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MATHEMATICAL MODELING FOR YES-NO VOTING IN POLITICAL SCIENCE

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ABSTRACT

The purpose of this paper is to define and study the concepts of political science in the context of mathematics. Some topics about various political voting systems have been proved using mathematics. Also limitations of these voting systems have been highlighted.

First I have introduced so called Yes-No voting system. Real world examples of Yes-No voting system have been given In particular. I define swap robust and weighted voting systems. I prove that every weighted voting system is swap robust but converse is not true. I see that some real world voting systems are not weighted. The concept of trade robust voting system is defined and it is proved that Voting system is trade robust if it is weighted. The concepts of equally desirable and more desirable elements in a Yes-No voting system have been defined and illustrated.

Keywords: *Yes-No voting system, swap robust voting system, weighted voting system, trade robust voting system.*

INTRODUCTION

Mathematics has found many areas of applications in science and social science. One such areal is political science. Elementary ideas of mathematics makes it possible to make the entities in political science more useful two such entities are voting system and political power. The ideas of mathematics make it possible to find various possibilities for analysis. It also shows the limitations of mathematics.

I believe that the purpose of highlighting the role of mathematics in political science has been fulfilled. There are many other aspects which can be considered.

YES – NO VOTING

INTRODUCTION

This voting systems in which a single alternative, such as a bill or an amendment, is pitted against the status quo. In these systems, each voter responds with a vote "yea" or "nay". A yes-no voting system is simply set of rules that specifies exactly which collections of "yea" votes yield passage of the issue at hand.

DEFINITION:

1) **YES-NO VOTING:** A YES-NO VOTING system is system in which voters vote either in favour of the bill or against the bill. A criteria is fixed whenever this criteria is satisfied bill is passed.
2) **Coalition:** In a Yes-No Voting System any collection of voters is called a coalition.
3) **Winning & Losing Coalition:** A coalition is said to be winning if passage is Guaranteed by "Yea" votes from exactly the voters in that coalition. Coalitions that are not winning are called losing.
4) **Weighted system:** A weighted system is voting system in which each voter has been assign a weight and also there is a quota such that a coalition is a winning coalition if and only if the sum of the weight of the members of coalition is equal to the quota or more than the quota.
5) **Swap-Robust:** Consider a Yes-No Voting System A and B be two winning coalition. If one element of A is sent to B and one element from B sent to A and if at least one of the resulting coalition is a winning coalition then we say that this Yes-No Voting System is Swap-Robust.

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6) **Trade Robust:** A Yes-No Voting system is said to be Trade Robust if there are winning coalition X1, X2...Xm and if there is arbitrary exchange of voters, among them, then at least one of the new coalitions is a winning coalition.
7) **Two-trade Robust:** A Yes-No Voting System is said to be Two-trade Robust if whenever, C1 & C2 are winning coalitions and if there is arbitrary exchange players between C1 & C2 then at least one of the new coalition is a winning coalitions.

EXAMPLE:

(1) The European Economic Community

In 1958, the treaty of Rome established the existence of a Yes-No Voting system called the European Economic Community. The voters in this system were the following six countries; France, Belgium, Germany, Netherlands, Italy, Luxembourg.

- France, Germany & Italy were given four votes each while Belgium and the Netherlands were given two votes and Luxembourg one.
- Passage required a total of at least twelve of the seventeen votes.

(2) The United Nation security Council:

- There are fifteen members in this council. Among these there are five permanent members there are United States, England, China, France, and Russia.
- To pass a bill nine votes out of fifteen are required if any nation among the permanent members issues a veto then the bill can- not be pass.

(3) The United States Federal system:

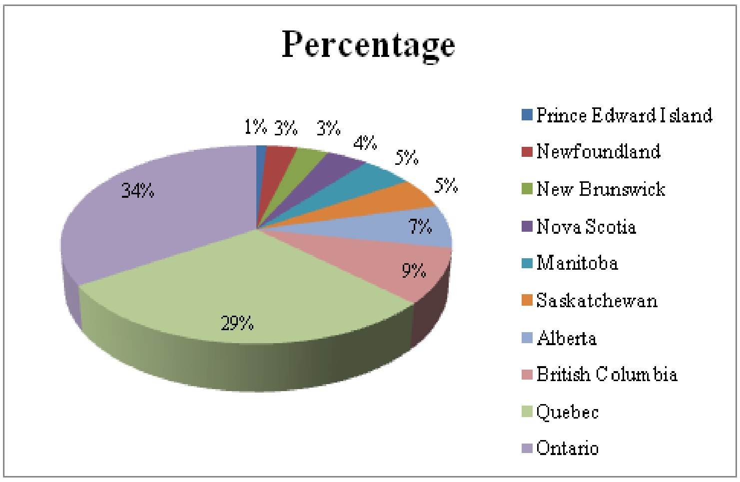
There are 537 voters in this Yes-No Voting System; 435 members of the House of Representatives, 100 members of the senate the Vice President and the President. The Vice President plays the role of tie-breaker in the senate and the President has veto power that can be overridden by a two-third vote of both the House and the senate. Thus, for a bill, it must be supported by either:

- 218 or more representatives and 51 or more senators (with or without the vice president) and the president.
- 218 or more representatives and 50 senators and the vice president and the president.
- 290 or more representatives and 67 or more senators (with or without the vice president) and the president.

(4) The System to Amend the Canadian Constitution:

Since 1982, an amendment to the Canadian constitution becomes law only if it approved by at least seven of the ten Canadian provinces subject to the proviso that the approving provinces have, among them, at least half of Canada's population. For our purposes, it will suffice to work with the following population percentages for the ten Canadian provinces:

Prince Edward Island (1%)
Newfoundland (3%)
New Brunswick (3%)
Nova Scotia (4%)
Manitoba (5%)
Saskatchewan (5%)
Alberta (7%)
British Columbia (9%)
Quebec (29%)
Ontario (34%)



EXAMPLE S OF WINNING COALITION

The coalition made up of France, Germany and Italy is a winning coalition, as is the coalition made up of France, Germany, Italy and Belgium. For most yes-No Voting Systems adding extra voters to a winning coalition again yields a winning coalition. Systems with this property are said to be monotone.

The above four examples of Yes-No voting systems suggest there are at least three distinct ways in which a Yes-No Voting System can be described.

(1) One can specify the number of votes each player has and how many votes are needed for passage. This is what was done for the European Economic Community. If we start with a set of voters, then we can construct a yes-no voting system by assigning real number "weights "to the voters and then set any real number q as the "quota." A coalition is winning when the sum of the weights of the voters is equal or exceeds the quota.

(2) One can explicitly list the winning coalitions, or, if the system is monotone, just the minimal winning coalitions. This is given in U.S. federal system, since the three cases given there describe the three kinds of winning coalitions in the U.S. federal system.

(3) One can use some combination of the above two, with provisos that often involve veto power. Both the U.N. Security Council and the procedure to amend the Canadian constitution are described in this way. Moreover, the description of the U.S.federal system in terms of the tie-breaking vote of the vice president, the presidential veto, and the Congressional override of this veto is another example of a description mixing weights with provisos and vetoes.

Now we prove that Security Council voting system is a weight system;

Let us assign weight '1' to each non-permanent member of the Security Council. Also let us assign weight 'x' to each permanent member of the Security Council. Let "q" be the quota is to be seceded if we take four permanent members and 10 Non-permanent members. Their total weight will be 4x+10. This coalition cannot be a winning coalition therefore $4x+10<q$.

If we take five permanent members and four non-permanent members then it is a winning coalition;

.'. $q \le 5x + 4$

.'. $4x + 10 < 5x + 4 \Rightarrow 6 < x$

Suppose we assign weight 7 to each permanent member and weight one for non-permanent members. If we assume that quota is 7 then this is a weighted voting system.

PROPOSITION: The U.N. security Council is a weighted voting system.

Proof: Now for $6 < x$, taking $x = 7$ from the definition and put $x = 7$ in $5x + 4$ so the quota be 39. Assign weight 7 to each permanent member and weight 1 to each non-permanent member. Let the quota be 39. We must now show that each winning coalition in the U.N. Security Council has weight at least 39, and that each losing coalition has weight at most38.

A winning coalition in the U.N. Security Council must contain all five permanent members (a total weight of 35) and at least four non-permanent members (an additional weight of 4). Hence any winning coalition meets or exceeds the quota of 39. A losing coalition on the other hand either omits a permanent member and thus, has weight at most;

$(7*4) + (1*10) = 28 + 10 = 38$

Or contains at most three non-permanent members and thus has weight at most;

$(7*5) + (1*3) = 35 + 3 = 38$.

Hence any losing coalition tall short of the quota of 39. This completes the proof.

SWAP ROBUSTNESS AND THE NON-WEIGHTEDNESS OF THE FEDERAL SYSTEM:

EXAMPLE:

Consider the e.g. of European Economic Community Council. It is a Weight Voting System there are France, Germany, Italy have been assign weight '4' to each of them Belgium and Netherlands have been assign weight '2' to each of them and Luxembourg has weight '1' quota q = 12.

A = {France, Germany, Italy}

B = {France, Germany, Belgium & Netherlands}

Let A' = {France, Germany, Belgium}

B' = {France, Germany, Belgium & Netherlands}

Note that A' is a losing coalition and B' is a winning coalition.

PROPOSITION: Every weighted Voting System is Swap Robust.

Proof: Let A and B be winning coalition in a Weight Voting System. Let a∈A, b∈B and suppose a & b are exchanged between A' & B'.

Let A' = (A-{a}) U {b}, B' = (B-{b}) U {a}

If weight of a = weight of b then the total weight of A' = total weight of A.
.'. A' is a winning coalition and B' is also a winning coalition. If weight of a > weight of b then, total weight of A' is < total weight of A and total weight of B' > total weight of B.

.'. B' is a winning coalition, similarly it can be proved that a' is a winning coalition,

if weight of a < weight of b. The above proposition provides a condition under which a Yes-no Voting System Cannot be weight voting system. If it can be proved that the given voting system is not Swap Robust then from the above proposition. It is proved that it is not Weighted Voting System.

Remark: It may be noted that if a voting system is Swap Robust Voting System then it may not be Weight Voting System.

Note that:

The Procedure to Amend Canadian Constitution is Swap Robust. Later on we will prove that it is not Weight Voting System. First we introduce the concept of trade Robustness.

It may be noted that there is difference between Trade Robustness and Swap Robustness ;

(1) The no. of voters which are exchange may not be one-one among the coalitions.
(2) The no. of coalitions involved in the exchange more than two

PROPOSITION: Every Weighted Voting System is Trade Robust.

Proof: Let C_1, $C_2 \ldots C_n$ be any collection of winning coalition in a Weight Voting System.Let W denote the weight f ^{n} then W (C_1) +W (C_2) +…+W (C_n) = P the average weight of a Coalition P/n.Suppose q is the quota therefore every i, W (i) ≥ q. Suppose the new coalitions after arbitrary exchange of voters are $C_{1'}$, $C_{2'} \ldots C_n$'. Note that, W ($C_{1'}$) +W ($C_{2'}$) +…+W ($C_{n'}$) =P.
.'. The average weight of the new coalition = P/n.

Note that:
q ≤ P/n the average of the new coalition is also P/n.
.'. There exist a new coalition C_j' such that W ($C_{j'}$)≥ P/n. ≥ q. So, C_j' is a winning coalition.

Note that:
The converse of the above then is also true. I.e. Every Trade Robust is a Weight Voting system.

EXAMPLE: There exists a Yes-no Voting System with nine voters that is Two-trade robust but not Trade Robust and thus not weighted.

Proof: Take the nine numbers. It is called "Magic Square" since the sum of every row and every column (as well as the diagonals) is the same (15).

4	3	8
9	5	1
2	7	6

Here the voters will be the nine numbers 1, 2 …9. Every coalition with four or more voters will be winning and every coalition with two or fewer voters will be losing. Far coalitions with exactly three voters the ones with sum greater than 15 will be winning and the ones with sum less than 15 will be losing. Now the only coalitions with exactly three Voters and sum exactly equal to 15 are rows the columns and the two diagonals. Here the rows to be winning and the columns and diagonals to be losing. This Yes-No Voting System is Two-trade Robust but not trade Robust, and thus not weighted.

To see that the system is Two-trade Robust, suppose that we have two winning coalitions. X & Y and a trade between them X' & Y', we must show that either X' or Y' is still a winning coalition we consider three cases:

Case-1: X or Y has four or more voters.
W.L.O.G. assumes it is X since Y is winning; Y has at least three voters. After the trade, the total number if voters is unchanged and so at least one of X' & Y' has four or more voters and is thus winning.

Case-2: X or Y has sum strictly greater than 15.
W.L.O.G. assumes it is X, since Y is winning; Y has sum at least 15. It is easy to see that the sum of X and Y is the same as the sum of X' and Y', thus in this case either X' has sum strictly greater than 15 or Y' has sum strictly greater than 15.

Case-3: X and Y have sum exactly equal to 15 and each contains exactly three voters.
Let X and Y must both be rows. If either X' or Y, has sum less than 15 then the other will have sum greater than 15 and so it will be a winning coalition, similarly if either X' or Y' has less than three voters then the other will have more than three and so it will be winning on the hand, if both X' and Y' have sum exactly equal to 15 and exactly three voters then the only way they could both be losing coalitions is if both were columns and/or diagonals. But one cannot convert two rows into two columns and/or diagonals by a Trade Robust.

To see that the system is not weighted it suffices to show that it is not trade Robust (by a than).For this, we start with the three rows R1, R2, &R3 which are winning coalitions.

Thus $R_1 = \{4, 3, 8\}$
$R_2 = \{9, 5, 1\}$
$R_3 = \{2, 7, 6\}$

Now consider the trade that send
3 from R_1 to R_2
8 from R_1 to R_3
1 from R_2 to R_3
9 from R_2 to R_1
2 from R_3 to R_1
7 from R_3 to R_2

These trades transform
R_1 into {4, 9, 2} which is the first column
R_2 into {3, 5, 7} which is the second column
R_3 into {8, 1, 6} which is the third column

Since the three columns are losing these shows that the system is not trade robust so it is not weighted.

CONCLUSION

In political science by using mathematical modeling of Yes-No voting system I concluded following interesting results
(1) The U.N. security council is weighted voting system
(2) Every weighted voting system is swap robust and Trade robust

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