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GENERALIZED gIC$_\lambda$-RATE SEQUENCE SPACES OF DIFFERENCE SEQUENCE OF MODAL INTERVAL NUMBERS DEFINED BY A MODULUS FUNCTION

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ABSTRACT

In this paper we introduce and study the concepts of generalized gIC$_\lambda$-Rate sequence spaces of difference sequence of modal interval numbers and prove some inclusion relations.

Keywords: *FK-spaces, Modulus Function, Rate sequence space, Modal Interval Numbers, Difference Sequence Space, C_λ-Summability Method.*

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1. INTRODUCTION

Many mathematical structures have been constructed with real or complex numbers. In recent years, these mathematical structures were replaced by fuzzy numbers or interval numbers and these mathematical structures have been very popular since 1965. Interval arithmetic is a tool in numerical computing where the rules for the arithmetic of intervals are explicity stated. Interval arithmetic was first suggested by P.S.Dwyer[3] in 1951. Development of interval arithmetic as a formal system and evidence of its value as a computational device was provided by R.E.Moore[13] in 1962. Probably the most important paper for the development of interval arithmetic has been published by the Japanese scientist Sunaga [19]. Recently, the sequence spaces of modal interval numbers $gIC^{\lambda}_{c_{0\pi}}(\Delta^m, f, p, q), gIC^{\lambda}_{c_\pi}(\Delta^m, f, p, q)$ and $gIC^{\lambda}_{(\ell_\infty)_\pi}(\Delta^m, f, p, q)$ using a modulus function f and more general C_λ-method in view of Armitage and Maddox [12]. Several properties of these spaces, and some inclusion relation have been examined.

2. PRELIMINARIES

Definition 2.1: A set consisting of a closed interval of real numbers x such that $a \le x \le b$ is called an interval number. A real interval can also be considered as a set. We denote the set of all real valued closed intervals by $I\Re$. Any elements of $I\Re$ is called closed interval and denoted by $\bar{x}$. That is $\bar{x} = [x_l, x_r] = \{x \in \Re : x_l \le x \le x_r\}$. An interval number $\bar{x}$ is a closed subset of real numbers. Let x_l *and* x_r be respectively referred to as the infimum (lower bound) and supremum (upper bound) of theinterval number $\bar{x}$. If $\bar{x} = [0,0]$, then $\bar{x}$ is said to be a zero interval. It is denoted by $\bar{0}$. Chiao [11] introduced sequence of interval numbers and defined usual convergence of sequences of interval number. When $\underline{x} > \bar{x}$, $\hat{x}$ is not an interval number. But in modal analysis, $[\bar{x}, \underline{x}]$ is a valid interval.

Definition 2.2: A modal interval number $\tilde{x} = \{[\underline{x}, \bar{x}] : \underline{x}, \bar{x} \in \Re\}$ is defined by a pair of real numbers $\bar{x}, \underline{x}$ and it is denoted by gI and $|\tilde{x}| = \max\{|\underline{x}|, |\bar{x}|\}$. If $\tilde{x} = [0,0]$, then $\tilde{x}$ is said to be a zero modal interval.

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Definition 2.3: For $\tilde{x}_1, \tilde{x}_2 \in gI$,

$\tilde{x}_1 = [a, a]$ (Degenerate modal interval number)

$\tilde{x}_1 = \tilde{x}_2$ if and only if $\underline{x}_1 = \underline{x}_2$ *and* $\overline{x}_1 = \overline{x}_2$.

$\tilde{x}_1 + \tilde{x}_2 = \{x \in \Re : \underline{x}_1 + \underline{x}_2 \le x \le \overline{x}_1 + \overline{x}_2)\}$

$\tilde{x}_1 \times \tilde{x}_2 = \{x \in \Re : \min(\underline{x}_1 \underline{x}_2, \underline{x}_1 \overline{x}_2, \overline{x}_1 \underline{x}_2, \overline{x}_1 \overline{x}_2) \le x \le \max(\underline{x}_1 \underline{x}_2, \underline{x}_1 \overline{x}_2, \overline{x}_1 \underline{x}_2, \overline{x}_1 \overline{x}_2)\}$

$\tilde{x}_1 / \tilde{x}_2 = \tilde{x}_1 \times \left[\frac{1}{\overline{x}_2}, \frac{1}{\underline{x}_2}\right]$

Definition 2.4: The distance between the two modal interval numbers $\tilde{x}_1, \tilde{x}_2$ is defined by $d(\tilde{x}_1, \tilde{x}_2) = \max\{|\underline{x}_1 - \underline{x}_2|, |\overline{x}_1 - \overline{x}_2|\}$. Clearly d is a metric on gI .

Definition 2.5: Let us define transformation $f : N \to gI, k \to f(k) = \tilde{x}_k$, then $\tilde{x} = (\tilde{x}_k)$ is called sequence of modal interval number. $\tilde{x}_k$ is called the k^{th} term of sequence $\tilde{x} = (\tilde{x}_k)$, $\omega(gI)$ denote the set of all sequence of modal interval number with real terms.

Definition 2.6: Let $\tilde{x} = (\tilde{x}_k) = ([\underline{x}_k, \overline{x}_k]) \in \omega(gI)$ .If $\underline{x}_k = \overline{x}_k$, for all $k \in N$, then the sequence $\tilde{x} = (\tilde{x}_k)$ is called degenerate sequence of modal interval number.

Definition 2.7: A sequence $\tilde{x} = (\tilde{x}_k)$ of modal interval number is said to be convergent to a modal interval number $\tilde{x}_0$ if for each $\varepsilon > 0$ there exists a positive integer k_0 such that $d(\tilde{x}_k, \tilde{x}_0) < \varepsilon$ for all $k \ge k_0$ and we denote it by $\lim_k \tilde{x}_k = \tilde{x}_0$. Equivalently $\lim_k \tilde{x}_k = \tilde{x}_0$ if and only if $\lim_k \underline{x}_k = \underline{x}_0$ *and* $\lim_k \overline{x}_k = \overline{x}_0$.

Definition 2.8: A sequence of modal interval numbers $\tilde{x} = (\tilde{x}_k)$ is said to be bounded if there exist a positive number A such that $d(\tilde{x}_k, \tilde{0}) \le A$ for all $k \in N$.

Definition 2.9: A modulus *f* is a function from [0, ∞) to [0,∞) such that

i) *f*(x) = 0 if and only if *x* = 0,
ii) *f*(x + y) ≤ *f(x)* + *f(y)* for *x*,*y* ≥ 0,
iii) *f* is increasing,
iv) *f* is continuous from the right at 0.

It follows that *f* must be continuous everywhere on [0, ∞). Maddox [12] used a modulus function to construct some sequence spaces. Later on using a modulus different sequence spaces have been studied by Altın and *et al.* [1], *et al.* [5], Nuray and Savas [15], Tripathy and Chandra [20] and many others.

Let $\pi = (\pi_n)$ be a sequence of positive number i.e., $\pi_n > 0, \forall n \in \mathbb{N}$ and *X* is *FK*- space. We shall consider the sets of sequences of modal interval numbers $\tilde{x} = (\tilde{x}_n)$

$$X_\pi(gI) = \left\{\tilde{x} \in \omega : (gI)\left(\frac{\tilde{x}_n}{\pi_n}\right) \in X(gI)\right\}.$$

The set $X_\pi(gI)$ may be considered as *FK*-space. We shall call them as rate spaces of modal interval numbers (see, [17]).

Let F be an infinite subset of ℕ and *F* as the range of a strictly increasing sequence of positive integers, say $F = \{\lambda(n)\}_{n=1}^{\infty}$. The Cesaro submethod C_λ is defined as

$$(C_\lambda \tilde{x})_n = \frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \tilde{x}_k , (n = 1,2,, \ldots)$$

where $\{\tilde{x}_k\}$ is a sequence of a modal interval numbers. Therefore, the C_λ-method yields a subsequence of the Cesaro method C_1 and hence it is regular for any λ. $C_\lambda$is obtained by deleting a set of rows from Cesaro matrix. If $\lambda(n) = n$ is taken, then $C_\lambda = C_1$ is obtained. On a range of sequences

$$\lim_n (C_\lambda \tilde{x})_n = \lim_n (C_1 \tilde{x})_n,$$

We will write $C_\lambda \sim C_1$. The basic properties of C_λ-method can be found in [2] and [18].

Let $p = (p_k)$ be a sequence of positive real numbers with $G = sup_k^{p_k}$ and $D = max(1, 2^{G-1})$. Then it is well known that for all $a_k, b_k \in \mathbb{R}$, the field of real numbers, for all $k \in \mathbb{N}$,

$$|a_k + b_k|^{p_k} \le D(|a_k|^{p_k} + |b_k|^{p_k}) \tag{1}$$

Also for any real μ,

$$\mu^{p_k} \le max(1, \mu^G) \tag{2}$$

Let X(gI) be a sequence space of modal interval numbers. Then X(gI) is called;

i) Solid (or normal) if $(\alpha_k \tilde{x}_k) \in X(gI)$ whenever $(\tilde{x}_k) \in X(gI)$ for all sequences (α_k) of scalars with $|\alpha_k| \le 1$.
ii) Symmetric if $(\tilde{x}_k) \in X(gI)$ implies $(\tilde{x}_{\pi(k)}) \in X(gI)$ where π is a permutation of $\mathbb{N}$,
iii) Sequence algebra if $X(gI)$ is closed under multiplication.

3. MAIN RESULTS

Let f be a modulus function, *X(g*I*)* be a locally convex Hausdorff topological linear space whose topology is determined by a set Q of continuous seminorms q and $p = (p_k)$ be a sequence of positive real numbers. Then defined the following sequence spaces of modal interval numbers

$$gIC^\lambda_{c_{0\pi}}(\Delta^m, f, p, q) = \left\{ \tilde{x} \in \omega(gI) : \frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left[f\left(q\left(\left| \Delta^m \frac{\tilde{x}_k}{\pi_k} \right| \right) \right) \right]^{p_k} \to 0 \; as \; n \to \infty \right\}$$

$$gIC^\lambda_{c_\pi}(\Delta^m, f, p, q) = \left\{ \begin{array}{c} \tilde{x} \in \omega(gI) : \frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left[f\left(q\left(\left| \Delta^m \frac{\tilde{x}_k}{\pi_k} - L \right| \right) \right) \right]^{p_k} \to 0 \\ as \; n \to \infty \; for \; some \; L \end{array} \right\}$$

$$gIC^\lambda_{(\ell_\infty)_\pi}(\Delta^m, f, p, q) = \left\{ \begin{array}{c} \tilde{x} \in \omega(gI) : \sup_n \frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left[f\left(\left(\left| \Delta^m \frac{\tilde{x}_k}{\pi_k} \right| \right) \right) \right]^{p_k} < \infty \\ as \; n \to \infty \end{array} \right\}$$

where $\Delta^0 \left(\frac{\tilde{x}_k}{\pi_k} \right) = \frac{x_k}{\pi_k}, \Delta^m \left(\frac{\tilde{x}_k}{\pi_k} \right) = \left(\Delta^{m-1} \frac{\tilde{x}_k}{\pi_k} - \Delta^{m-1} \frac{\tilde{x}_{k+1}}{\pi_{k+1}} \right)$ and $\Delta^m \left(\frac{\tilde{x}_k}{\pi_k} \right) = \sum_{v=0}^m (-1)^v \binom{m}{v} \frac{\tilde{x}_{k+v}}{\pi_{k+v}}$.

For $f(x) = x$ we shall write $gIC^\lambda_{c_{0\pi}}(\Delta^m, p, q), gIC^\lambda_{c_\pi}(\Delta^m, p, q)$and $gIC^\lambda_{(\ell_\infty)_\pi}(\Delta^m, p, q)$ instead of $gIC^\lambda_{c_{0\pi}}(\Delta^m, f, p, q)$, $gIC^\lambda_{c_\pi}(\Delta^m, f, p, q)$ and $gIC^\lambda_{(\ell_\infty)_\pi}(\Delta^m, f, p, q)$ respectively.

Theorem 3.1: Let $p = (p_k)$ be bounded, then $gIC^\lambda_{c_{0\pi}}(\Delta^m, f, p, q), gIC^\lambda_{c_\pi}(\Delta^m, f, p, q)$ and $gIC^\lambda_{(\ell_\infty)_\pi}(\Delta^m, f, p, q)$ are linear spaces of modal interval numbers.

Theorem 3.2: $gIC^\lambda_{c_{0\pi}}(\Delta^m, f, p, q)$is a paranormed space of modal interval number (*not totally paranormed*), paranormed by

$$g_\Delta(\tilde{x}) = \sup_n \left\{ \frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} f\left[q\left(\left| \Delta^m \frac{\tilde{x}_k}{\pi_k} \right| \right) \right]^{p_k} \right\}^{\frac{1}{M}}$$

where $M = \max(1, G = sup \; p_k)$.

Theorem 3.3: Let f, f_1, f_2 are modulus function and $0 < h = \inf p_k \le \sup_k p_k = G$ then

(i) $gIC^\lambda_{c_{0\pi}}(\Delta^m, f_1, p, q) \subseteq gIC^\lambda_{c_{0\pi}}(\Delta^m, f \circ f_1, p, q)$
(ii) $gIC^\lambda_{c_{0\pi}}(\Delta^m, f_1, p, q) \cap gIC^\lambda_{c_{0\pi}}(\Delta^m, f_2, p, q) \subseteq gIC^\lambda_{c_{0\pi}}(\Delta^m, f_1 + f_2, p, q)$

Proof: (i) Let $\tilde{x} = \left(\frac{\tilde{x}_k}{\pi_k} \right) \in gIC^\lambda_{c_{0\pi}}(\Delta^m, f_1, p, q)$. Let $\varepsilon > 0$ and choose δ with $0 < \delta < 1$ such that $f(t) < \varepsilon$ for $0 \le t \le \delta$.Write $y_k = f_1\left(q\left(\left| \Delta^m \frac{\tilde{x}_k}{\pi_k} \right| \right) \right)$ and consider

$$\sum_{k=1}^{\lambda(n)} [f(y_k)]^{p_k} = \sum_1 [f(y_k)]^{p_k} + \sum_2 [f(y_k)]^{p_k}$$

where the first summation is over $y_k \le \delta$ and the second over $y_k > \delta$. Since f is continuous, we get

$$\sum_1 [f(y_k)]^{p_k} < \lambda(n) \max(\varepsilon^h, \varepsilon^G) \tag{3}$$

and for $y_k > \delta$ we use the fact that

$$y_k < \frac{y_k}{\delta} < 1 + \frac{y_k}{\delta}.$$

By the definition of f, we have $y_k > \delta$,

$$f(y_k) \leq f(1)\left[1 + \left(\frac{y_k}{\delta}\right)\right] \leq 2f(1)\frac{y_k}{\delta}.$$

Hence

$$\frac{1}{\lambda(n)}\sum\nolimits_2 [f(y_k)]^{p_k} < \max\left(1, \left(\frac{2f(1)}{\delta}\right)^G\right)\frac{1}{\lambda(n)}\sum\nolimits_{k=1}^{\lambda(n)}[y_k]^{p_k} \to 0 \tag{4}$$

By (3) and (4) we have $gIC_{c_{0\pi}}^{\lambda}(\Delta^m, f_1, p, q) \subseteq gIC_{c_{0\pi}}^{\lambda}(\Delta^m, f \circ f_1, p, q)$.

(ii) Let $\tilde{x} = \left(\frac{\tilde{x}_k}{\pi_k}\right) \in gIC_{c_{0\pi}}^{\lambda}(\Delta^m, f_1, p, q) \cap gIC_{c_{0\pi}}^{\lambda}(\Delta^m, f_2, p, q)$. Then there exist f_1 and f_2 such that

$$\frac{1}{\lambda(n)}\sum\nolimits_{k=1}^{\lambda(n)}\left[f_1\left(q\left(\left|\Delta^m\frac{\tilde{x}_k}{\pi_k}\right|\right)\right)\right]^{p_k} \to 0 \ as\ n \to \infty \tag{5}$$

$$\frac{1}{\lambda(n)}\sum\nolimits_{k=1}^{\lambda(n)}\left[f_2\left(q\left(\left|\Delta^m\frac{\tilde{x}_k}{\pi_k}\right|\right)\right)\right]^{p_k} \to 0 \ as\ n \to \infty \tag{6}$$

Then using (i) it can be shown that

$$\begin{aligned}\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left[(f_1+f_2)\left(q\left(\left|\Delta^m\frac{\tilde{x}_k}{\pi_k}\right|\right)\right)\right]^{p_k} &= \frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left[f_1\left(q\left(\left|\Delta^m\frac{\tilde{x}_k}{\pi_k}\right|\right)\right)\right]^{p_k} + \frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left[f_2\left(q\left(\left|\Delta^m\frac{\tilde{x}_k}{\pi_k}\right|\right)\right)\right]^{p_k}\\ &\leq D\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left[f_1\left(q\left(\left|\Delta^m\frac{\tilde{x}_k}{\pi_k}\right|\right)\right)\right]^{p_k} + D\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left[f_2\left(q\left(\left|\Delta^m\frac{\tilde{x}_k}{\pi_k}\right|\right)\right)\right]^{p_k}\\ &\to 0 \ as\ n \to \infty\ .\end{aligned}$$

Therefore $\tilde{x} = \left(\frac{\tilde{x}_k}{\pi_k}\right) \in gIC_{c_{0\pi}}^{\lambda}(\Delta^m, f_1 + f_2, p, q)$.

Hence, $gIC_{c_{0\pi}}^{\lambda}(\Delta^m, f_1, p, q) \cap gIC_{c_{0\pi}}^{\lambda}(\Delta^m, f_2, p, q) \subseteq gIC_{c_{0\pi}}^{\lambda}(\Delta^m, f_1 + f_2, p, q)$.

The proof of the following result is a routine work of the above theorem.

Corollary 3.4: Let f, f_1, f_2 are modulus function then

(i) $gIC_{c_\pi}^{\lambda}(\Delta^m, f_1, p, q) \subseteq gIC_{c_\pi}^{\lambda}(\Delta^m, f \circ f_1, p, q)$,

(ii) $gIC_{c_\pi}^{\lambda}(\Delta^m, f_1, p, q) \cap gIC_{c_\pi}^{\lambda}(\Delta^m, f_2, p, q) \subseteq gIC_{c_\pi}^{\lambda}(\Delta^m, f_1 + f_2, p, q)$,

(iii) $gIC_{(\ell_\infty)_\pi}^{\lambda}(\Delta^m, f_1, p, q) \subseteq gIC_{(\ell_\infty)_\pi}^{\lambda}(\Delta^m, f \circ f_1, p, q)$,

(iv) $gIC_{(\ell_\infty)_\pi}^{\lambda}(\Delta^m, f_1, p, q) \cap gIC_{(\ell_\infty)_\pi}^{\lambda}(\Delta^m, f_2, p, q) \subseteq gIC_{(\ell_\infty)_\pi}^{\lambda}(\Delta^m, f_1 + f_2, p, q)$.

Proposition 3.5: $gIC_{c_\pi}^{\lambda}(\Delta^{m-1}, f, p, q) \subseteq gIC_{c_\pi}^{\lambda}(\Delta^m, f, p, q)$.

Theorem 3.6: Let $m \geq 1$, then the following inclusion are strict

(i) $gIC_{c_{0\pi}}^{\lambda}(\Delta^{m-1}, f, q) \subseteq gIC_{c_{0\pi}}^{\lambda}(\Delta^m, f, q)$,

(ii) $gIC_{c_\pi}^{\lambda}(\Delta^{m-1}, f, q) \subseteq gIC_{c_\pi}^{\lambda}(\Delta^m, f, q)$,

(iii) $gIC_{(\ell_\infty)_\pi}^{\lambda}(\Delta^{m-1}, f, q) \subseteq gIC_{(\ell_\infty)_\pi}^{\lambda}(\Delta^m, f, q)$.

Example 3.7: Let $f(x) = x$ and $q(x) = |x|$. Consider the sequences $(\tilde{x}_k) = ([0, k^{m+\alpha}])$ and $(\pi^k) = (k^{\alpha+1})$, where $\tilde{x} = \left(\frac{\tilde{x}_k}{\pi_k}\right)$ and $m \in \mathbb{N}, \alpha \in \mathbb{R}$. Then $\tilde{x} \in gIC_{c_{0\pi}}^{\lambda}(\Delta^m, f, q)$ but $\tilde{x} \notin gIC_{c_{0\pi}}^{\lambda}(\Delta^{m-1}, f, q)$, since $\Delta^m\frac{\tilde{x}_k}{\pi_k} = 0$, $\Delta^{m-1}\frac{\tilde{x}_k}{\pi_k} = [0, (-1)^{m-1}(m-1)!]\ \forall k \in \mathbb{N}$.

Theorem 3.8: For any two sequence $p = (p_k)$ and $t = (t_k)$ of positive real numbers and any two seminorms q_1, q_2 we have

(i) $gIC_{c_{0\pi}}^{\lambda}(\Delta^m, f, p, q_1) \cap gIC_{c_{0\pi}}^{\lambda}(\Delta^m, f, p, q_2) \neq \emptyset$,

(ii) $gIC_{c_\pi}^{\lambda}(\Delta^m, f, p, q_1) \cap gIC_{c_\pi}^{\lambda}(\Delta^m, f, p, q_2) \neq \emptyset$,

(iii) $gIC_{(\ell_\infty)_\pi}^{\lambda}(\Delta^m, f, p, q_1) \cap gIC_{(\ell_\infty)_\pi}^{\lambda}(\Delta^m, f, p, q_2) \neq \emptyset$.

Proof: Since the zero element belongs to each of the above classes of sequences, thus the intersection is nonempty.

The following result is a consequence of Theorem 3.3 (i) and Corollary 3.4 (i) and (iii).

Theorem 3.9: Let *f* be a modulus function. Then

(i) $gIC^{\lambda}_{c_{0\pi}}(\Delta^m, p, q) \subseteq gIC^{\lambda}_{c_{0\pi}}(\Delta^m, f, p, q)$,
(ii) $gIC^{\lambda}_{c_{\pi}}(\Delta^m, p, q) \subseteq gIC^{\lambda}_{c_{\pi}}(\Delta^m, f, p, q)$,
(iii) $gIC^{\lambda}_{(\ell_\infty)_\pi}(\Delta^m, p, q) \subseteq gIC^{\lambda}_{(\ell_\infty)_\pi}(\Delta^m, f, p, q)$.

Theorem 3.10: The sequence spaces $gIC^{\lambda}_{c_{0\pi}}(\Delta^m, f, p, q), gIC^{\lambda}_{c_{\pi}}(\Delta^m, f, p, q)$ and $gIC^{\lambda}_{(\ell_\infty)_\pi}(\Delta^m, f, p, q)$ are neither solid nor symmetric, nor sequence algebras for m$\geq$ 1.

Proof: Let m = 1, p_k= 1 for all k$\in \mathbb{N}, f(x) = x \; and \; q(x) = |x|$. If the sequences $(\tilde{x}_k) = ([0, k^{n+1}])$ and $(\pi^k) = (k^n)$ are taken, then the sequence $\left(\frac{\tilde{x}_k}{\pi_k}\right)$ belongs to $gIC^{\lambda}_{(\ell_\infty)_\pi}(\Delta)$ and $gIC^{\lambda}_{c_\pi}(\Delta)$ where n$\in \mathbb{R}$. Let $\alpha_k = (-1)^k$, then $(\alpha_k \tilde{x})$ does not belong to $gIC^{\lambda}_{(\ell_\infty)_\pi}(\Delta)$ and $gIC^{\lambda}_{c_\pi}(\Delta)$. Hence $gIC^{\lambda}_{c_\pi}(\Delta^m, f, p, q)$ and $gIC^{\lambda}_{(\ell_\infty)_\pi}(\Delta^m, f, p, q)$ are not solid. The other cases can be proved on considering similar examples.

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