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On *v*-Separation Axioms

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ABSTRACT

In this paper by using v-open sets we define almost v-normality and mild v-normality also we continue the study of further properties of v-normality. We show that these three axioms are regular open hereditary. We also define the class of almost v-irresolute mappings and show that v-normality is invariant under almost v-irresolute M-v-open continuous surjection.

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***Key words and Phrases:** v-open, semiopen, semipreopen, almost normal, midly normal, M-v-closed,M-v-open, rc-continuous.*

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1. Introduction

In 1967, A. Wilansky has introduced the concept of US spaces. Next, in 1982, S.P. Arya et al have introduced and studied the concept of semi-US spaces and also they made study of s-convergence, sequentially semi-closed sets, sequentially s-compact notions. Recently G.B. Navalagi studied P-Normal Almost-P-Normal, Mildly-P-Normal and Pre-US spaces. *v*-open sets and *v*-continuous mappings were introduced in 2006 and 2008 by V. K. Sharma and S. Balasubramanian et, al. The purpose of this paper is to examine the normality and *v*-US spaces, *v*-convergence, sequentially *v*-compact, sequentially *v*-continuous maps, and sequentially sub *v*-continuous maps in the context of these new concepts. The topological spaces are not assumed to satisfy any separation axioms unless explicitly stated. All notions and symbols which are not defined in this paper may be found in the appropriate references. Throughout the paper X and Y denote Topological spaces on which no separation axioms are assumed explicitly stated.

2. Preliminaries

Definition 2.1:
(i) A subset A of a topological space X is said to be preopen (semiopen, and semipreopen) if $A \subset \text{intcl}A$ (resp. $A \subset \text{clint}A$, and, $A \subset \text{cl int cl}A$).

(ii) $A \subset X$ is said to be *v*-open if there exists a regular open set U such that $U \subset A \subset \text{cl } U$.

(iii) The family of all *v*-open sets of X containing point x is denoted by *v*O(X, x).

Definition 2.2: A function *f* is said to be almost–preirresolute if for each x in X and each pre-neighbourhood V of *f*(x), $(f^{-1}(V))^*$ is a pre-neighborhood of x.

Clearly every preirresolute map [16] is almost preirresolute.

Definition 2.3: A space X is said to be
(i) v-T_1 (v-T_2) v-T_2 if for any $x \neq y$ in X, there exist (disjoint) U; $V \in v O(X)$ such that $x \in U$ and $y \in V$.

(ii) weakly Hausdorff if each point of X is the intersection of regular closed sets of X.

(iii) *v*-normal if for any pair of disjoint closed sets F_1 and F_2, there exist disjoint *v*-open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

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(iv) almost *v*-normal if for each closed set A and each regular closed set B such that $A \cap B = \emptyset$, there exist disjoint *v*-open sets U and V such that $A \subset U$ and $B \subset V$.

(v) mildly *v*-normal[mildly normal] if for every pair of disjoint regular closed sets F_1 and F_2 of X, there exist disjoint *v*-open[open] sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

(vi) weakly regular if for each pair consisting of a regular closed set A and a point x such that $A \cap \{x\} = \emptyset$, there exist disjoint open sets U and V such that $x \in U$ and $A \subset V$.

(vii) weakly p-regular if for each point x and a regular open set U containing {x}, there is a preopen set V such that $x \in V \subset clV \subset U$.

(viii) A subset A of a space X is S-closed relative to X if every cover of A by semiopen sets of X has a finite subfamily whose closures cover A.

(ix) *v*- R_1 iff for x, y ∈ X with *v*cl{x} ≠ *v*cl{y}, there exist disjoint *v*-open sets U and V such that vcl{x} ⊂ U and *v*cl{y} ⊂ V.

(x) US-space if every convergent sequence has exactly one limit point to which it converges.

(xi) pre-US if every sequence $<x_n>$ in X p-converges to a unique point.

(xii) pre-S_1 if it is pre-US and every sequence $<x_n>$ p-converges with subsequence of $<x_n>$ pre-side points.

(xiii) per-S_2 if it is pre-US and every sequence $<x_n>$ in X p-converges which has no pre-side point.

Definition 2.4:
(i) A sequence $<x_n>$ is said to be p-converges to a point x in X, written as $<x_n> \to^p x$, if $<x_n>$ is eventually in every pre open set containing x.

(ii) A point y is a pre-cluster point of sequence $<x_n>$ iff $<x_n>$ is frequently in every preopen set containing x. The set of all pre-cluster points of $<x_n>$ will be denoted by $pcl(x_n)$.

(iii) A point y is pre-side point of a sequence $<x_n>$ if y is a pre-cluster point of $<x_n>$ but no subsequence of $<x_n>$ p-converges to y.

Clearly, if a sequence $<x_n>$ p-converges to a point x of X, then $<x_n>$ converges to x.

Definition 2.5: A set B is said to be sequentially pre closed if every sequence in B p-converges to a point in B.

Definition 2.6: A subset Y of a space X is said to be sequentially p-compact if every sequence in Y has a subsequence which p-converges to point in Y.

Definition 2.7: A function *f* is said to be
(i) sequentially nearly continuous if for each point x in X and each sequence $<x_n>$ in X converging to x, there exists a subsequence $<x_{nk}>$ of $<x_n>$ such that $f(x_{nk}) \to f(x)$.

(ii) sequentially subcontinuous if for each point x in X and each sequence $<x_n>$ in X converging to x, there exists a subsequence $<x_{nk}>$ of $<x_n>$ and a point y ∈ Y such that $f<x_{nk}> \to y$.

(iii) sequentially compact preserving[l] if the image f(K) of every sequentially compact set K is sequentially compact in Y.

(iv) sequentially precontinuous at x ∈ X if $f(x_n)$ p-converges to f(x) whenever $<x_n>$ is a sequence p-converging to x. If f is sequentially *v*-continuous at all x∈X, then f is said to be sequentially precontinuous.

(v) sequentially nearly precontinuous if for each point x ∈ X and each sequence $<x_n>$ in X p-converging to x, there exists a subsequence $<x_{nk}>$ of $<x_n>$ such that $<f(x_{nk})> \to^p f(x)$.

(vi) sequentially sub-precontinuous if for each x∈X and each sequence $<x_n>$ in X p-converging to x, there exists a subsequence $<x_{nk}>$ of $<x_n>$ and a point y ∈ Y such that $<f(x_{nk})> \to^p y$.

(vii) sequentially p-compact preserving if the image f(K) of every sequentially p-compact set K of X is sequentially p-compact in Y

3. *v*-Normal spaces

Definition 3.1: A space X is said to be *v*-normal if for any pair of disjoint closed sets F_1 and F_2, there exist disjoint *v*-open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Note 1: From the above definition we have the following implication diagram.

$\quad\rightarrow\ \rightarrow\ r\alpha\text{-}T_4\ \rightarrow\rightarrow\ v\text{-}T_4\rightarrow\rightarrow$

$\uparrow \qquad\qquad\qquad\qquad\qquad\qquad\qquad \downarrow$

$r\text{-}T_4 \rightarrow \pi\text{-}T_4 \rightarrow T_4 \rightarrow\rightarrow \alpha\text{-}T_4 \rightarrow s\text{-}T_4 \rightarrow \beta\text{-}T_4$

$\qquad\qquad\qquad \downarrow$

$\qquad\qquad\qquad p\text{-}T_4$

Example 1: Let X = {a, b, c} and τ = {∅, {a}, {b, c}, X}. Then X is *v*-normal.

Example 2: Let X = {a, b, c, d} and τ = {∅,{b, d},{a, b, d},{b, c, d}, X}. Then X is *v*-normal and is not normal.

We have the following characterization of *v*-normality.

Theorem 3.1: For a space X the following are equivalent:
(a) X is *v*-normal.
(b) For every pair of open sets U and V whose union is X, there exist *v*-closed sets A and B such that $A\subset U$, $B \subset V$ and $A\cup B = X$.
(c) For every closed set F and every open set G containing F, there exists a *v*-open set U such that $F \subset U \subset vcl(U) \subset G$.

Proof: (a)⇒(b): Let U and V be a pair of open sets in a *v*-normal space X such that $X = U\cup V$. Then X-U,X-V are disjoint closed sets. Since X is *v*-normal there exist disjoint *v*-open sets U_1 and V_1 such that $X\text{-}U \subset U_1$ and $X\text{-}V \subset V_1$. Let $A = X\text{-}U_1$, $B = X - V_1$. Then A and B are *v*-closed sets such that $A \subset U$, $B \subset V$ and $A\cup B = X$.

(b) ⇒(c): Let F be a closed set and G be an open set containing F. Then X-F and G are open sets whose union is X.

Then by (b), there exist *v*-closed sets W_1 and W_2 such that $W_1 \subset X\text{-}F$ and $W_2 \subset G$ and $W_1 \cup W_2 = X$. Then $F\subset X\text{-}W_1$, $X\text{-}G \subset X\text{-}W_2$ and $(X\text{-}W_1)\cap(X\text{-}W_2)= \emptyset$. Let $U= X\text{-}W_1$ and $V= X\text{-}W_2$. Then U and V are disjoint *v*-open sets such that $F\subset U\subset X\text{-}V\subset G$.As X-V is *v*-closed set, we have $vcl(U) \subset X\text{-}V$ and $F\subset U\subset vcl(U)\subset G$.

(c) ⇒ (a): Let F_1 and F_2 be any two disjoint closed sets of X. Put $G = X\text{-}F_2$, then $F_1\cap G = \emptyset$. $F_1 \subset G$ where G is an open set. Then by (c), there exists a *v*-open set U of X such that $F_1 \subset U \subset vcl(U) \subset G$. It follows that $F_2 \subset X\text{-}\ vcl(U) = V$, say, then V is *v*-open and $U \cap V = \emptyset$.Hence F_1 and F_2 are separated by *v*-open sets U and V. Therefore X is *v*-normal.

Theorem 3.2: A regular open subspace of a *v*-normal space is *v*-normal.

Proof: Let Y be a regular open subspace of a *v*-normal space X. Let A and B be disjoint closed subsets of Y. As Y is regular open, A,B are closed sets of X. By *v*-normality of X, there exist disjoint *v*-open sets U and V in X such that $A \subset U$ and $B\subset V$, $U\cap Y$ and $V\cap Y$ are *v*-open in Y such that $A\subset U\cap Y$ and $B\subset V\cap Y$. Hence Y is *v*-normal.

Example 3: Let X= {a, b, c} and τ ={∅, {a},{b},{a, b}, X}. Then X is not *v*-normal and *v*-regular.

Now, we define the following.

Definition 3.2: A function f: X → Y is said to be almost –*v*-irresolute if for each x in X and each *v*-neighborhood V of $f(x)$, $v\text{cl}(f^{-1}(V))$ is a *v*-neighborhood of x.

Clearly every *v*-irresolute map is almost *v*-irresolute.

The Proof of the following lemma is straightforward and hence omitted.

Lemma 3.1: f is almost *v*-irresolute iff $f^{1}(V) \subset v\text{-int}(vcl(f^{1}(V))))$ for every $V\in vO(Y)$.

Now we prove the following.

Lemma 3.2: f is almost *v*-irresolute iff $f(vcl(U)) \subset vcl(f(U))$ for every $U\in vO(X)$.

Proof: Let U∈ *v*GO(X).Suppose y∉ *vcl*(*f*(U)). Then there exists V∈ *v*GO(y) such that $V\cap f(U) = \varnothing$. Hence $f^{-1}(V)\cap U = \varnothing$. Since U∈*v*GO(X), we have v-int($vcl(f^{-1}(V))) \cap vcl(U) = \varnothing$. Then by lemma 3.1, $f^{-1}(V)\cap vcl(U) = \varnothing$ and hence $V\cap f(vcl(U)) = \varnothing$.This implies that y∉*f*(*v*cl(U)).

Conversely, if V∈*v*GO(Y), then W = X- $vcl(f^{-1}(V))$∈ vGO(X). By hypothesis, $f(vcl(W)) \subset vcl(f(W))$ and hence X- v-int($vcl(f^{-1}(V))) = vcl(W) \subset f^{-1}(vcl(f(W))) \subset f^{-1}(vcl[f(X\text{-}f^{-1}(V))]) \subset f^{-1}[vcl(Y\text{-}V)] = f^{-1}(Y\text{-}V) = X\text{-}f^{-1}(V)$. Therefore, $f^{-1}(V) \subset v$-int($vcl(f^{-1}(V))$). By lemma 3.1, *f* is almost *v*-irresolute.

Now we prove the following result on the invariance of *v*-normality.

Theorem 3.3: If *f* is an M-*v*-open continuous almost *v*-irresolute function from a *v*-normal space X onto a space Y, then Y is *v*-normal.

Proof: Let A be a closed subset of Y and B be an open set containing A. Then by continuity of *f*, $f^{-1}(A)$ is closed and $f^{-1}(B)$ is an open set of X such that $f^{-1}(A) \subset f^{-1}(B)$. As X is *v*-normal, there exists a *v*-open set U in X such that $f^{-1}(A) \subset U \subset vcl(U) \subset f^{-1}(B)$. Then $f(f^{-1}(A)) \subset f(U) \subset f(vcl(U)) \subset f(f^{-1}(B))$. Since *f* is M-*v*-open almost *v*-irresolute surjection, we obtain $A \subset f(U) \subset vcl(f(U)) \subset B$. Then again by Theorem 3.1 the space Y is *v*-normal.

Lemma 3.3: A mapping *f* is M-*v*-closed if and only if for each subset B in Y and for each *v*-open set U in X containing $f^{-1}(B)$, there exists a *v*-open set V containing B such that $f^{-1}(V)\subset U$.

Now we prove the following:

Theorem 3.4: If *f* is an M-*v*-closed continuous function from a *v*-normal space onto a space Y, then Y is *v*-normal.

Now in view of lemma 2.2 [19] and lemma 3.3, we prove that the following result.

Theorem 3.5: If *f* is an M-*v*-closed map from a weakly Hausdorff *v*-normal space X onto a space Y such that $f^{-1}(y)$ is S-closed relative to X for each y∈ Y, then Y is v-T_2.

Proof: Let y_1 and y_2 be any two distinct points of Y. Since X is weakly Hausdorff, $f^{-1}(y_1)$ and $f^{-1}(y_2)$ are disjoint closed subsets of X by lemma 2.2 [19]. As X is *v*-normal, there exist disjoint *v*-open sets V_1 and V_2 such that $f^{-1}(y_i) \subset V_i$, for i = 1, 2. Since *f* is M-*v*-closed, there exist *v*-open sets U_1 and U_2 containing y_1 and y_2 such that $f^{-1}(U_i) \subset V_i$ for i = 1, 2. Then it follows that $U_1\cap U_2 = \varnothing$. Hence Y is v-T_2.

4. Almost *v*-normal spaces

Definition 4.1: A space X is said to be almost *v*-normal if for each closed set A and each regular closed set B such that $A\cap B = \varnothing$, there exist disjoint *v*-open sets U and V such that $A\subset U$ and $B\subset V$.

Note 2: From the above definition we have the following implication diagram.

$$\begin{array}{ccccccccccc} & \rightarrow & \rightarrow \text{Al-r}\alpha\text{-}T_4 & \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow & \text{Al-}v\text{-}T_4 \rightarrow & \\ \uparrow & & & & \downarrow & \\ \text{Al-r-}T_4 \rightarrow & \text{Al-}\pi\text{-}T_4 \rightarrow & \text{Al-}T_4 \rightarrow & \text{Al-}\alpha\text{-}T_4 \rightarrow & \text{Al-s-}T_4 \rightarrow & \text{Al-}\beta\text{-}T_4 \\ & \downarrow & & & & \\ & \text{Al-p-}T_4 & & & & \end{array}$$

Example 4: Let X = {a, b, c} and τ = {∅, {a}, {a, b}, {a, c}, X}.Then X is almost *v*-normal and not *v*-normal.

Now, we have characterization of almost *v*-normality in the following.

Theorem 4.1: For a space X the following statements are equivalent:
(a) X is almost *v*-normal
(b) For every pair of sets U and V, one of which is open and the other is regular open whose union is X, there exist *v*-closed sets G and H such that $G\subset U$, $H\subset V$ and $G\cup H = X$.
(c) For every closed set A and every regular open set B containing A, there is a *v*-open set V such that $A \subset V\subset vcl(V) \subset B$.

Proof: (a)⇒(b) :Let U be an open set and V be a regular open set in an almost *v*-normal space X such that $U \cup V = X$. Then (X-U) is closed set and (X-V) is regular closed set with $(X\text{-}U)\cap(X\text{-}V) = \phi$. By almost v-normality of X, there exist disjoint *v*-open sets U_1 and V_1 such that $X\text{-}U \subset U_1$ and $X\text{-}V \subset V_1$. Let G = X- U_1 and H = X-V_1. Then G and H are *v*-closed sets such that $G\subset U$, $H\subset V$ and $G\cup H = X$.

(b) ⇒ (c) and (c) ⇒ (a) are obvious.

One can prove that almost *v*-normality is also regular open hereditary.

Almost *v*-normality does not imply almost *v*-regularity as the following example shows.

However, we observe that every almost *v*-normal R_0 space is almost *v*-regular.

Next, we prove the following.

Theorem 4.2: Every almost regular, *v*-compact space X is almost *v*-normal.

Recall that a function *f*: X→ Y is called rc-continuous if inverse image of regular closed set is regular closed.

Now, we state the invariance of almost *v*-normality in the following.

Theorem 4.3: If *f* is continuous M-*v*-open rc-continuous and almost *v*-irresolute surjection from an almost *v*-normal space X onto a space Y, then Y is almost *v*-normal.

5. Mildly *v*-normal spaces

Definition 5.1: A space X is said to be mildly *v*-normal if for every pair of disjoint regular closed sets F_1 and F_2 of X, there exist disjoint *v*-open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Note 3: From the above definition we have the following implication diagram.

→ → Mild-rα-T_4 → → → → Mild-*v*-T_4 → →
↑ ↓
Mild-r-T_4 → Mild-π-T_4 → Mild-T_4 → Mild-α-T_4 → Mild-s-T_4 → Mild-β-T_4
↓
Mild-p-T_4

We have the following characterization of mild *v*-normality.

Theorem 5.1: For a space X the following are equivalent.
(a) X is mildly *v*-normal.
(b) For every pair of regular open sets U and V whose union is X, there exist *v*-closed sets G and H such that $G \subset U$, $H \subset V$ and $G \cup H = X$.
(c) For any regular closed set A and every regular open set B containing A, there exists a *v*-open set U such that $A \subset U \subset vcl(U) \subset B$.
(d) For every pair of disjoint regular closed sets, there exist *v*-open sets U and V such that $A \subset U$, $B \subset V$ and $vcl(U) \cap vcl(V) = \varnothing$.

This theorem may be proved by using the arguments similar to those of Theorem 4.1.

Also, we observe that mild *v*-normality is regular open hereditary.

We define the following

Definition 5.2: A space X is weakly *v*-regular if for each point x and a regular open set U containing {x}, there is a *v*-open set V such that $x \in V \subset clV \subset U$.

Theorem 5.2: If *f* : X → Y is an M-*v*-open rc-continuous and almost *v*-irresolute function from a mildly *v*-normal space X onto a space Y, then Y is mildly *v*-normal.

Proof: Let A be a regular closed set and B be a regular open set containing A. Then by rc-continuity of *f*, $f^{-1}(A)$ is a regular closed set contained in the regular open set $f^{1}(B)$. Since X is mildly *v*-normal, there exists a *v*-open set V such that $f^{1}(A) \subset V \subset vcl(V) \subset f^{-1}(B)$ by Theorem 5.1. As *f* is M-*v*-open and almost *v*-irresolute surjection, it follows that $f(V) \in vGO(Y)$ and $A \subset f(V) \subset vcl(f(V)) \subset B$. Hence Y is mildly *v*-normal.

Theorem 5.3: If *f*: X → Y is rc-continuous, M-*v*-closed map from a mildly *v*-normal space X onto a space Y, then Y is mildly *v*-normal.

Theorem 5.4: The following are equivalent for a space X:
(a) X is mildly normal;
(b) for any disjoint H, K∈RC(X), there exist disjoint r-open sets U, V such that H⊂U and K⊂V;
(c) for any disjoint H, K∈RC(X), there exist disjoint *v*-open sets U, V such that H⊂U and K⊂V;
(d) for any H∈RC(X) and any V∈RO(X) containing H, there exists an *v*-open set U of X such that H⊂U⊂*v*cl(U)⊂V;
(e) for any H∈RC(X) and any V∈RO(X) containing H, there exists an *v*-open set U of X such that H⊂U⊂*v*cl(U)⊂V;
(f) for any disjoint H, K∈RC(X), there exist disjoint *v*-open sets U, V such that H⊂U and K⊂V.

6. *v*-US spaces:

Definition 6.1:A sequence $<x_n>$ is said to be *v*-converges to a point x of X, written as $<x_n> \to^v x$ if $<x_n>$ is eventually in every *v*-open set containing x.

Clearly, if a sequence $<x_n>$ *v*-converges to a point x of X, then $<x_n>$ *v*-converges to x.

Definition 6.2: A space X is said to be *v*-US if every sequence $<x_n>$ in X *v*-converges to a unique point.

Theorem 6.1: Every *v*-US space is *v*-T_1.

Proof: Let X be *v*-US space. Let x and y be two distinct points of X. Consider the sequence $<x_n>$ where $x_n = x$ for every n. Cleary, $<x_n>$ *v*-converges to x. Also, since x ≠ y and X is *v*-US, $<x_n>$ cannot *v*-converge to y, i.e, there exists a *v*-open set V containing y but not x. Similarly, if we consider the sequence $<y_n>$ where $y_n = y$ for all n, and proceeding as above we get a *v*-open set U containing x but not y. Thus, the space X is *v*-T_1.

Theorem 6.2: Every *v*-T_2 space is *v*-US.

Proof: Let X be *v*-T_2 space and $<x_n>$ be a sequence in X. If possible suppose that $<x_n>$ *v*-converge to two distinct points x and y. That is, $<x_n>$ is eventually in every *v*-open set containing x and also in every *v*-open set containing y. This is contradiction since X is *v*-T_2 space. Hence the space X is *v*-US.

Definition 6.3: A set F is sequentially *v*-closed if every sequence in F *v*-converges to a point in F.

Theorem 6.3: X is *v*-US iff the diagonal set is a sequentially *v*-closed subset of X x X.

Proof: Let X be *v*-US. Let $<x_n, x_n>$ be a sequence in Δ. Then $<x_n>$ is a sequence in X. As X is *v*-US, $<x_n> \to^v x$ for a unique x ∈ X. i.e., if $<x_n>$ *v*-converges to x and y. Thus, x = y. Hence Δ is sequentially *v*-closed set.

Conversely, let Δ be sequentially *v*-closed. Let a sequence $<x_n>$ *v*-converge to x and y. Hence sequence $<x_n, x_n>$ *v*-converges to (x, y). Since Δ is sequentially *v*-closed, (x, y) ∈ Δ which means that x = y implies space X is *v*-US.

Definition 6.4: A subset G of a space X is said to be sequentially *v*-compact if every sequence in G has a subsequence which *v*-converges to a point in G.

Theorem 6.4: In a *v*-US space every sequentially *v*-compact set is sequentially *v*-closed.

Proof: Let X be *v*-US space. Let Y be a sequentially *v*-compact subset of X. Let $<x_n>$ be a sequence in Y. Suppose that $<x_n>$ *v*-converges to a point in X-Y. Let $<x_{np}>$ be subsequence of $<x_n>$ that *v*-converges to a point y ∈ Y since Y is sequentially *v*-compact. Also, let a subsequence $<x_{np}>$ of $<x_n>$ *v*-converge to x ∈ X-Y. Since $<x_{np}>$ is a sequence in the *v*-US space X, x= y. Thus, Y is sequentially *v*-closed set.

Next, we give a hereditary property of *v*-US spaces.

Theorem 6.5: Every regular open subset of a *v*-US space is *v*-US.

Proof: Let X be a *v*-US space and Y ⊂ X be an regular open set. Let $<x_n>$ be a sequence in Y. Suppose that $<x_n>$ *v*-converges to x and y in Y. We shall prove that $<x_n>$ *v*-converges to x and y in X. Let U be any *v*-open subset of X containing x and V be any *v*-open set of X containing y. Then, U∩Y and V∩Y are *v*-open sets in Y. Therefore, $<x_n>$ is eventually in U∩Y and V∩Y and so in U and V. Since X is *v*-US, this implies that x = y. Hence the subspace Y is *v*-US.

Theorem 6.6: A space X is *v*-T_2 iff it is both *v*-R_1 and *v*-US.

Proof: Let X be *v*-T_2 space. Then X is *v*-R_1 and *v*-US by Theorem 6.2.

Conversely, let X be both v-R_1 and v-US space. By Theorem 6.1, X is both v-T_1 and v-R_1 and, it follows that space X is v-T_2.

Definition 6.5: A point y is a v-cluster point of sequence $<x_n>$ iff $<x_n>$ is frequently in every v-open set containing x.

The set of all v-cluster points of $<x_n>$ will be denoted by v-cl(x_n).

Definition 6.6: A point y is v-side point of a sequence $<x_n>$ if y is a v-cluster point of $<x_n>$ but no subsequence of $<x_n>$ v-converges to y.

Now, we define the following.

Definition 6.7: A space X is said to be v-S_1 if it is v-US and every sequence $<x_n>$ v-converges with subsequence of $<x_n>$ v-side points.

Definition 6.8: A space X is said to be v-S_2 if it is v-US and every sequence $<x_n>$ in X v-converges which has no v-side point.

Lemma 6.1: Every v-S_2 space is v-S_1 and Every v-S_1 space is v-US.

Now using the notion of sequentially continuous functions, we define the notion of sequentially v-continuous functions.

Definition 6.9: A function f is said to be sequentially v-continuous at $x \in X$ if $f(x_n)$ v-converges to $f(x)$ whenever $<x_n>$ is a sequence v-converging to x. If f is sequentially v-continuous at all $x \in X$, then f is said to be sequentially v-continuous.

Theorem 6.7: Let f and g be two sequentially v-continuous functions. If Y is v-US, then the set $A = \{x \mid f(x) = g(x)\}$ is sequentially v-closed.

Proof: Let Y be v-US and suppose that there is a sequence $<x_n>$ in A v-converging to $x \in X$. Since f and g are sequentially v-continuous functions, $f(x_n) \to^v f(x)$ and $g(x_n) \to^v g(x)$. Hence $f(x) = g(x)$ and $x \in A$. Therefore, A is sequentially v-closed.

Next, we prove the product theorem for v-US spaces.

Theorem 6.8: Product of arbitrary family of v-US spaces is v-US.

Proof: Let $X = \prod_{\lambda \in \wedge} X_\lambda$ where X_λ is v-US. Let a sequence $<x_n>$ in X v-converges to x (= x_λ) and y (= y_λ). Then the sequence $<x_{n\lambda}>$ v-converges to x_λ and y_λ for all $\lambda \in \wedge$. For suppose there exists a $\mu \in \wedge$ such that $<x_{n\mu}>$ does not v-converges to x_μ. Then there exists a τ_μ-v-open set U_μ containing x_μ such that $<x_{n\mu}>$ is not eventually in U_μ. Consider the set $U = \prod_{\lambda \in \wedge} X_\lambda \times U_\mu$. Then U is a v-open subset of X and $x \in U$. Also, $<x_n>$ is not eventually in U, which contradicts the fact that $<x_n>$ v-converges to x. Thus we get $<x_{n\lambda}>$ v-converges to x_λ and y_λ for all $\lambda \in \wedge$. Since X_λ is v-US for each $\lambda \in \wedge$. Thus x = y. Hence X is v-US.

7. Sequentially sub-v-continuity

In this section we introduce and study the concepts of sequentially sub-v-continuity, sequentially nearly v-continuity and sequentially v-compact preserving functions and study their relations and the property of v-US spaces.

Definition 7.1: A function f is said to be sequentially nearly v-continuous if for each point $x \in X$ and each sequence $<x_n>$ in X v-converging to x, there exists a subsequence $<x_{nk}>$ of $<x_n>$ such that $<f(x_{nk})> \to^v f(x)$.

Definition 7.2: A function f is said to be sequentially sub-v-continuous if for each point $x \in X$ and each sequence $<x_n>$ in X v-converging to x, there exists a subsequence $<x_{nk}>$ of $<x_n>$ and a point $y \in Y$ such that $<f(x_{nk})> \to^v y$.

Definition 7.3: A function f is said to be sequentially v-compact preserving if $f(K)$ is sequentially v-compact in Y for every sequentially v-compact set K of X.

Lemma 7.1: Every function f is sequentially sub-v-continuous if Y is a sequentially v-compact.

Proof: Let $<x_n>$ be a sequence in X v-converging to a point x of X. Then $\{f(x_n)\}$ is a sequence in Y and as Y is sequentially v-compact, there exists a subsequence $\{f(x_{nk})\}$ of $\{f(x_n)\}$ v-converging to a point $y \in Y$. Hence f is sequentially sub-v-continuous.

Theorem 7.1: Every sequentially nearly *v*-continuous function is sequentially *v*-compact preserving.

Proof: Suppose *f* is a sequentially nearly *v*-continuous function and let K be any sequentially *v*-compact subset of X. Let $<y_n>$ be any sequence in *f* (K). Then for each positive integer n, there exists a point $x_n \in$ K such that $f(x_n) = y_n$. Since $<x_n>$ is a sequence in the sequentially *v*-compact set K, there exists a subsequence $<x_{nk}>$ of $<x_n>$ *v*-converging to a point x ∈ K. By hypothesis, *f* is sequentially nearly *v*-continuous and hence there exists a subsequence $<x_j>$ of $<x_{nk}>$ such that $f(x_j) \to^v f(x)$. Thus, there exists a subsequence $<y_j>$ of $<y_n>$ *v*-converging to $f(x) \in f(K)$. This shows that *f* (K) is sequentially *v*-compact set in Y.

Theorem 7.2: Every sequentially *v*-continuous function is sequentially *s*-continuous.

Proof: Let *f* be a sequentially *v*-continuous and $<x_n>$ be a sequence in X which *s*-converges to a point x∈X. Then $<x_n>$ *s*-converges to x. Since *f* is sequentially *s*-continuous, $f(x_n) \to^v f(x)$. But we know that $<x_n>$ *v*-converges to x implies $<x_n>$ *s*-converges to x and hence $f(x_n) \to^s f(x)$ implies *f* is sequentially *v*-continuous.

Note 4: From the above Theorem we have the following implication diagram.

→ → seq.rα.c → seq.*v*.c → →
↑ ↓
seq.r.c → seq.π.c → seq.c → seq.α.c → seq.s.c. → seq.β.c
↓
seq.p.c

Theorem 7.3: Every sequentially *v*-compact preserving function is sequentially sub-*v*-continuous.

Proof: Suppose *f* is a sequentially *v*-compact preserving function. Let x be any point of X and $<x_n>$ any sequence in X *v*-converging to x. We shall denote the set $\{x_n \mid n = 1, 2, 3 \ldots\}$ by A and $K = A \cup \{x\}$. Then K is sequentially *v*-compact since $x_n \to^v x$. By hypothesis, *f* is sequentially *v*-compact preserving and hence *f* (K) is a sequentially *v*-compact set of Y. Since $\{f(x_n)\}$ is a sequence in $f(K)$, there exists a subsequence $\{f(x_{nk})\}$ of $\{f(x_n)\}$ *v*-converging to a point $y \in f(K)$. This implies that *f* is sequentially sub-*v*-continuous.

Theorem 7.4: A function f: X→ Y is sequentially *v*-compact preserving iff $f_{/K}$: K → *f* (K) is sequentially sub-*v*-continuous for each sequentially *v*-compact subset K of X.

Proof: Suppose *f* is a sequentially *v*-compact preserving function. Then $f(K)$ is sequentially *v*-compact set in Y for each sequentially *v*-compact set K of X. Therefore, by Lemma 7.4 above, $f_{/K}$: K→ $f(K)$ is sequentially *v*-continuous function.

Conversely, let K be any sequentially *v*-compact set of X. Let $<y_n>$ be any sequence in *f* (K). Then for each positive integer n, there exists a point $x_n \in K$ such that $f(x_n) = y_n$. Since $<x_n>$ is a sequence in the sequentially *v*-compact set K, there exists a subsequence $<x_{nk}>$ of $<x_n>$ *v*-converging to a point x ∈ K. By hypothesis, $f_{/K}$: K→ *f* (K) is sequentially sub-*v*-continuous and hence there exists a subsequence $<y_{nk}>$ of $<y_n>$ *v*-converging to a point $y \in f(K)$.This implies that *f* (K) is sequentially *v*-compact set in Y. Thus, *f* is sequentially *v*-compact preserving function.

The following corollary gives a sufficient condition for a sequentially sub-*v*-continuous function to be sequentially *v*-compact preserving.

Corollary 7.1: If *f* is sequentially sub-*v*-continuous and $f(K)$ is sequentially *v*-closed set in Y for each sequentially *v*-compact set K of X, then *f* is sequentially *v*-compact preserving function.

Proof: Omitted.

Conclusion

Properties of *v*-normality, Almost *v*-normality, Mildly *v*-normality and *v*-US spaces are discussed in this paper.

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