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THE MINUS PARTIAL ORDER IN INTERVAL VALUED FUZZY MATRICES

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ABSTRACT

In this paper, we study the minus ordering for Interval Valued Fuzzy Matrices, analogous to that of minus partial ordering for complex matrices and prove that the minus ordering is a partial ordering in the set of all Interval Valued Fuzzy Matrices. Some properties of minus ordering are derived.

Keywords: *Fuzzy matrix, Regular Fuzzy matrix, Interval Valued Fuzzy Matrices, Partial Ordering.*

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1. INTRODUCTION

We deal with Interval Valued Fuzzy Matrices (IVFM) that is, matrices whose entries are intervals and all the intervals are subintervals of the interval [0,1]. Thomason introduced fuzzy matrices and discussed about the convergence of powers of a fuzzy matrix [7]. Kim and Roush have developed a theory for fuzzy matrices analogous to that for Boolean Matrices [1]. Recently the concept of IVFM a generalization of fuzzy matrix was introduced and developed by Shyamal and Pal [6], by extending the max.min operations on fuzzy algebra $\mathcal{F}$ =[0,1], for elements a,b$\in\mathcal{F}$, a+b = max{a, b} and a.b = min{a, b}. Each element a$\in\mathcal{F}$ is regular. Since axa = a holds under the fuzzy multiplication for all x $\geq$a ('$\geq$' is the usual ordering on real numbers). Hence $\mathcal{F}$ is regular. Let $\mathcal{F}_{m,n}$ be the set of all mxn fuzzy matrices over the fuzzy algebra with support [0,1] is not regular. A matrix A$\in\mathcal{F}_{m,n}$ is said to be regular if there exists X$\in\mathcal{F}_{m,n}$ such that AXA =A. In this case, X is called a generalized(g^-) inverse of A and is denoted by A^-. Let A{1} denotes the set of all g-inverses of A.

In this paper, we study the minus ordering for Interval Valued Fuzzy Matrices, analogous to that of minus partial ordering for complex matrices and as an extension of that of fuzzy matrices [3]. In section 2, we present the basic definitions notations on IVFM and required results on regular fuzzy matrices and regular IVFM. In section 3, we discuss the minus ordering on regular IVFM and prove that minus ordering is a partial ordering in the set of all regular IVFM. In section 4, some properties of minus ordering are derived.

2. PRELIMINARIES

In this section, some basic definitions and results needed are given. Let $(IVFM)_{mn}$ denotes the set of all mxn Interval Valued Fuzzy Matrices(IVFM)s.

Definition: 2.1
An Interval Valued Fuzzy Matrix (IVFM) of order mxn is defined as A= $(a_{ij})_{mxn}$, where $a_{ij} = [a_{ijL}, a_{ijU}]$, the ij^{th} element of A is an interval representing the membership value. All the elements of an IVFM are intervals and all the intervals are the subintervals of the interval [0, 1].

Let A and B be any two IVFMs. The following operations are defined for any two elements x $\in$ A and y$\in$ B, where x = $[x_L, x_U]$ and y = $[y_L, y_U]$ are intervals in [0, 1] such that $x_L < x_U$ and $y_L < y_U$.

(i) x + y = [max { x_L, y_L}, max{x_U, y_U}]

(ii) x. y = [min { x_L, y_L}, min{x_U, y_U}]

Here we shall follow the basic operations on IVFM as given in [6].

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For $A = (a_{ij}) = ([a_{ijL}, a_{ijU}])$ and $B = (b_{ij}) = ([b_{ijL}, b_{ijU}])$ of order mxn their sum denoted as A+B defined as,

$$A + B = (a_{ij} + b_{ij}) = ([(a_{ijL} + b_{ijL}), (a_{ijU} + b_{ijU})]) \tag{2.1}$$

For $A = (a_{ij})_{mxn}$ and $B = (b_{ij})_{nxp}$ their product denoted as AB is defined as,

$$AB = (c_{ij}) = \left(\sum_{k=1}^{n} a_{ik}\, b_{kj} \right) \qquad i=1, 2\ldots m \text{ and } j=1, 2\ldots p$$

$$= [\sum_{k=1}^{n} (a_{ikL} \,.\, b_{kjL}),\ \sum_{k=1}^{n} (a_{ikU} \,.\, b_{kjU})] \qquad i=1, 2, \ldots, m \text{ and } j=1, 2, \ldots, p \tag{2.2}$$

$A \leq B$ if and only if $a_{ijL} \leq b_{ijL}$ and $a_{ijU} \leq b_{ijU}$

In particular if $a_{ijL} = a_{ijU}$ and $b_{ijL} = b_{ijU}$ then (2.2) reduces to the standard max. min composition of Fuzzy Matrices [2, 4].

Definition: 2.2 For a pair of Fuzzy Matrices $E = (e_{ij})$ and $F=(f_{ij})$ in $\mathcal{F}_{m,n}$ such that $E \leq F$, let us define the interval matrix denoted as [E, F], whose ij[th] entry is the interval with lower limit e_{ij} and upper limit f_{ij}, that is $([e_{ij}, f_{ij}])$. In particular for E = F, IVFM [E,E] reduces to $E \in \mathcal{F}_{m,n}$.

For $A = (a_{ij}) = ([a_{ijL}, a_{ijU}]) \in (IVFM)_{mn}$, let us define $A_L = (a_{ijL})$ and $A_U = (a_{ijU})$.

Clearly A_L and A_U belongs to $\mathcal{F}_{m,n}$ such that $A_L \leq A_U$ and from Definition (2.2) A can be written as $A = [A_L, A_U]$ (2.3).

This representation for an IVFM, as an interval matrices of its lower and upper limit fuzzy matrices has been introduced and developed in [5].

For $A \in (IVFM)_{mn}$, A^T, A_{i*}, A_{*j}, R (A), C (A) denotes the transpose of A, i[th] row of A, j[th] column of A, row space of A and column space of A respectively. Let $\mathcal{F}^-_{m,n}$ be the set of all regular fuzzy matrices of order mxn.

In the sequel we shall make use of the following results on Fuzzy Matrices found in [2] and [3].

Lemma: 2.3 For $A, B \in \mathcal{F}_{m,n}$, we have the following:
(i) $R(B) \subseteq R(A) \Leftrightarrow B = XA$ for some $X \in \mathcal{F}_m$.
(ii) $C(B) \subseteq C(A) \Leftrightarrow B = AY$ for some $Y \in \mathcal{F}_n$.

Definition: 2.4 For $A \in \mathcal{F}^-_{mn}$ and $B \in \mathcal{F}_{mn}$; the minus ordering denoted as $\bar{<}$ is defined as
$A \bar{<} B \Leftrightarrow A^- A = A^- B$ and $A A^- = B A^-$ for some $A^- \in A\{1\}$.

Lemma: 2.5 For $A \in \mathcal{F}^-_{mn}$ and $B \in \mathcal{F}_{mn}$; the following are equivalent.
(i) $A \bar{<} B$
(ii) $A = A A^- B = B A^- A = B A^- B$ for some $A^- \in A\{1\}$

In the sequel we shall make use of the following results on IVFM and regular IVFM obtained in [5].

Lemma: 2.6 For $A = [A_L, A_U] \in (IVFM)_{mn}$ and $B = [B_L, B_U] \in (IVFM)_{np}$, the following hold.
(i) $A^T = [A_L^T, A_U^T]$
(ii) $AB = [A_LB_L, A_UB_U]$

Theorem: 2.7 Let $A = [A_L, A_U] \in (IVFM)_{mn}$.
Then A is regular IVFM $\Leftrightarrow A_L$ and $A_U \in \mathcal{F}_{mn}$ are regular.

Theorem: 2.8 Let $A = [A_L, A_U]$ be an $(IVFM)_{mn}$ Then,
(i) $R(A) = [R(A_L), R(A_U)] \in (IVFM)_{1n}$
(ii) $C(A) = [C(A_L), C(A_U)] \in (IVFM)_{1m}$

Theorem: 2.9 For $A, B \in (IVFM)_{mn}$
(i) $R(B) \subseteq R(A) \Leftrightarrow B = XA$ for some $X \in (IVFM)_m$
(ii) $C(B) \subseteq C(A) \Leftrightarrow B = AY$ for some $Y \in (IVFM)_n$

Theorem: 2.10 Let $A \in (IVFM)_{mn}$, $P \in (IVFM)_m$, $Q \in (IVFM)_n$. A is regular $\Leftrightarrow$ PAQ is regular for permutation matrices P and Q.

3. MINUS ORDERING FOR IVFM

In this section, we define the minus ordering for Interval Valued Fuzzy Matrices as an extension of that for fuzzy matrices studied in [3] and characterize the class of all IVFMs under minus ordering. We prove that the minus ordering is a partial ordering in $(IVFM)^{-}_{mn}$, the set of all regular $(IVFM)_{mn}$. For $A \in (IVFM)^{-}_{mn}$ **,** let A{1} be the set of all g-inverses of A.

Definition: 3.1 For $A \in (IVFM)^{-}_{mn}$ and $B \in (IVFM)_{mn}$; the minus ordering denoted as $\overline{<}$ is defined as $A \overline{<} B \Leftrightarrow A^{-} A = A^{-} B$ and $A A^{-} = B A^{-}$ for some $A^{-} \in A\{1\}$.

Remark: 3.2 In particular for $A \in \mathcal{F}^{-}_{mn}$ and $B \in \mathcal{F}_{mn}$, Definition (3.1) reduces to minus ordering in Definition (2.4) on fuzzy matrices.

Remark: 3.3 In the above Definition of minus ordering $A \overline{<} B$, B need not be regular. This is illustrated in the following.

Example: 3.4

$$\text{or } A = \begin{pmatrix} [0.4,1] & [0.4,1] & [0.4,1] \\ [0.4,1] & [0.4,1] & [0.4,1] \\ [0.4,1] & [0.4,1] & [0.4,1] \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} [0.5,\ 1] & [0.4,0.5] & [0.3,0.5] \\ [0.4,0.5] & [0.3,0.5] & [0,\ 0] \\ [0.4,0.5] & [0.4,0.5] & [0.4,0.5] \end{pmatrix}$$

be an IVFM of order3x3. Since A is idempotent, A itself is a g- inverse of A. Here $A \overline{<} B$ for $A^{-} \in A\{1\}$. B is not regular, since there is no $X \in (IVFM)_{3,3}$ such that BXB = B.

Lemma: 3.5 Let $A = [A_L, A_U] \in (IVFM)^{-}_{mn}$ and $B = [B_L, B_U] \in (IVFM)_{mn}$
Then $A \overline{<} B \Leftrightarrow A_L \overline{<} B_L$ and $A_U \overline{<} B_U$ for $A_L, A_U, B_L, B_U \in \mathcal{F}_{mn}$.

Proof: Let $A = [A_L, A_U]$ and $B = [B_L, B_U]$

$A \overline{<} B \Leftrightarrow A^{-} A = A^{-} B$ and $A A^{-} = B A^{-}$ (By Definition (3.1))

$A^{-} A = A^{-} B \Leftrightarrow [A_L^{-}, A_U^{-}]\ [A_L, A_U] = [A_L^{-}, A_U^{-}]\ [B_L, B_U]$

$\Leftrightarrow [A_L^{-} A_L, A_U^{-} A_U] = [A_L^{-} B_L, A_U^{-} B_U]$ (By Lemma (2.6) (ii))

$\Leftrightarrow A_L^{-} A_L = A_L^{-} B_L$ and $A_U^{-} A_U = A_U^{-} B_U$

Similarly we have,
$A A^{-} = B A^{-} \Leftrightarrow A_L A_L^{-} = B_L A_L^{-}$ and $A_U A_U^{-} = B_U A_U^{-}$

Hence $A \overline{<} B \Leftrightarrow A_L \overline{<} B_L$ and $A_U \overline{<} B_U$ (By Definition (3.1))

Lemma 3.6
For $A \in (IVFM)^{-}_{mn}$ and $B \in (IVFM)_{mn}$; the following are equivalent:
(i) $A \overline{<} B$
(ii) $A = A A^{-} B = B A^{-} A = B A^{-} B$

Proof: (i) $\Rightarrow$ (ii)
$A \overline{<} B \Rightarrow A^{-} A = A^{-} B$ and $A A^{-} = B A^{-}$ for some $A^{-} \in A\{1\}$.

Now,
$A = A (A^{-} A) = A A^{-} B$ (By Definition (3.1))
$A = (A A^{-}) A = B (A^{-} A) = B A^{-} B$

Thus (ii) holds.
(ii) $\Rightarrow$ (i)
Let $X = A^{-} A A^{-}$
$AXA = A(A^{-} A A^{-})A = (AA^{-} A) A^{-} A = A A^{-} A = A \Rightarrow X \in A\{1\}$.

Now, $XA = (A^- A A^-)(A A^- B) = A^-(A A^- A) A^- B = (A^- A A^-)B = XB$

Similarly $AX = BX$. Hence $A \overset{-}{<} B$ for $X \in A\{1\}$.

Theorem 3.7
For $A = [A_L, A_U] \in (IVFM)^-_{mn}$ and $B=[B_L, B_U] \in (IVFM)_{mn}$;the following are equivalent:
(i) $A \overset{-}{<} B$
(ii) $A_L \overset{-}{<} B_L$ and $A_U \overset{-}{<} B_U$
(iii) $A_L = A_L A_L^- B_L = B_L A_L^- A_L = B_L A_L^- B_L$ and $A_U = A_U A_U^- B_U = B_U A_U^- A_U = B_U A_U^- B_U$
(iv) $A = A A^- B = B A^- A = B A^- B$

Proof:
(i)$\Leftrightarrow$(ii) Precisely Lemma (3.5).
(i) $\Leftrightarrow$ (iv) This follows from Lemma (3.6)
(ii) $\Leftrightarrow$ (iii) Since A is regular, By Theorem (2.7), both A_L and A_U are regular fuzzy matrices. Therefore, this equivalence follows from Theorem (2.5) for the fuzzy regular matrices A_L and A_U and fuzzy matrices B_L and B_U.

Proposition 3.8
For $A, B \in (IVFM)^-_{mn}$, If $A \overset{-}{<} B$ then $B\{1\} \subseteq A\{1\}$.

Proof: By Lemma (3.6), $A \overset{-}{<} B \Leftrightarrow A = A A^- B = B A^- A = B A^- B$

For $B^- \in B\{1\}$,

$A B^- A = (A A^- B) B^- (B A^- A) = A A^-(B B^- B) A^- A = (A A^- B) A^- A = A A^- A = A.$

Hence $A B^- A = A$ for each $B^- \in B\{1\}$. Therefore $B\{1\} \subseteq A\{1\}$.

Corollary: 3.9 For $A, B \in (IVFM)^-_{mn}$, If $A \overset{-}{<} B$ with B idempotent then $B \in A\{1\}$.

Proof: Since B is idempotent, B regular and B itself is a generalized inverse of B. Here $B \in B\{1\}$.By Proposition (3.8), $B\{1\} \subseteq A\{1\}$. Hence $B \in A\{1\}$.

Lemma: 3.10 For $A, B \in (IVFM)^-_{mn}$,
(i) $R(A) \subseteq R(B) \Leftrightarrow A = A B^- B$ for each B^- of B.
(ii) $C(A) \subseteq C(B) \Leftrightarrow A = B B^- A$ for each B^- of B.

Proof: By Theorem (2.9),

$$R(A) \subseteq R(B) \Leftrightarrow A = XB = (XB) B^- B = A B^- B \quad \text{for each } B^- \in B\{1\}.$$

and $$C(A) \subseteq C(B) \Leftrightarrow A = BY = B B^- (BY) = B B^- A \quad \text{for each } B^- \in B\{1\}.$$

Theorem: 3.11 For $A, B \in (IVFM)^-_{mn}$, the following are equivalent:
(i) $A \overset{-}{<} B$
(ii) $R(A) \subseteq R(B)$ and $C(A) \subseteq C(B)$ and $AB^- A = A$.

Proof: (i) $\Rightarrow$ (ii)
$A = B A^- B$ (By Lemma (3.6))
$= B A^- (B B^- B) = (B A^- B) B^- B = A B^- B.$
$A = A B^- B$ for each $B^- \in B\{1\}$. $\Rightarrow R(A) \subseteq R(B)$ (By Lemma (3.10)(i)).

Similarly, $A = B B^- A$ for each $B^- \in B\{1\}$. $\Rightarrow C(A) \subseteq C(B)$ (By Lemma (3.10) (ii)).

Also $A=AB^- A$ (By Proposition (3.8)).

(ii) $\Rightarrow$ (i)
Let $X = B^- A B^-$
$AXA = A(B^- A B^-)A = (AB^- A) B^- A = AB^- A = A. \Rightarrow X \in A\{1\}$.

Now, $AX = A(B^- A B^-) = B B^- A(B^- A B^-)$ (By Lemma (3.10)(ii))
$= B B^- (AB^- A) B^- = B (B^- A B^-) = BX.$

Similarly, $XA = XB$ (By Lemma (3.10) (i) and $AB^- A = A$). Hence $A \overset{-}{<} B$.

Theorem: 3.12 In $(IVFM)^{-}_{mn}$, the minus ordering $\overline{<}$ is a partial ordering.

Proof:
(i) $A \overline{<} A$ is Obvious.

Hence $<$ reflexive.

(ii) $A \overline{<} B \Rightarrow A = B A^{-} B$ (By Lemma (3.6))
$B \overline{<} A \Rightarrow \overline{B} = B B^{-} A = A B^{-} B$ (By Lemma (3.6))

Now,
$A = B A^{-} B = (B B^{-} A) A^{-} (A B^{-} B) = B B^{-} (A A^{-} A) B^{-} B = B B^{-} (A B^{-} B) = B B^{-} B = B$

Hence $A \overline{<} B$ and $B \overline{<} A \Rightarrow A = B$. Hence $<$ Antisymmetric.

(iii) $A \overline{<} B \Rightarrow A = A B^{-} A$ and $A = A B^{-} B = B B^{-} A$ (By Theorem (3.11)

$B \overline{<} C \Rightarrow B = C B^{-} B = B B^{-} C$ (By Lemma (3.6))

Let $X = B^{-} A B^{-}$

Then $A X A = A(B^{-} A B^{-})A = (A B^{-} A) B^{-} A = A B^{-} A = A. \Rightarrow X \in A\{1\}$.

Since $A \overline{<} B$ and $B \overline{<} C$, by applying Theorem (3.11) repeatedly, we have,

$AX = A(B^{-} A B^{-}) = B B^{-} A (B^{-} A B^{-}) = B B^{-} (A B^{-} A) B^{-} = B B^{-} A B^{-} = C B^{-} B (B^{-} A B^{-})$

$= C B^{-} (B B^{-} A) B^{-} = C (B^{-} A B^{-}) = CX.$

Similarly $XA = XC$. Since $X \in A\{1\}$, with $AX = CX$ and $XA = XC$, it follows that $A \overline{<} C$

4. PROPERTIES OF MINUS PARTIAL ORDERING

In this section, we shall derive some basic properties of minus partial ordering on regular IVFM.

Proposition: 4.1 For $A \in (IVFM)^{-}_{mn}$, and $B \in (IVFM)^{-}_{mn}$, $A \overline{<} B \Leftrightarrow A^{T} \overline{<} B^{T}$

Proof: Let $A = [A_L, A_U]$ and $B = [B_L, B_U]$

For $A^{-} = [A_L^{-}, A_U^{-}] \in A\{1\}$.

$A \overline{<} B \Leftrightarrow A A^{-} = B A^{-}$ and $A^{-} A = A^{-} B$ for some $A^{-} \in A\{1\}$

$$AA^{-} = BA^{-} \Leftrightarrow [A_L, A_U][A_L^{-}, A_U^{-}] = [B_L, B_U][A_L^{-}, A_U^{-}]$$
$$\Leftrightarrow [A_L A_L^{-}, A_U A_U^{-}] = [B_L A_L^{-}, B_U A_U^{-}] \quad \text{(By Lemma (2.6)(ii)}$$
$$\Leftrightarrow [A_L A_L^{-}, A_U A_U^{-}]^{T} = [B_L A_L^{-}, B_U A_U^{-}]^{T}$$
$$\Leftrightarrow [(A_L A_L^{-})^{T}, (A_U A_U^{-})^{T}] = [(B_L A_L^{-})^{T}, (B_U A_U^{-})^{T}]$$
(By Lemma (2.6)(i)
$$\Leftrightarrow [(A_L^{-})^{T} A_L^{T}, (A_U^{-})^{T} A_U^{T}] = [(A_L^{-})^{T} B_L^{T}, (A_U^{-})^{T} B_U^{T}]$$
$$\Leftrightarrow [(A_L^{T})^{-} A_L^{T}, (A_U^{T})^{-} A_U^{T}] = [(A_L^{T})^{-} B_L^{T}, (A_U^{T})^{-} B_U^{T}]$$
$$\Leftrightarrow (A_L^{T})^{-} A_L^{T} = (A_L^{T})^{-} B_L^{T} \text{ and } (A_U^{T})^{-} A_U^{T} = (A_U^{T})^{-} B_U^{T}$$

Similarly we have,

$$A^{-}A = A^{-}B \Leftrightarrow A_L^{T}(A_L^{T})^{-} = B_L^{T}(A_L^{T})^{-} \text{ and } A_U^{T}(A_U^{T})^{-} = B_U^{T}(A_U^{T})^{-}$$

Hence $A \overline{<} B \Leftrightarrow A_L^{T} \overline{<} B_L^{T} \Leftrightarrow A_U^{T} \overline{<} B_U^{T} \Leftrightarrow A^{T} \overline{<} B^{T}$

Proposition: 4.2. For $A \in (IVFM)^{-}_{mn}$, and $B \in (IVFM)_{mn}$, $A \overline{<} B \Leftrightarrow PAQ \overline{<} PBQ$ for some permutation matrix P and Q.

Proof: By Theorem (2.10), A is regular, then PAQ is regular (IVFM) for permutation matrices P and Q and $Q^TA^-P^T$ is a g-inverse of PAQ.

$$\begin{aligned}\text{Now, } (PAQ)^- (PAQ) &= (Q^TA^-P^T)(PAQ)\\ &= Q^TA^-(P^TP)AQ\\ &= Q^TA^-AQ\\ &= Q^TA^-BQ \qquad \text{(By Definition (3.1))}\\ &= (Q^TA^-P^T)(PBQ)\\ &= (PAQ)^-(PBQ)\end{aligned}$$

Similarly, $(PAQ)(PAQ)^- = (PBQ)(PAQ)^-$

Hence $A \bar{<} B \Leftrightarrow PAQ \bar{<} PBQ$

Conversely, $PAQ \bar{<} PBQ \Rightarrow P^T(PAQ)Q^T \bar{<} P^T(PBQ)Q^T \Rightarrow A \bar{<} B$

Proposition: 4.3 For $A\in (IVFM)^-_{mn}$, and $B\in (IVFM)_{mn}$, if $A \bar{<} B$ with B is idempotent, then A is idempotent.

Proof: Since $A \bar{<} B$, By Lemma (3.6),

$$A^2 = A\,A = (A\,A^-B)(B\,A^-A) = A\,A^-B^2\,A^-A = (A\,A^-B)\,A^-A = AA^-A = A$$

Remark: 4.4. In the above Proposition 3.3, if $A \bar{<} B$ with A^2 idempotent then B^2 need not be idempotent. This is illustrated in the following.

Example 4.5

Consider $A = \begin{pmatrix} [0.5, 1] & [0.5, 1] \\ [0.5, 1] & [0.5,1] \end{pmatrix}$ and $B = \begin{pmatrix} [0, 0.5] & [0.6, 1] \\ [0.5,1] & [0,0.5] \end{pmatrix} \in (IVFM)_{2x2}$.

Here $A \bar{<} B$ for $A^-=A$, But B is not idempotent.

Proposition: 4.6 For $A\in (IVFM)^-_{mn}$, and $B\in (IVFM)_{mn}$, if $A \bar{<} B$ then $B^2 = 0$ implies $A^2 = 0$.

Proof: Since $A \bar{<} B$, By Lemma (3.6),

$$A^2 = A\,A = (A\,A^-B)(B\,A^-A) = A\,A^-B^2\,A^-A = 0$$

Proposition: 4.7 For $A, B\in (IVFM)^-_{mn}$. If $A \bar{<} B$ then A+B is regular and $(A^- + B^-)$ is a g-inverses of (A+B).

Proof: By using Proposition (3.6), Theorem (3.11), and Definition (3.1), we have

$(A+B)(A^- + B^-)(A+B) = (A+B)$

Hence $(A^- + B^-) \in (A+B)\{1\}$ and A+B is regular.

REFERENCES:

[1] Kim, K. H., (1982), Boolean Matrix Theory and Applications Marcel Dekker. Inc. New york.
[2] Kim, K. H., and Roush,F. N.,(1980), Generalized fuzzy matrices, Fuzzy sets and systems, 4, 293 – 315.
[3]Meenakshi, AR and Inbam. C, (2004), The Minus Partial Order in Fuzzy Matrices, Int. Journal of Fuzzy Mathematics vol.12, No.3, 695 – 700.
[4] Meenakshi, AR., (2008), Fuzzy Matrix, Theory and Applications, MJP Publishers, Chennai.
[5]Meenakshi, AR., and Kaliraja, M., (2010), Regular Interval Valued Fuzzy Matrices, Advances in Fuzzy Mathematics, Vol 5, No 1, 7 -15.
[6] Shyamal.A. K., and Pal. M., (2006), Interval Valued Fuzzy Matrices, Journal of Fuzzy Mathematics, Vol 14, No 3, 582 – 592.
[7] Thomason, M. G., (1977), Convergence of powers of fuzzy matrix, J. Math Anal Appl. 57, 476 – 480.

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