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Slightly *v*g-continuousfunctions

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Abstract

In this paper we discuss new type of continuous functions called slightly vg-continuous functions; its properties and interrelation with other continuous functions are studied.

Keywords: *slightly continuous functions; slightly semi-continuous functions; slightly pre-continuous; slightly β-continuous functions; slightly γ-continuous functions and slightly ν-continuous functions.*

AMS-classification Numbers: *54C10; 54C08; 54C05*

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1. Introduction

In 1995 T. M. Nour introduced slightly semi-continuous functions. After him T. Noiri and G. I. Chae further studied slightly semi-continuous functions in 2000. T. Noiri individually studied about slightly β–continuous functions in 2001. C. W. Baker introduced slightly precontinuous functions in 2002. Erdal Ekici and M. Caldas studied slightly γ-continuous functions in 2004. Arse Nagli Uresin and others studied slightly δ–continuous functions in 2007. Recently S. Balasubramanian and P. A. S. Vyjayanthi studied slightly *v*-continuous functions in 2011. Inspired with these developments I introduce in this paper slightly *v*g-continuous function and study its basic properties and interrelation with other type of such functions available in the literature. Throughout the paper a space X means a topological space (X, τ).

2. Preliminaries

Definition 2.1: A⊂ X is called

(i) closed if its complement is open.

(ii) rα-open [*v*-open] if ∃ U∈ αO(X)[RO(X)] such that U⊂ A⊂ αcl(U)[U⊂ A⊂cl(U)].

(iii) semi-θ-open if it is the union of semi-regular sets and its complement is semi-θ-closed.

(iv) Regular closed[α-closed; pre-closed; β—closed] if $A = cl\{A^o\}$[resp:$(cl(A^o))^o\subseteq A$; $cl(A^o)\subseteq A$; $cl((cl\{A\}))^o\subseteq A$].

(v) Semi closed [*v*-closed] if its complement if semi open [*v*-open].

(vi) g-closed [rg-closed] if cl A⊆ U whenever A⊆ U and U is open in X.

(vii) sg-closed [gs-closed] if s(cl A) ⊆ U whenever A⊆ U and U is semi-open{open} in X.

(viii) pg-closed [gp-closed; gpr-closed] if pcl(A) ⊆U whenever A⊆U and U is pre-open[open; regular-open] in X.

(ix) αg-closed [gα-closed; rgα-closed; rαg-closed] if αcl(A) ⊆ U whenever A⊆ U and U is open[α-open; rα-open; r-open] in X.

(x) *v*g-closed if *v*cl(A) ⊆ U whenever A⊆ U and U is *v*-open in X.

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Definition 2.2: A function *f*:X→ Y is said to be

(i) continuous [resp**:** nearly-continuous; rα-continuous; *v*-continuous; α-continuous; semi-continuous; β–continuous; pre-continuous] if inverse image of each open set is open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β -open; preopen].

(ii) nearly-irresolute [resp**:** rα-irresolute; *v*-irresolute; α-irresolute; irresolute; β–irresolute; pre-irresolute] if inverse image of each regular-open[resp**:** rα-open; *v*-open; α-open; semi-open; β-open; preopen] set is regular-open[resp**:** rα-open; *v*-open; α-open; semi-open; β-open; preopen].

(iii) almost continuous[resp**:** almost nearly-continuous; almost rα-continuous; almost *v*-continuous; almost α-continuous; almost semi-continuous; almost β–continuous; almost pre-continuous] if for each x in X and each open set (V, *f*(x)), ∃ an open [resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β-open; preopen] set (U, x) such that $f(U) \subset (cl(V))^o$.

(iv) weakly continuous[resp**:** weakly nearly-continuous; weakly rα-continuous; weakly *v*-continuous; weakly α-continuous; weakly semi-continuous; weakly β–continuous; weakly pre-continuous] if for each x in X and each open set (V, *f*(x)), ∃ an open [resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β-open; preopen] set (U, x) such that *f* (U) ⊂ cl(V).

(v) slightly continuous[resp**:** slightly semi-continuous; slightly pre-continuous; slightly β–continuous; slightly γ–continuous; slightly α-continuous; slightly r-continuous; slightly *v*-continuous] at x in X if for each clopen subset V in Y containing *f*(x), ∃ U∈ τ (X)[∃ U∈ SO(X); ∃ U∈ PO(X); ∃ U∈ βO(X); ∃ U∈ γ O(X); ∃ U∈ α O(X); ∃ U∈ RO(X); ∃ U∈ *v* O(X)] containing x such that *f*(U) ⊆ V.

(vi) slightly continuous[resp**:** slightly semi-continuous; slightly pre-continuous; slightly β–continuous; slightly γ-continuous; slightly α-continuous; slightly r-continuous; slightly *v*-continuous] if it is slightly-continuous [resp**:**slightly semi-continuous; slightly pre-continuous; slightly β–continuous; slightly γ-continuous; slightly α-continuous; slightly r-continuous; slightly *v*-continuous] at each x in X.

(vii) almost strongly θ-semi-continuous[resp**:** strongly θ-semi-continuous] if for each x in X and for each V∈ σ(Y, *f*(x)), ∃ U∈ SO(X, x) such that *f*(scl(U)) ⊂ scl(V)[resp**:** *f*(scl(U)) ⊂ V].

Note 1: From the above Definitions we have the following interrelations among the closed sets.

g-closed gs-closed ← ← ←
↓ ↓ ↑
rαg-closed → rgα-closed → rg-closed → ***v*g-closed** ← sg-closed ← βg-closed
↑ ↑ ↑ ↑ ↑
↑ → → rα-closed → → → → *v*-closed → → ↓ ↑ ↑
Regular closed → π-closed → closed → α-closed → semi closed → β-closed
↓ ↓ ↓
πg-closed pre-closed → ω-closed ≠ gα-closed
↓ ↓
gp-closed ← pg-closed rω-closed

Definition 2.3: X is said to be a

(i) compact [resp**:** nearly-compact; rα-compact; *v*-compact; α-compact; semi-compact; β–compact; pre-compact; mildly-compact] space if every open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β-open; preopen; clopen] cover has a finite subcover.

(ii) countably-compact[resp**:** countably-nearly-compact; countably-rα-compact; countably-*v*-compact; countably-α-compact; countably-semi-compact; countably-β–compact; countably-pre-compact; mildly-countably compact] space if every countable open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β-open; preopen; clopen] cover has a finite subcover.

(iii) closed-compact [resp**:** closed-nearly-compact; closed-rα-compact; closed-*v*-compact; closed-α-compact; closed-semi-compact; closed-β–compact; closed-pre-compact] space if every closed [resp**:** regular-closed; rα-closed; *v*-closed; α-closed; semi-closed; β-closed; preclosed] cover has a finite subcover.

(iv) Lindeloff[resp**:** nearly-Lindeloff; rα-Lindeloff; *v*-Lindeloff; α-Lindeloff; semi-Lindeloff; β–Lindeloff; pre-Lindeloff; mildly-Lindeloff] space if every open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β-open; preopen; clopen] cover has a countable subcover.

(v) Extremally disconnected [briefly e.d] if the closure of each open set is open.

Definition 2.4: X is said to be a

(i) T_0[resp: r-T_0; rα-T_0; v-T_0; α-T_0; semi-T_0; β-T_0; pre-T_0; Ultra T_0] space if for each $x \neq y \in X$ $\exists$ $U \in \tau(X)$[resp: rO(X); rαO(X); vO(X); αO(X); SO(X); βO(X); PO(X); CO(X)] containing either x or y.

(ii) T_1[resp: r-T_1; rα-T_1; v-T_1; α-T_1; semi-T_1; β-T_1; pre-T_1; Ultra T_1] space if for each $x \neq y \in X$ $\exists$ U, $V \in \tau(X)$ [resp: rO(X); rαO(X); vO(X); αO(X); SO(X); βO(X); PO(X): CO(X)] such that $x \in U - V$ and $y \in V - U$.

(iii) T_2[resp: r-T_2; rα-T_2; v-T_2; α-T_2; semi-T_2; β-T_2; pre-T_2; Ultra T_2] space if for each $x \neq y \in X$ $\exists$ U, $V \in \tau(X)$ [resp: rO(X); rαO(X); vO(X); αO(X); SO(X); βO(X); PO(X); CO(X)] such that $x \in U$; $y \in V$ and $U \cap V = \phi$.

(iv) C_0[resp: r-C_0; rα-C_0; v-C_0; α-C_0; semi-C_0; β-C_0; pre-C_0; Ultra C_0] space if for each $x \neq y \in X$ $\exists$ $U \in \tau(X)$ [resp: rO(X); rαO(X); vO(X); αO(X); SO(X); βO(X); PO(X); CO(X)]whose closure contains either x or y

(v) C_1[resp: r-C_1; rα-C_1; v-C_1; α-C_1; semi-C_1; β-C_1; pre-C_1; Ultra C_1] space if for each $x \neq y \in X$ $\exists$ U, $V \in \tau(X)$ [resp: rO(X); rαO(X); vO(X); αO(X); SO(X); βO(X); PO(X); CO(X)]whose closure contains x and y.

(vi) C_2[resp: r-C_2; rα-C_2; v-C_2; α-C_2; semi-C_2; β-C_2; pre-C_2; Ultra C_2] space if for each $x \neq y \in X$ $\exists$ disjoint U, $V \in \tau(X)$[resp: rO(X); rαO(X); vO(X); αO(X); SO(X); βO(X); PO(X); CO(X)]whose closure contains x and y.

(vii) D_0[resp: r-D_0; rα-D_0; v-D_0; α-D_0; semi-D_0; β-D_0; pre-D_0; Ultra D_0] space if for each $x \neq y \in X$ $\exists$ $U \in D(X)$ [resp: rD(X); rαD(X); vD(X); αD(X); SD(X); βD(X); PD(X); COD(X)] containing either x or y.

(viii) D_1[resp: r-D_1; rα-D_1; v-D_1; α-D_1; semi-D_1; β-D_1; pre-D_1; Ultra D_1] space if for each $x \neq y \in X$ $\exists$ U, $V \in D(X)$ [resp: rD(X); rαD(X); vD(X); αD(X); SD(X); βD(X); PD(X); COD(X)]such that $x \in U-V$ and $y \in V-U$.

(ix)D_2[resp: r-D_2; rα-D_2; v-D_2; α-D_2; semi-D_2; β-D_2; pre-D_2; Ultra D_2] space if for each $x \neq y \in X$ $\exists$ U, $V \in D(X)$ [resp: rD(X); rαD(X); vD(X); αD(X); SD(X); βD(X); PD(X); CD(X)] such that $x \in U$; $y \in V$ and $U \cap V = \phi$.

(x) R_0[resp: r-R_0; rα-R_0; v-R_0; α-R_0; semi-R_0; β-R_0; pre-R_0; Ultra R_0] space if for each x in X $\exists$ $U \in \tau(X)$[resp: RO(X); rαO(X); vO(X); αO(X); SO(X); βO(X); PO(X); CO(X)]cl{x}$\subseteq$ U[resp: rcl{x}$\subseteq$ U; $v$cl{x}$\subseteq$ U; αcl{x}$\subseteq$ U; scl{x}$\subseteq$ U] whenever $x \in U \in \tau(X)$[resp: $x \in U \in$ RO(X); $x \in U \in v$ O(X); $x \in U \in \alpha$ O(X); $x \in U \in$ SO(X)]

(xi) R_1[resp: r-R_1; rα-R_1; v-R_1; α-R_1; semi-R_1; β-R_1; pre-R_1; Ultra R_1] space if for x,y$\in$ X such that cl{x} $\neq$ cl{y}[resp: such that rcl{x} $\neq$ rcl{y}; such that rαcl{x} $\neq$ rαcl{y}; such that $v$cl{x} $\neq$ $v$cl{y}; such that αcl{x} $\neq$ αcl{y}; such that scl{x} $\neq$ scl{y}; such that βcl{x} $\neq$ βcl{y}; such that pcl{x} $\neq$ pcl{y}; such that COcl{x} $\neq$ COcl{y};] $\exists$ disjoint U; $V \in \tau(X)$ such that cl{x}$\subseteq$ U[resp: RO(X) such that rcl{x}$\subseteq$ U;Rα O(X) such that rαcl{x}$\subseteq$ U; v O(X) such that $v$cl{x}$\subseteq$ U; RO(X) such that αcl{x}$\subseteq$ U; SO(X) such that scl{x}$\subseteq$ U; β O(X) such that βcl{x}$\subseteq$ U; PO(X) such that pcl{x}$\subseteq$ U; CO(X) such that COcl{x}$\subseteq$ U] and cl{y}$\subseteq$ V [resp: RO(X) such that rcl{y}$\subseteq$V;RαO(X) such that rαcl{y}$\subseteq$V; vO(X) such that $v$cl{y}$\subseteq$V; RO(X) such that αcl{y}$\subseteq$V; SO(X) such that scl{y}$\subseteq$V; β O(X) such that βcl{y}$\subseteq$V; PO(X) such that pcl{y}$\subseteq$V; CO(X) such that COcl{y}$\subseteq$V]

Lemma 2.1:

(i) Let A and B be subsets of a space X, if $A \in v$GO(X) and $B \in$ RO(X), then $A \cap B \in v$GO(B).

(ii)Let $A \subset B \subset X$, if $A \in v$GO(B) and $B \in$ RO(X), then $A \in v$GO(X).

3. Slightly *vg*-continuous functions:

Definition 3.0: A function f:X$\to$ Y is said to be

(i) slightly g-continuous[resp: slightly sg-continuous; slightly pg-continuous; slightly βg-continuous; slightly γg-continuous; slightly α g-continuous; slightly rg-continuous] at x in X if for each clopen subset V in Y containing f(x), $\exists$ $U \in$ GO(X)[$\exists$ $U \in$ SGO(X); $\exists$ $U \in$ PGO(X); $\exists$ $U \in \beta$GO(X); $\exists$ $U \in \gamma$GO(X); $\exists$ $U \in \alpha$GO(X); $\exists$ $U \in$ RGO(X)] containing x such that $f(U) \subseteq V$.

(ii) slightly g-continuous[resp: slightly sg-continuous; slightly pg-continuous; slightly β g-continuous; slightly γ g-continuous; slightly α g-continuous; slightly rg-continuous] if it is slightly g-continuous[resp:slightly sg-continuous; slightly pg-continuous; slightly βg-continuous; slightly γg-continuous; slightly αg-continuous; Slightly rg-continuous] at each x in X.

Definition 3.1: A function f**:** X→ Y is said to be

(i) slightly *vg*-continuous function at x in X if for each clopen subset V in Y containing f(x), ∃ U∈ *v*GO(X) containing x such that $f(U) \subseteq V$.

(ii) slightly *vg*-continuous function if it is slightly *vg*-continuous at each x in X.

Note 2: Here after we call slightly *vg*-continuous function as sl.*v* g.c function shortly.

Example 3.1: X = Y = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ, {a}, {b, c}, Y}. Let f**:**X→ Y be identity function, then f is sl.*v*g.c.

Example 3.2: X = Y = {a, b, c}; τ = {ϕ, {a}, {a, b}, X} and σ = {ϕ, {a}, {b, c}, Y}. Let f**:**X→ Y be identity function, then f is not sl.*v*g.c.

Theorem 3.1: The following are equivalent:

(i) f**:** X→ Y is sl.*vg*.c.

(ii) $f^{-1}(V)$ is *vg*-open for every clopen set V in Y.

(iii) $f^{-1}(V)$ is *vg*-closed for every clopen set V in Y.

(iv) $f(v\text{gcl}(A)) \subseteq v\text{gcl}(f(A))$.

Corollary 3.1: The following are equivalent.

(i) f**:**X→ Y is sl.*vg*.c.

(ii) For each x in X and each clopen subset V∈ (Y, f(x)) ∃ U∈ *v*GO(X, x) such that $f(U) \subseteq V$.

Theorem 3.2: Let $\Sigma = \{U_i : i \in I\}$ be any cover of X by regular open sets in X. A function f is sl.*vg*.c. iff $f_{/U_i}$**:** is sl.*vg*.c., for each i∈ I.

Proof: Let i∈ I be an arbitrarily fixed index and $U_i \in$ RO(X). Let $x \in U_i$ and V∈CO(Y, $f_{U_i}(x)$) Since f is sl.*vg*.c,
∃ U∈*v*GO(X, x) such that $f(U) \subset V$. Since $U_i \in$ RO(X), by Lemma 2.1 $x \in U \cap U_i \in v\text{GO}(U_i)$
and $(f_{/U_i})U \cap U_i = f(U \cap U_i) \subset f(U) \subset V$. Hence $f_{/U_i}$ is sl.*vg*.c.

Conversely Let x in X and V∈ CO(Y, f(x)), ∃ i∈ I such that $x \in U_i$. Since $f_{/U_i}$ is sl.*v* g.c, ∃ U∈*v*GO(U_i, x) such that $f_{/U_i}(U) \subset V$. By Lemma 2.1, U∈ *v* GO(X) and $f(U) \subset V$. Hence f is sl.*vg*.c.

Theorem 3.3:

(i) If f**:** X→ Y is *vg*-irresolute and g**:** Y→ Z is sl.*vg*.c.[slightly-continuous], then $g \bullet f$ is sl.*vg*.c.

(ii) If f**:** X→ Y is *vg*-irresolute and g**:** Y→ Z is *vg*.-continuous, then $g \bullet f$ is sl.*vg*.c.

(iii) If f**:** X→ Y is *vg*-continuous and g**:** Y→ Z is slightly-continuous, then $g \bullet f$ is sl.*vg*.c.

(iv) If f**:** X→ Y is rg-continuous and g**:** Y→ Z is sl.*vg*.c. [slightly-continuous], then $g \bullet f$ is sl.*vg*.c.

Theorem 3.4: If f**:**X→ Y is *vg*-irresolute, *vg*-open and *v*GO(X) = τ and g**:**Y→ Z be any function, then $g \bullet f$**:** X→ Z is sl.*vg*.c iff g**:**Y→ Z is sl.*vg*.c.

Proof: If part**:** Theorem 3.3(i)
Only if part**:** Let A be clopen subset of Z. Then $(g \bullet f)^{-1}(A)$ is a *vg*-open subset of X and hence open in X
[by assumption]. Since f is *vg*-open $f\,(g \bullet f)^{-1}(A)$ is *vg*-open Y ⇒ $g^{-1}(A)$ is *vg*-open in Y. Thus g**:**Y→ Z is sl.*vg*.c.

Corollary 3.2: If f**:** X→ Y is *vg*-irresolute, *vg*-open and bijective, g**:** Y→ Z is a function. Then g**:** Y→ Z is sl.*vg*.c. iff $g \bullet f$ is sl.*vg*.c.

Theorem 3.5: If g**:** X → X× Y, defined by $g(x) = (x, f(x))$ for all x in X be the graph function of f**:** X → Y. Then g**:**X→ X×Y is sl.*v* g.c iff f is sl.*v* g.c.

Proof: Let V∈ CO(Y), then X× V is clopen in X× Y. Since g**:**X→ Y is sl.*vg*.c., $f^{-1}(V) = f^{-1}(X \times V) \in v$ GO(X). Thus f is sl.*vg*.c.

Conversely, let x in X and F be a clopen subset of X× Y containing g(x). Then F∩ ({x}× Y) is clopen in {x}× Y containing g(x). Also {x}× Y is homeomorphic to Y. Hence {y∈ Y**:**(x, y) ∈ F} is clopen subset of Y. Since f is sl.*vg*.c., $\cup\{f^{-1}(y):(x, y) \in F\}$ is *vg*-open in X. Further $x \in \cup\{f^{-1}(y):(x, y) \in F\} \subseteq g^{-1}(F)$. Hence $g^{-1}(F)$ is *vg*-open. Thus g**:**X→ Y is sl.*vg*.c.

Theorem 3.6:
(i) If $f: X\rightarrow \Pi Y_\lambda$ is sl.*v* g.c, then $P_\lambda \bullet f : X\rightarrow Y_\lambda$ is sl.*v* g.c for each $\lambda\in\Gamma$, where P_λ is the projection of ΠY_λ onto Y_λ.
(ii) $f: \Pi X_\lambda\rightarrow \Pi Y_\lambda$ is sl.*v* g.c, iff $f_\lambda: X_\lambda\rightarrow Y_\lambda$ is sl.*v* g.c for each $\lambda\in\Gamma$.

Remark:
(i) Composition of two sl.*v* g.c functions is not in general sl.*v*g.c.
(ii) Algebraic sum and product of sl.*v* g.c functions is not in general sl.*v*g.c.
(iii) The pointwise limit of a sequence of sl.*v* g.c functions is not in general sl.*v*g.c.

Example 3.3: Let X = Y = [0, 1]. Let $f_n: X\rightarrow Y$ is defined as follows $f_n(x) = x_n$ for n = 1, 2, 3, . ., then f:X→ Y defined by $f(x) = 0$ if $0 \le x < 1$ and $f(x) = 1$ if x = 1. Therefore each f_n is sl.*v*g.c but f is not sl.*v*g.c. For (1/2, 1] is clopen in Y, but $f^{-1}((1/2, 1]) = \{1\}$ is not *vg*-open in X.

However we can prove the following**:**

Theorem 3.7: The uniform limit of a sequence of sl.*v*g.c functions is sl.*v*g.c.

Note: Pasting Lemma is not true for sl.*v*g.c functions. However we have the following weaker versions.

Theorem 3.8: Let X and Y be topological spaces such that $X = A\cup B$ and let $f_{/A}: A\rightarrow Y$ and $g_{/B}: B \rightarrow Y$ are sl.r.c maps such that $f(x) = g(x)$ for all $x\in A\cap B$. Suppose A and B are r-open sets in X and RO(X) is closed under finite unions, then the combination $\alpha: X\rightarrow Y$ is sl.*v*g.c continuous.

Theorem 3.9: Pasting Lemma Let X and Y be spaces such that $X = A\cup B$ and let $f_{/A}: A\rightarrow Y$ and $g_{/B}: B \rightarrow Y$ are sl.*v*g.c maps such that $f(x) = g(x)$ for all $x\in A\cap B$. Suppose A, B are r-open sets in X and *v*GO(X) is closed under finite unions, then the combination $\alpha: X\rightarrow Y$ is sl.*v*g.c.

Proof: Let $F\in CO(Y)$, then $\alpha^{-1}(F) = f^{-1}(F)\cup g^{-1}(F)$, where $f^{-1}(F)\in v GO(A)$ and $g^{-1}(F)\in vGO(B) \Rightarrow f^{-1}(F)$;
$g^{-1}(F)\in vGO(X) \Rightarrow f^{-1}(F)\cup g^{-1}(F)\in vGO(X)$[by assumption]. Therefore $\alpha^{-1}(F)\in vGO(X)$. Hence $\alpha: X\rightarrow Y$ is sl.*v*g.c.

4. Comparisons:

Theorem 4.1:
(i) If f is sl.rg.c, then f is sl.*vg*.c.
(ii) If f is sl.sg.c, then f is sl.*vg*.c.
(iii) If f is sl.g.c, then f is sl.*vg*.c.
(iv) If f is sl.s.c, then f is sl.*vg*.c.
(v) If f is sl.*v*.c, then f is sl.*vg*.c.
(vi) If f is sl.r.c, then f is sl.*vg*.c.
(vii) If f is sl.c, then f is sl.*vg*.c.
(viii) If f is sl. ω.c, then f is sl.*vg*.c.
(ix) If f is sl.rgα.c, then f is sl.rg.c.
(x) If f is sl. ω-irresolute, then f is sl.*vg*.c.
(xi) If f is sl.rω.c, then f is sl.*vg*.c.
(xii) If f is sl. π.c, then f is sl.*vg*.c.
(xiii) If f is sl. α.c, then f is sl.*vg*.c.
(xiv) If f is sl.gα.c, then f is sl.*vg*.c.

Note 3: By note 1 and from the above Theorem we have the following implication diagram.

```
                              sl.g.c    sl.gs.c     ←
                                ↓          ↓              ↑
sl.rαg.c → sl.rgα.c → sl.rg.c  → sl.vg.c  ←  sl.sg.c ←  sl.βg.c
   ↑            ↑            ↑         ↑              ↑          ↑
   ↑ → → sl.rα.c           →          sl.v.c →     ↓  ↑          ↑
sl.r.c → sl.π.c        → sl.c    →        sl.α.c → sl.s.c. →  sl.β.c
            ↓                ↓                     ↓
         sl.πg.c     ≠   sl.p.c →    sl.ω.c ≠ sl.gα.c
                              ↓          ↓
               sl.gp.c ← sl.pg.c ≠   sl.rω.c
```


Theorem 4.2:
(i) If RαO(X) = RO(X) then *f* is sl.rα.c. iff *f* is sl.r.c.
(ii) If RαO(X) = *v*GO(X) then *f* is sl.rα.c. iff *f* is sl.*vg*.c.
(iii) If *v*GO(X) = RO(X) then *f* is sl.rα.c. iff *f* is sl.*vg*.c.
(iv) If *v*GO(X) = αO(X) then *f* is sl. α.c. iff *f* is sl.*vg*.c.
(v) If *v*GO(X) = SO(X) then *f* is sl.s.c. iff *f* is sl.*vg*.c.
(vi) If *v*GO(X) = βO(X) then *f* is sl.β.c. iff *f* is sl.*vg*.c.

Theorem 4.3: If *f* is sl.*vg*.c., from a discrete space X into a e.d space Y, then *f* is w.s.c.

Corollary 4.1: If *f* is sl.*vg*.c., from a discrete space X into a e.d space Y, then**:**
(i) *f* is w.s.c. (ii) *f* is w.β.c. (iii) *f* is w.p.c.

Theorem 4.4: If *f* is sl.*vg*.c., and X is e.d, then *f* is sl.c.

Proof: Let x in X and V∈ CO(Y, *f*(x)). Since *f* is sl.*vg*.c, ∃ U∈ *v* GO(X, x) such that *f*(U) ⊂ V ⇒ U∈ SR(X, x) such that *f*(U) ⊂ V. Since X is e.d. U∈ CO(X). Hence *f* is sl.c.

Corollary 4.2: If *f* is sl.*vg*.c.,*v*GO(X) = *v*O(X) and X is *v*-$T_{1/2}$ and e.d, then**:**
(i) *f* is sl.c. (ii) *f* is sl. α.c. (iii)*f* is sl.s.c. (iv) *f* is sl.β.c. (v) *f* is sl.p.c.

Theorem 4.5: If *f* is sl.*vg*.c., from a discrete space X into a e.d space Y, then *f* st. θ.s.c.

Proof: Let x in X and V∈ σ (Y, *f*(x)), then scl(V) ⊂ (cl V)^{o}∈ RO(Y). Since Y is e.d, scl(V) ∈ CO(Y). Since *f* is sl.*vg*.c, *f* is sl.s.c, [by Thm 4.1[iv]] ∃ U∈ SO(X, x) such that *f*(scl(U)) ⊂ scl(V), so *f* is a.st. θ.s.c.

Theorem 4.6: If *f* is sl.*vg*.c from a discrete space X into a T_3 space Y, then *f* st. θ.s.c.

Proof: Let x in X and V∈ σ (Y, *f*(x)). Since Y is Ultra regular, ∃ W∈ CO(Y) such that *f*(x) ∈ W⊂ V. Since *f* is sl.*vg*.c, by Thm 4.1(iv) ∃ U∈ SO(X, x) such that *f*(scl(U)) ⊂ W and *f*(scl(U)) ⊂ V. Thus *f* is st. θ.s.c.

Example 4.1: In Example 3.1 above *f* is sl.*v* g.c; sl.sg.c; sl.gs.c; sl.rα.c; sl.*v*.c; sl.s.c. and sl.β.c; but not sl.g.c; sl.rg.c; sl.gr.c; sl.pg.c; sl.gp.c; sl.gpr.c; sl.gα.c; sl. αg.c; sl.rgα.c; sl.r.c; sl.p.c; sl. α.c; and sl.c;

Example 4.2: In Example 3.2 above *f* is sl.rα.c; and sl.gpr.c; but not sl.*v* g.c; sl.sg.c; sl.gs.c; sl.*v*.c; sl.s.c; sl.β.c; sl.g.c; sl.rg.c; sl.gr.c; sl.pg.c; sl.gp.c; sl.gα.c; sl. αg.c; sl.rgα.c; sl.r.c; sl.p.c; sl. α.c; and sl.c;

Remark 4.1: sl.rα.c; sl.gpr.c; and s.c. are independent of sl.*vg*.c..

5. Covering and Separation properties of slightly *v*g continuous functions:

Theorem 5.1: If *f***:**X→ Y is sl.*vg*.c.[resp**:** sl.rg.c] surjection and X is *vg*-compact, then Y is compact.

Proof: Let {G_i**:**i∈ I} be any clopen cover for Y. Then each G_i is clopen in Y and hence each G_i is open in Y. Since *f***:** X→ Y is sl.*vg*.c., $f^{-1}(G_i)$ is *vg*-open in X. Thus {$f^{-1}(G_i)$} forms a *vg*-open cover for X and hence have a finite subcover, since X is *vg*-compact. Since *f* is surjection, Y = *f*(X) = $\cup^n_{i=1} G_i$. Therefore Y is compact.

Corollary 5.1: If *f***:** X→ Y is sl.*v*.c.[resp**:** sl.r.c] surjection and X is *vg*-compact, then Y is compact.

Theorem 5.2: If *f***:** X→ Y is sl.*vg*.c., surjection and X is *vg*-compact[*vg*-lindeloff] then Y is mildly compact[mildly lindeloff].

Proof: Let {U_i**:**i∈ I} be clopen cover for Y. For each x in X, ∃ α_x∈ I such that *f*(x) ∈ $U_{\alpha x}$ and ∃ V_x∈ *v*GO(X, x) such that *f*(V_x)⊂ $U_{\alpha x}$. Since the family {V_i**:**i∈ I} is a cover of X by *v*g-open sets of X, ∃ a finite subset I_0 of I such that X⊂ ∪{V_x**:**x∈ I_0}. Therefore Y⊂ ∪ {*f*(V_x)**:**x∈ I_0}⊂ ∪ {$U_{\alpha x}$**:**x∈ I_0}. Hence Y is mildly compact.

Corollary 5.2:
(i) If *f***:** X→ Y is sl.rg.c[resp**:** sl.*v*.c.; sl.r.c] surjection and X is *vg*-compact[*vg*-lindeloff] then Y is mildly compact[mildly lindeloff].
(ii) If *f***:**X→ Y is sl.*vg*.c.[resp**:** sl.rg.c; sl.*v*.c.; sl.r.c] surjection and X is locally *vg*-compact{resp**:***vg*-Lindeloff; locally *vg*-lindeloff}, then Y is locally compact{resp**:** Lindeloff; locally lindeloff}.

(iii)If f: X→ Y is sl.vg.c., surjection and X is semi-compact[semi-lindeloff] then Y is mildly compact[mildly lindeloff].

(iv) If f: X→ Y is sl.vg.c., surjection and X is β−compact[β−lindeloff] then Y is mildly compact[mildly lindeloff].

(v) If f: X→ Y is sl.vg.c.[sl.r.c.], surjection and X is locally vg-compact{resp: vg-lindeloff; locally vg-lindeloff} then Y is locally mildly compact{resp: locally mildly lindeloff}.

Theorem 5.3: If f:X→ Y is sl.vg.c., surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

Proof: Let $\{V_i : V_i \in CO(Y); i \in I\}$ be a cover of Y, then $\{f^{-1}(V_i) : i \in I\}$ is vg-open cover of X[by Thm 3.1] and so there is finite subset I_0 of I, such that $\{f^{-1}(V_i): i \in I_0\}$ covers X. Therefore $\{V_i : i \in I_0\}$ covers Y since f is surjection.

Hence Y is mildly compact.

Corollary 5.3: If f:X→ Y is sl.rg.c[resp: sl.v.c.; sl.r.c.] surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

Theorem 5.3: If f: X→ Y is sl.vg.c.,[resp: sl.rg.c.; sl.v.c.; sl.r.c.] surjection and X is vg-connected, then Y is connected.
Proof: If Y is disconnected, then Y = A∪ B where A and B are disjoint clopen sets in Y. Since f is sl.vg.c. surjection, $X = f^{-1}(Y) = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ $f^{-1}(B)$ are disjoint vg-open sets in X, which is a contradiction for X is vg-connected. Hence Y is connected.

Corollary 5.4: The inverse image of a disconnected space under a sl.vg.c.,[resp: sl.rg.c.; sl.v.c.; sl.r.c.] surjection is vg-disconnected.

Theorem 5.4: If f: X→ Y is sl.vg.c.sl.vg.c.[resp: sl.rg.c.; sl.v.c.], injection and Y is UT_i, then X is vg_i i = 0, 1, 2.

Proof: Let $x_1 \neq x_2 \in X$. Then $f(x_1) \neq f(x_2) \in Y$ since f is injective. For Y is UT_2 ∃ $V_j \in CO(Y)$ such that $f(x_j) \in V_j$ and $\cap V_j = \phi$ for j = 1,2. By Theorem 3.1, $x_j \in f^{-1}(V_j) \in vGO(X)$ for j = 1,2 and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is vg_2.

Theorem 5.5: If f:X→ Y is sl.vg.c.[resp: sl.rg.c.; sl.v.c.], injection; closed and Y is UT_i, then X is vgg_i i = 3, 4.

Proof:(i) Let x in X and F be disjoint closed subset of X not containing x, then f(x) and f(F) be disjoint closed subset of Y not containing f(x), since f is closed and injection. Since Y is ultraregular, f(x) and f (F) are separated by disjoint clopen sets U and V respectively. Hence $x \in f^{-1}(U)$; $F \subseteq f^{-1}(V)$, $f^{-1}(U)$; $f^{-1}(V) \in vGO(X)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Thus X is vgg_3.

(ii) Let F_j and $f(F_j)$ are disjoint closed subsets of X and Y respectively for j = 1,2, since f is closed and injection. For Y is ultranormal, $f(F_j)$ are separated by disjoint clopen sets V_j respectively for j = 1,2. Hence $F_j \subseteq f^{-1}(V_j)$ and $f^{-1}(V_j) \in vGO(X)$ and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is vgg_4.

Theorem 5.6: If f: X→ Y is sl.vg.c.[resp: sl.rg.c.; sl.v.c.], injection and
(i) Y is UC_i[resp: UD_i] then X is $v\,gC_i$[resp: vgD_i] i = 0, 1, 2.
(ii)Y is UR_i, then X is vg-R_i i = 0, 1.

Theorem 5.7: If f:X→ Y is sl.vg.c.[resp: sl.v.c.; sl.rg.c; sl.r.c] and Y is UT_2, then the graph G(f) of f is vg-closed in the product space X× Y.

Proof: Let $(x_1, x_2) \notin G(f)$ implies $y \neq f(x)$ implies ∃ disjoint V; W∈ CO(Y) such that $f(x) \in V$ and $y \in W$. Since f is sl.vg.c., ∃ U∈ vGO(X) such that x∈ U and $f(U) \subset W$ and $(x, y) \in U \times V \subset X \times Y - G(f)$. Hence G($f$) is vg-closed in X×Y.

Theorem 5.8: If f:X→ Y is sl.vg.c.[resp: sl.v.c.; sl.rg.c; sl.r.c] and Y is UT_2, then $A = \{(x_1, x_2) | f(x_1) = f(x_2)\}$ is vg-closed in the product space X× X.

Proof: If $(x_1, x_2) \in X \times X - A$, then $f(x_1) \neq f(x_2)$ implies ∃ disjoint $V_j \in CO(Y)$ such that $f(x_j) \in V_j$, and since f is sl.vg.c., $f^{-1}(V_j) \in vGO(X, x_j)$ for j = 1,2. Thus $(x_1, x_2) \in f^{-1}(V_1) \times f^{-1}(V_2) \in vGO(X \times X)$ and $f^{-1}(V_1) \times f^{-1}(V_2) \subset X \times X - A$. Hence A is vg-closed.

Theorem 5.9: If f: X→ Y is sl.r.c.[resp: sl.c.]; g: X→ Y is sl.vg.c[resp: sl.rg.c; sl.v.c]; and Y is UT_2, then E = {x in X: f(x) = g(x)} is vg-closed in X.

CONCLUSION

In this paper we defined slightly-*vg*-continuous functions, studied its properties and their interrelations with other types of slightly-continuous functions.

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