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Pre* regular generalized closed sets in Topological spaces

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ABSTRACT

*In this paper, we introduce a new class of sets namely, p*rg-closed sets in topological spaces. Also we study some of its basic properties and investigate the relationship with the other existing closed sets in topological space. It has been proved that the class of pre* regular generalized closed set lies between the class of regular closed set and g-closed set.*

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Keywords: *Regular closed, pre* closed, pre* regular generalised closed sets.*

1. INTRODUCTION

In 1937, Stone.M [17] introduced regular open subsets of a topological spaces. In [8] he also defined the concept of generalized closed set. In 1982, Mashhour *et al.* [13], defined the concept of pre closed. S.P. Arya and N. Tour [4] defined gs-closed sets in 1990. Gnanambal and Palaniappan [7] and Rao [6] introduced gsp-closed sets, gpr-closed sets respectively. In 1993, Palaniappan and Chandra SekranRao [15] introduced rg-closed sets. In 1996, H. Maki, J.Umehara and T. Noiri [11] introduced the class of pre generalized closed sets Quite recently, Selvi *et al.* [16], introduced pre* closed sets.

In this paper, the concept of pre* regular generalized closed sets is to be introduced and studied some of their properties. It has been proved that the class of pre* regular generalized closed set lies between the class of regular closed set and g-closed set.

2. PRELIMINARIES

Throughout this paper (X, τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, cl(A), int(A) and X\A denotes the closure, the interior and the complement of A in X respectively.

We recall the following definitions and results.

Definition 2.1: Let (X, τ) be a topological space. A subset A of X is said to be generalized closed [8] (briefly g-closed) set if cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

Definition 2.2: Let (X, τ) be a topological space and $A \subseteq X$. The generalized closure of A is denoted by cl*(A) and is defined by the intersection of all g-closed sets containing A and the generalized interior of A denoted by int*(A) and is defined by union of all g-open sets contained in A.

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Definition 2.3: Let (X, τ) be a topological space. A subset A of X is said to be

1. a pre-open [13] set if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$.
2. a semi-open [2] set if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.
3. a regular-open [17] set if $int(cl(A)) = A$ and a regular-closed set if $cl(int(A)) = A$.
4. a semi pre- open [2] set if $A \subseteq cl(int(cl(A)))$ and a semi pre closed set if $int(cl(int(A))) \subseteq A$.
5. an α-open[14] set if $A \subseteq int(cl(int(A)))$ and a α-closed set if $cl(int(cl(A))) \subseteq A$.
6. b-open [3] set if $A \subseteq cl(int(A)) \cup int(cl(A))$ and b-closed set [2] if $int(cl(A)) \cap cl(int(A)) \subseteq A$.
7. a pre* open [16] set if $A \subseteq int^*(cl(A))$ and a pre* closed set if $cl^*(int(A)) \subseteq A$.

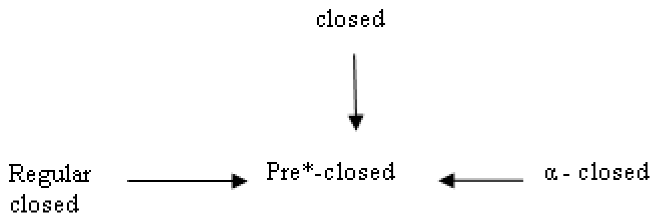
Definition 2.4: Let (X, τ) be a topological space and $A \subseteq X$. The regular closure [17] of A is denoted by rcl(A) and is defined by intersection of all regular closed sets containing A .

Definition 2.5: Let (X, τ) be a topological space. A subset A of X is said to be a

1) generalized semi closed [4] (briefly gs-closed) set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
2) α- generalized closed [10] (briefly αg-closed) set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
3) regular generalized closed [15] (briefly rg-closed) set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).
4) generalized pre closed [11] (briefly gp-closed) set if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
5) generalized regular closed [5] (briefly gr-closed) set if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
6) generalized semi pre closed [6] (briefly gsp-closed) set if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
7) generalized pre regular closed [7] (briefly gpr-closed) set if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) .
8) generalized α-closed [10](briefly gα-closed) set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).
9) generalized b- closed [1] (briefly gb- closed) set if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
10) generalized α b- closed [18] (briefly gαb- closed) set if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ) .
11) regular generalized b-closed [12] (briefly rgb-closed) set if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).

The complement of the above mentioned closed sets are their respective open sets.

Remark 2.6:



Lemma 2.7: A subset A of a topological space X is pre* closed if and only if $p^* cl(A) = A$.

3. Pre* regular generalized closed sets

Definition 3.1: Let (X, τ) be a topological space. A subset A of X is called pre* regular generalized closed (briefly p*rg-closed) set if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre*-open in (X, τ).

Definition 3.2: For a subset A of (X, τ),

(i) the intersection of all p*rg-closed sets containing A is called the p*r -closure of A and is denoted by p*rg-cl(A). That is, p*rg- cl(A) = $\cap$ {F | F is p*rg-closed in X, $A \subseteq F$}.

(ii) the union of all p*rg–open sets contained in A is called the p*rg –interior of A and is denoted by p*rg-int (A) . That is, p*rg-int (A) = $\cup$ {G | G is p*rg-open in X, $A \subseteq G$}.

Theorem 3.3: Every regular closed set is p*rg-closed.

Proof: Straight forward.

Remark 3.3.1: The converse of the theorem need not be true as seen from the following example.

Example 3.3.2: Let X={a, b, c, d} with topology τ ={ ϕ ,{a},{a, b},{c, d}, {a, c, d}, X}. Here the set {b, c, d} is p*rg-closed but not regular closed.

Theorem 3.4: For a topological space (X, τ), the following conditions are hold.

(i) Every p*rg-closed set is gs-closed set.
(ii) Every p*rg-closed set is αg-closed set.
(iii) Every p*rg-closed set is gp-closed set.
(iv) Every p*rg-closed set is gsp-closed set.
(v) Every p*rg-closed set is rg-closed set.
(vi) Every p*rg-closed set is rgb-closed set.

Proof: proof is straight forward.

Remark 3.4.1: The converse of the theorem need not be true as seen from the following example.

Example 3.4.2: Let X = {a, b, c} with topology τ = { ϕ, {a}, {a, b}, X}. Here the set {c} is gs-closed, gα-closed, gp-closed, gr-closed, gsp-closed and rgb-closed but not p*rg-closed .

Theorem 3.5: For a topological space (X, τ), the following conditions are hold.

(i) Every p*rg-closed set is gr-closed set.
(ii) Every p*rg-closed set is gb-closed set.
(iii) Every p*rg-closed set is gαb -closed set.
(iv) Every p*rg-closed set is gα-closed set.

Proof: proof is straight forward.

Remark 3.5.1: The converse of the theorem need not true as seen from the following example.

Example 3.5.2: Let X= {a, b, c} with topology τ = { ϕ, {a}, {a, b}, X}. Here the set {b} is gα-closed, gαb-closed, gr-closed, gb-closed but not p*rg-closed .

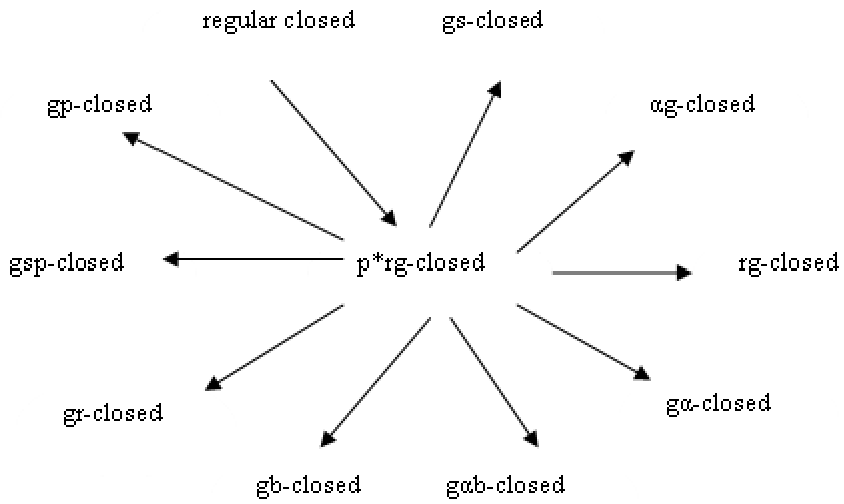
Theorem 3.6: Every p*rg-closed set is gsp-closed.

Proof: Since every open set is pre* open and from the definition 2.5 (6), we have the proof .

Remark 3.6.1: The converse of the theorem need not true as seen from the following example.

Example 3.6.2: Let X = {a, b, c} with topology τ = { ϕ, {a}, {a, b}, X}. Here the set {b} is gsp- closed but not p*rg-closed.

Remark 3.8: The above discussions are summarized in the following implications.



4. Characterisation of p*rg closed sets

Theorem 4.1: A subset A of X is p*rg-closed if and only if rcl(A)\A does not contain any non-empty pre*-closed sets.

Proof: Necessity: Suppose F is a non-empty pre* closed subset of X such that F $\subseteq$ rcl(A)\A. Then A $\subseteq$ X\F and X\F is pre*-open in (X, τ). Since A is p*rg-closed in X, rcl(A) $\subseteq$ X \F which implies F $\subseteq$ X\rcl(A). Thus, F $\subseteq$ (rcl(A) ∩ (X\rcl(A)) = ϕ.

Sufficiency: LetA $\subseteq$ U and U is pre* open. Suppose rcl(A) does not contained in U, then rcl(A) ∩(X\U) is a non-empty pre*closed set of rcl(A)\A, which is contradiction. Therefore, rcl(A) $\subseteq$ U. Hence A is p*rg -closed.

Corollary 4.1.1: For any subset A of X, if A is p*rg-closed then rcl(A)\A does not contain any non-empty regular closed set.

Proof: Let A be a subset of X. Then by theorem 4.1, we have rcl(A)\A does not contain any non-empty pre* closed set. Since every regular closed set is pre* closed, we have rcl(A)\A does not contain any non-empty regular closed set.

Theorem 4.2: If A is both pre*-open and p*rg-closed set in X, then A is pre closed set.

Proof: Since A is pre* open and p*rg-closed in X, rcl(A) $\subseteq$ A. Since every regular closed set is pre closed, pcl(A) $\subseteq$ rcl(A) $\subseteq$ A. But always A $\subseteq$ pcl(A). Therefore, A = pcl(A). Hence A is pre closed set.

Theorem 4.3: If A is both pre* open and p*rg-closed set in X, then A is a closed set.

Proof: Since A is pre* open and p*rg-closed in X, rcl(A) $\subseteq$ A. Since every regular closed set is closed, we have cl(A) $\subseteq$ rcl(A) =A. Hence cl(A) $\subseteq$ A. But always A $\subseteq$ cl(A) and hence A = cl(A). Therefore, A is closed.

Theorem 4.4: Let X be a topological space, if a subset A of X is open and gr-closed then A is p*rg-closed set in X.

Proof: Assume that A is open and gr-closed. Let U be pre* open in X containing A. Then we have rcl(A) $\subseteq$ U. Hence A is p*rg-clolsed set in X.

Theorem 4.5: Let X be a topological space. If a subset A of X is π-open and πgr-closed then A is p*rg-closed set in X.

Proof: Assume that A is π-open and πgr-closed. Let U be pre* open in X containing A. Then we have rcl(A) $\subseteq$ U. Thus rcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is pre* open. Hence A is p*rg-closed set in X.

Theorem 4.6: If A is regular closed and p*rg-closed, then A is pre* closed.

Proof: Suppose that A is regular closed and p*rg-closed. Since every regular closed set is pre*closed, p*cl(A) $\subseteq$ rcl(A). Since A is regular closed, we have rcl(A) =A. This implies p*cl(A) $\subseteq$ A. We know that A $\subseteq$ p*cl(A) and hence p*cl(A)=A. Therefore, A is pre* closed.

Theorem 4.7: Let A be any p*rg closed set. If A is regular closed then rcl(A)\A is pre* closed.

Proof: Necessity: Since A is regular closed set in (X, τ), rcl(A) =A. Then rcl(A)\A = ϕ, which is a pre* closed set in (X, τ).

Remark 4.7.1: The converse of the above theorem need not be true.

Example 4.7.2: Let X = {a, b, c, d} with topology τ = { ϕ, {a}, {b}, {a, b}, X}. Let A = {c, d}. Here rcl(A)\A = ϕ, which is pre* closed but A is not regular closed.

Theorem 4.8: The union of any two p*rg-closed set is p*rg-closed

Proof: Let A and B be p*rg-closed sets. Let U be pre*-open set in X such that A∪B $\subseteq$ U. Then A $\subseteq$ U and B $\subseteq$ U. Then rcl(A) $\subseteq$ U and rcl(B) $\subseteq$ U. Then rcl(A∪B) = rcl(A)∪rcl(B) $\subseteq$ U. Therefore, A∪B is p*rg-closed set.

Theorem 4.9: Arbitrary intersection of p*rg-closed set in a topological space X is p*rg-closed.

Proof: Let $\{A_i | i \in I\}$ be any collection of p*rg-closed set. Let U be a pre*-open set containing each A_i, $i \in I$. Since each A_i is p*rg-closed, we have $rcl(A_i) \subseteq U$ for each $i \in I$. Thus $\bigcap_{i \in I} rcl(A_i) \subseteq U$ But $rcl(\bigcap_i A_i) \subseteq \bigcap_{i \in I} rcl(A_i) \subseteq U$, where U is pre*-open. Hence $\bigcap_{i \in I} A_i$ is p*rg-closed.

Theorem 4.10: Let A and B be subsets such that $A \subseteq B \subseteq rcl(A)$. If A is p*rg-closed set then B is p*rg-closed set.

Proof: Let A and B be subsets such that $A \subseteq B \subseteq rcl(A)$. Suppose that A is p*rg-closed. Let $B \subseteq U$ and U be a pre* open in X. Since $A \subseteq B$ and $B \subseteq U$, we have $A \subseteq U$. Hence $A \subseteq U$ and U is pre*open in X. Since A is p*rg-closed, we have $rcl(A) \subseteq U$. Since $B \subseteq rcl(A)$, we have $rcl(B) \subseteq rcl(rcl(A)) = rcl(A) \subseteq U$. Hence $rcl(B) \subseteq U$. Hence B is p*rg-closed set.

Theorem 4.11: For every element x in the space X, {x} is pre*-closed or X\{x} is p*rg-closed.

Proof: Suppose {x} is not pre*-closed. Then X\{x} is not pre*-open implies the only pre*- open set containing X\{x} is X. This implies $rcl(X\backslash\{x\}) \subseteq X$. Hence X\{x} is p*rg-closed.

Theorem 4.12: Suppose $B \subseteq A \subseteq X$. B is p*rg-closed set relative to A and A is p*rg-closed set in Y, then B is p*rg-closed set relative to X.

Proof: Let $B \subseteq U$ and U be a pre* open set in X. Since $B \subseteq A$ and $B \subseteq U$ then $B \subseteq A \cap U$. Since B is p*rg closed set relative to A, $rcl(B) \subseteq A \cap U \subseteq U$. Therefore $A \cup rcl(B) \subseteq U$. Since A is pre* closed and $B \subseteq A$ gives $rcl(B) \subseteq U \cap (rcl(B))^c$. This implies that rcl(B) contained in U but not contained in $(rcl(B))^c$. Therefore, B is p*rg - closed set relative to X.

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