathrm{I}_{q(w)-\mathrm{DHS}} \tag{14}$$

where $$\mathrm{I}_{q(w)-\mathrm{DHS}} = \int_{-\infty}^{\infty} \frac{\log(e^{\pi\varphi/2} + q(w)e^{-\pi\varphi/2})}{e^{\pi\varphi/2} + q(w)e^{-\pi\varphi/2}}\, dx\ .$$

Proof: The previous results forms in (11), (12), (13) and (14) in this theorem can be directly derived by applying the form (2) of the entropy and the rest of the proof is similar to the proof of theorem 1.

Theorem 3: For the *pq*-DHS variable $X_{pq-\mathrm{DHS}}$ and the *p(w)q(w)*-DHS variable $X_{p(w)q(w)-\mathrm{DHS}}$ the following statements are verified:

$$\mathrm{H}(f_{pq-\mathrm{DHS}}) = \mathrm{H}(f_{qp-\mathrm{DHS}}) \quad \text{and} \quad \mathrm{H}(f_{p(w)q(w)-\mathrm{DHS}}) = \mathrm{H}(f_{q(w)p(w)-\mathrm{DHS}})\ .$$

Proof: These statements are satisfied directly since

$$\mathrm{I}_{pq-\mathrm{DHS}} = \int_{-\infty}^{\infty} \frac{\log(p\,e^{\pi x/2} + q\,e^{-\pi x/2})}{p\,e^{\pi x/2} + q\,e^{-\pi x/2}}\, dx = \mathrm{I}_{qp-\mathrm{DHS}}$$

and

$$\mathrm{I}_{p(w)q(w)-\mathrm{DHS}} = \int_{-\infty}^{\infty} \frac{\log(p(w)\,e^{\pi x/2} + q(w)\,e^{-\pi x/2})}{p(w)\,e^{\pi x/2} + q(w)\,e^{-\pi x/2}}\, dx = \mathrm{I}_{q(w)p(w)-\mathrm{DHS}}$$

are verified computationally.

Theorem 4: For $p = q = 1,\ p(w) = q(w) = 1,\ p = 1,\ q = 1,\ p(w) = 1$ and $q(w) = 1$ in (10), (11), (12), (13), (14) and (15) respectively the HS distribution with pdf $f_{\mathrm{HS}}(x)$ given in (1) is recovered and in this case we find that the entropy

$$\mathrm{H}(f_{(1)(1)-\mathrm{DHS}}) = \mathrm{H}(f_{\mathrm{HS}}) \approx 1.38629436111989$$

of the HS distribution can be obtained computationally.

Proof: The proof of this theorem can be directly achieved computationally.

4. CALCULATION OF ENTROPY FOR SOME SPECIAL DHS DISTRIBUTIONS

In this section we give the corresponding values or forms of some special cases of the *pq*-DHS distribution and the *p(w)q(w)*-DHS distribution according to the parameters p, q, p (w) and q (w). These obtained values or forms of the entropy can easily be worked out with the help of MAPLE or MATHEMATICA. We give the following cases:

1. Case: for $p = q = 1$ and $p(w) = q(w) = 1$, we find that

$$\mathrm{H}(f_{pq-\mathrm{DHS}}) = \mathrm{H}(f_{p(w)q(w)-\mathrm{DHS}}) = 1.38629436111989$$

2. Case: for $p > q > 1$, we find that

$$\mathrm{H}(f_{pq-\mathrm{DHS}}) = \mathrm{H}(f_{(9)(2)-\mathrm{DHS}}) \approx 1.38629436111989\,,$$
$$\mathrm{H}(f_{pq-\mathrm{DHS}}) = \mathrm{H}(f_{(19)(10)-\mathrm{DHS}}) \approx 0.972572797379024$$

3. Case: for $1 > p > q$, we find that

$$\mathrm{H}(f_{pq-\mathrm{DHS}}) = \mathrm{H}(f_{(1/2)(1/9)-\mathrm{DHS}}) \approx 1.05071101239882\,,$$
$$\mathrm{H}(f_{pq-\mathrm{DHS}}) = \mathrm{H}(f_{(1/10)(1/19)-\mathrm{DHS}}) \approx 1.38629436111989$$

4. Case: for $q > p \geq 1$, we find that

$$\mathrm{H}(f_{pq-\mathrm{DHS}}) = \mathrm{H}(f_{(2)(9)-\mathrm{DHS}}) \approx 1.38629436111989,$$

$$\mathrm{H}(f_{pq-\mathrm{DHS}}) = \mathrm{H}(f_{(10)(19)-\mathrm{DHS}}) \approx 0.972572797379024$$

5. Case: for $1 > q > p$, we find that

$$\mathrm{H}(f_{pq-\mathrm{DHS}}) = \mathrm{H}(f_{(1/9)(1/2)-\mathrm{DHS}}) \approx 1.05071101239882,$$

$$\mathrm{H}(f_{pq-\mathrm{DHS}}) = \mathrm{H}(f_{(1/19)(1/10)-\mathrm{DHS}}) \approx 1.38629436111989$$

6. Case: Based on [3], for the $p(w)q(w)$-DHS distribution we consider the case when $p(w) = 1$, $q(w) = \exp(w)$ and $\varphi = \cosh(w)\ x + 3$. In this case, the pdf is given in the following closed form:

$$f_{p(w)q(w)-\mathrm{DHS}}(\varphi; 1, \exp(w)) = \frac{\exp(w/2)}{2\,\mathrm{sech}(w)}\ \mathrm{sech}_{p(w)q(w)}\left(\frac{\pi\,(\cosh(w)\ x + 3)}{2}\right); \quad x, w \in \mathrm{R},$$

Therefore, we can find that,

$$\mathrm{H}(f_{p(w)q(w)-\mathrm{DHS}}) = \mathrm{H}(f_{(1)(\exp(w))-\mathrm{DHS}})$$

$$= \cosh(w)\sqrt{e^{w}}\,(\log(\cosh(w)) + \frac{w}{2})\ \mathrm{I}_{(1)(\exp(w))-\mathrm{DHS}}$$

where

$$\mathrm{I}_{(1)(\exp(w))-\mathrm{DHS}} = \int_{-\infty}^{\infty} \frac{\log(e^{\pi(x\cosh(w)+3)/2} + e^{w-\pi(x\cosh(w)+3)/2})}{e^{\pi(x\cosh(w)+3)/2} + e^{w-\pi(x\cosh(w)+3)/2}}\,d\,x$$

For a fixed real value of the parameter w we can obtain on $\mathrm{I}_{(1)(\exp(w))-\mathrm{DHS}}$ and so we can determine the entropy functional in this case.

5. CONCLUSION

In this paper, we concerned to derive some closed forms of the entropy functional for the deformed hyperbolic secant variables for various cases. These closed forms are studied based on some constructed families of the deformed hyperbolic secant distributions. It is found that these forms of entropy functional depend on the introduced parameters or parametric functions. Samples of the entropy functional of the *pq*-DHS distribution and the *p(w)q*(w)-DHS distribution for some special cases of p and q and some forms of *p(w)* and *q*(w) have been computed and discussed.

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