International Journal of Mathematical Archive-8(5), 2017, 80-86
Available online through www.ijma.info ISSN 2229 – 5046

ON NOVEL APPROACH TO ANALYZE THE MINIMUM FUZZY COST IN RSA METHOD WITH STATSTICAL PREDICTION

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(Received On: 18-04-17; Revised & Accepted On: 19-05-17)

ABSTRACT

In this paper, we propose a new algorithm to obtain the Fuzzy Assignment Cost with the help of the ranking function. The initial fuzzy optimal solution is obtained by One's Termination Algorithm. Later the fuzzy optimal solutions are analyzed in RSA (Revised Sensitivity Analysis).Our interval optimal solutions are analyzed in statistical behavior. Here the parameters are considered as Triangular Fuzzy numbers. A numerical example is included.

Key Words: *Triangular Fuzzy Number, One's Termination Algorithm, Solution Algorithm, FLP, RSAT, Statistical implementation, t- test.*

1. INTRODUCTION

Assignment problem plays an important role in industry and other applications. The assignment problem refers to another special class of Linear Programming Problem, the objectives are assign to number of resources to equal number of activities. Already many researchers are discussed the classical assignment problem [1, 2, 3]. In general for the situation parameters of the FAP is not deterministic. In this case Fuzzy set theory is most helpful to analyze the system optimality level. [6, 4] investigated the classical assignment problem in fuzzy environment. Generally, the system parameters are affected the cost, time and availability of the product etc. So the fuzzy Linear Programming Problem is flexibility to recovered the above parameters. [5, 15, 14] formulated the FLP with the coefficient of cost and time discussed through the sensitive analysis Technique. [9] proposed the additive model of FAP. [7] stated the invariance Sensitivity analysis of FAP. We extend the work in [7, 9]. In this paper we proposed a new algorithm to analyze the minimal fuzzy assignment with the help of RSA Technique. A numerical example is included. This paper is organized as follows: In section 2, some basic definitions of fuzzy set theory and Mathematical formulation of Fuzzy Assignment Problem can be formulated. In section 3, new algorithm namely, One's Termination algorithm is used to find the initial fuzzy assignment cost and to verify the initial fuzzy optimal assignment cost, the Proposed Algorithm is used. In section 4, numerical example is discussed with the help of algorithms.

PRELIMINARIES

Definition 2.1[18]: A Fuzzy set $\tilde{A}$ in a universe of discourse X is defined by $\tilde{A} = \{x, \mu_{\bar{A}}(x) / x \in X\}$, where $\mu_{\bar{A}}(x) : X \to [0,1]$ is called the membership function of $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is the degree of membership to which $x \in \tilde{A}$

Definition 2.2 [12] Triangular Fuzzy Number & Membership function
For a Triangular Fuzzy Number A(x) can be represented as $A(a_1,a_2,a_3)$ and its membership function is defined as follows:

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$$\mu_{\tilde{A}}(x)=\begin{Bmatrix} 0 & x \le a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1 \le x < a_2 \\ \dfrac{a_3-x}{a_3-a_2} & a_2 \le x \le a_3 \\ 0, & x \ge a_3 \end{Bmatrix}$$

Definition 2.3 [12]: Addition of two Triangular Fuzzy Numbers
If A= (a_1, a_2, a_3) and B = (b_1, b_2, b_3) be two Triangular fuzzy numbers ,then the addition is defined as follows.
A + B = (a_1+b_1, a_2+b_2, a_3+b_3)

Definition 2.4 [10]: Ranking function
If A = (a_1, a_2, a_3; ω) be a triangular fuzzy number then the ranking function is defined as follows:

$$R(A)=\frac{\omega}{4}\{\alpha(a_1+a_3)+4a_2(1-\alpha)\}$$ with $\omega = 1$ and $\alpha = 0.5$

2.5 Mathematical Formulation of Fuzzy Assignment Problem
Mathematically, Fuzzy assignment problem can be stated as follows:

$$Min \quad \overline{Z}=\sum_{i=1}^{n}\sum_{j=1}^{n}\overline{c}_{ij}x_{ij} \qquad subject\ \ to$$

$$\sum_{i=1}^{n}x_{ij}=1 \qquad \sum_{j=1}^{n}x_{ij}=1$$

$$where \quad x_{ij}=\begin{Bmatrix} 1, if\ \ the\, i^{th}\ \ person\ \ is\, assigned\ \ to\ j^{th}\ \ job \\ 0, \quad otherwise, \end{Bmatrix}$$

and $\overline{c}_{ij}$ = [c_{ij}^{l} ,c_{ij} , c_{ij}^{u}] is fuzzy cost

3. **PROPOSED ALGORITHMS**

Here we proposed two kinds of algorithms to obtain the optimal solution of FAP.
(i) One's Termination Algorithm
(ii) Solution Algorithm

3.1 One's Termination Algorithm:

Step-1: Determine the fuzzy cost table:
(i) If the number of Row is equal to the number of Column, then go to step (3)
(ii) If the number of Row is not equal to the number of Column, then go to step (2)

Step-2: Add dummy Row or dummy Column, so that the cost table become a square matrix. The cost entries of dummy row / columns are always zero.

Step-3: Find the maximum element of each row (column) in the assignment matrix. Then divide each element i^{th} row and j^{th} column of the matrix, at least exists one ones exists in each row or column.

Step-4: (O_T) = Sum of the costs of all cells adjacent to one's cell is divided by the sum of the all non –zero cells.

Step-5: Assign the maximum possible to the cell having maximum (O_T). If (O_T) attains maximum values in more than one cell, choose the cell having minimum Assignment cost.

Step-6: Draw the minimum number of lines (horizontal or vertical) to cover all the ones of the matrix.
(i) If the number of assignments is equal to "n" then the fuzzy optimal solution is obtained.
(ii) If the number of assignments is less than "n" then the fuzzy optimal solutions is does not obtained then go to step 7.

Step-7: Draw the new revised fuzzy cost matrix as follows:

(i) Find the smallest element of the reduced matrix not covered by any of the lines.

(ii) Divide this element from all the uncovered elements and add the same to all the elements lying at the intersection on any two lines.

Step-8: Go to Step 3 to 7 and repeat the procedure until fuzzy optimum solution is obtained.

3.2 Solution Algorithm:

It is most help to easily obtain the Exact Fuzzy Optimal cost. The procedure is as follows:

Step-1: Obtained the Initial Solution of FAP using One's Termination Algorithm.

Step-2: From step1, convert it to LPP using simplex method.

Step-3: in a table of step 2, take a row for each constraint and a column for each variable. Enter the reduced cost in the last row of the table under the corresponding columns and negative entry for the cost under rhs column.

Step-4: Identify the unit column vector and label the row containing entry "ONE", label remaining the row as open and placing the "? "notation.

Step-5: Suppose the identity element "one" does not exist, choose the another minimum element and label the column vector.

Step-6: Find the smallest cost from the last row, the column corresponding to the smallest cost is a pivot element and using simplex method, we obtained the optimal solution of FAP.

Step-7: Delete corresponding Column to the initial FAP table from step 4.

Step-8: If the ratio of rhs value is positive then the procedure is stopped or, otherwise go to step 4.

Step-9: The Necessity condition is satisfied (one job is assigned to exactly one person) and the fuzzy Optimal Assignment solution exist or repeat the procedure go to step 3.

4. STASTISTICAL IMPLEMENTATION

Statistics deals with gathering, classifying and analyzing data, it is different from probability and fully defined probabilistical problems have unique solutions. Statistical theory can be split up into 2 branches, one is descriptive and another one is inductive. Descriptive statistics deals with the collection of data such as summarizing the available data by such variables as mean, median, mode and SD etc. The interference uses the data draw the diagrams and conclusions about the environment from which the data came .Now the Statistical concept using fuzzy assignment concept because we analyzed the significance level of each job to assigned to each machine. Using T-distribution the observed data collection ,then we obtain the mean and variance of the interval $[X_l^{\alpha}, X_U^{\alpha}]$ AND t-calculated value compare with the tabulated value of T in 5% LOS. If $T_{cal} > T_{tab}$, then the null hypothesis is rejected or otherwise accepted. The test statistic value is calculated as follows:

$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

The interval of Fuzzy Optimal Assignment Cost as follows:

$$\overline{X}_l \pm 1.96\,(\sigma / \sqrt{n}) \;\; \& \;\; \overline{X}u \pm 1.96\,(\sigma / \sqrt{n})$$

4.1 Numerical Example:

A bus renting company has one bus at each of the five sheds S_1, S_2, S_3, S_4 and S_5. A customer in each of the five places P_1, P_2, P_3, P_4 and P_5 requires a bus for a tour. The distance in kilometers between the sheds and places where the assigned to the customers so as to minimize the distance traveled? Find the minimum cost of the above assignment. Here the distances are considered as Triangular Fuzzy Numbers.

Table-1

Buses/Places	P1	P2	P3	P4	P5
S1	(11,14,15)	(6,7,9)	(15,17,20)	(`8,20,22)	(19,21,23)
S2	(2,4,8)	(10,12,15)	(20,24,26)	(15,17,20)	(6,8,12)
S3	(11,14,15)	(8,10,11)	(1,3,4)	(1,3,4)	(1,3,4)
S4	(6,7,9)	(12,151,16)	(8,10,11)	(8,10,11)	(14,16,19)
S5	(8,10,11)	(10,12,15)	(2,6,8)	(2,6,8)	(14,16,19)

Solution: Using the ranking function, the Fuzzy assignment problem can be converted into crisp assignment problem.

Table-2

Buses/ places	P1	P2	P3	P4	P5
S1	10	5	13	15	16
S2	3	9	18	13	6
S3	10	7	2	2	2
S4	5	11	9	7	12
S5	7	9	10	4	12

Using the One's Termination algorithm, we get the initial Fuzzy optimal assignment cost is

Table-3

Buses/places	P1	P2	P3	P4	P5
S1	4	$\mathbf{1^{(2)}}$	3	6	3
S2	$1^{(3)}$	$1^{(3)}$	3	4	$\mathbf{1^{(3)}}$
S3	7	4	$\mathbf{1^{(2)}}$	2	$1^{(2)}$
S4	$\mathbf{1^{(3)}}$	$1^{(2)}$	$1^{(2)}$	$1^{(1)}$	$1^{(1)}$
S5	2	$1^{(1)}$	$1^{(1)}$	$\mathbf{1^{(1)}}$	$1^{(1)}$

The optimal assignment cost is $s_1 \rightarrow p_2$, $s_2 \rightarrow p_5$, $s_3 \rightarrow p_3$, $S_4 \rightarrow p_1$, $s_5 \rightarrow p_4$.

The fuzzy optimal assignment optimal cost $R(\tilde{Z})$ = (6,7,9) + (6,8,12) + (1,3,4) + (6,7,9) + (2,6,8)
= (21, 31, 42)

The fuzzy optimal assignment cost = 23.375

5. TO VERIFY THE INITIAL FUZZY OPTIMAL ASSIGNMENT COST

We analyze the fuzzy optimal assignment cost with the help of RSAT (Revised Sensitivity analysis Technique). The reduced cost c_{ij} 's are entered in cost row and sum of reductions from row and column as (5+3+2+5+4+1+1+1+1+1=24) as negative entry under the rhs column is entered. The initial table after incorporating this change in table1 is shown below.

Table-4

bvs	X11	X12	X13	X14	X15	X21	X22	X23	X24	X25	X31	X32	X33	X34	X35	X41	X42	X43	X44	X45	X51	X52	X53	X54	X55	rhs
?	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
?	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
?	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
?	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1
?	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	1
?	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	1
?	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	1
?	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	1
Cij	2	1	3	3	3	1	3	6	4	2	5	4	1	1	1	1	2	2	1	2	2	2	3	1	3	-24

Fuzzy optimal assignment cost table is

Table-5

Bvs	x11	X13	X14	X15	X22	X23	X24	X25	X31	X32	X42	X43	X44	X51	X52	X54	X55	rhs
X12	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
X21	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	1
X53	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
X33	1	0	0	0	-1	-1	-1	-1	1	0	0	0	0	1	0	0	0	0
X34	-1	-1	-1	0	1	0	0	0	0	01	0	0	0	0	1	0	0	0
X35	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1
X41	0	0	1	0	0	0	0	1	0	0	0	0	0	0	-1	-1	-1	0
X44	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	1	1
c_{ij}	2	3	3	3	3	6	4	2	5	4	2	2	2	2	2	3	3	24

We observe that bvs is complete as well as rhs column has positive values and hence optimum solution is reached.

The optimum solution is $x_{12} = 1$; $x_{21} = 1$; $x_{35} = 1$; $x_{44} = 1$; $x_{53} = 1$

The fuzzy optimal assignment optimal cost $R(\tilde{Z}) = (6, 7, 9) + (2, 4, 8) + (1, 3, 4) + (8, 10, 11) + (2, 6, 8)$
$= (19, 30, 40)$
$= 22.375$

Existing Method (Hungarian Method), the Fuzzy Optimal Assignment Cost = 23.375

Determination of α- cut of optimal fuzzy assignment cost α-cut fuzzy assignment values are computed from their membership functions as follows:

$$\mu_{c12}(x) = \begin{cases} \dfrac{x-6}{7-6} & 6 \le x < 7 \\ \dfrac{9-x}{9-7} & 7 \le x \le 9 \end{cases}$$

Similarly $C_{12}^{(\alpha)} = [6 + \alpha, 9-2\alpha]$ (1)

$C_{21}^{(\alpha)} = [2+2\alpha, 8-4\alpha]$ (2)

$C_{35}^{(\alpha)} = [1+2\alpha, 4-\alpha]$ (3)

$C_{53}^{(\alpha)} = [8+2\alpha, 11-\alpha]$ (4)

$C_{44}^{(\alpha)} = [2+4\alpha, 8-2\alpha]$ (5)

$= [19+11\alpha, 40-10\alpha]$

Uncertainty Range Table:

Table-6

α-degree of uncertainty	Interval of Fuzzy assignment cost= [19+11α, 40-10 α]	Average range of α
0	[19,40]	29.5
0.1	[20.1,39]	29.55
0.2	[21.2,38]	29.6
0.3	[22.3,37]	29.65
0.4	[23.4,36]	29.7
0.5	[24.5,35]	29.75
0.6	[25.6,34]	29.8
0.7	[26.7,33]	29.85
0.8	[27.8,32]	29.9
0.9	[28.9,31]	29.95
1.0	[30,30]	30.00

Case-(1): Weighted Average :

Weighted average assignment value

$W(Z_{ij})$ = (0 x 29.5) + (0.1 x 29.55) + (0.2x 29.6)+ (0.3 x 29.65) + (0.4 x 29.70) + (0.5 x 29.75)
+ (0.6 x 29.80) + (0.7 x 29.85) +(0.8 x 29.90) + (0.9x 29.95) + (1.0 x 30)
= 0+2.955 + 5.92 + 8.895 + 11.88 + 14.875 +17.88+ 20.895 +23.92 +26.955 +30
= 164.175/ 10

$W(Z_{ij})$ = 16.4175.

Weighted Average of Lower bound:

$W_L(Z_{ij})$ = (0 x 19) + (0.1 x 20.1) + (0.2x 21.2) + (0.3x 22.3) + (0.4x 23.4) + (0.5x 24.5) + (0.6 x25.6)
+ (0.7 x 26.7) + (0.8x 27.8) + (0.9 x 28.9) + (1.0 x 30)
= 0+2.01+4.24+ 6.69 + 9.36 + 12.25+15.36+18.69+22.24+26.24+26.01+30
= 146.85/10
=14.685

Weighted Average of Lower bound $W_L(Z_{ij})$ = 14.685.

Weighted Average of upper bound

$W_U(Z_{ij})$ = (0 X 40) +(0.1X 39)+(0.2X 38) + (0.3 X 37) + (0.4X 36) + (0.5 X 35) + (0.6 X 34)
+ (0.7 X 33) + (0.8 X 32) + (0.9 X 31) + (1.0 X 30)
= 3.9 +7.6+11.1+14.4+17.5+20.4+23.1+25.6+27.9+30
=181.5/10

$W_U(Z_{ij})$ =18.15

Case-(i): Weighted Average

(I) Lower bound value = 14.685

(II) Upper bound value = 18.15

(III) Average value = 16.4175

Case-(ii): The certainty levels are splitted into 3 type's viz., (a) low level – 0 to 0.3 (b) Medium level 0.4 to 0.6 and (c) high level 0.7 to 1.0

(a) Low level 0 to 0.3
Lower bound lies between the interval [19.56, 21.74]
Upper bound lies between the interval [37.40, 39.6]

(b) **Medium level 0.4 to 0.6**
Lower bound lies between the interval [23.48, 25.52]
Upper bound lies between the interval [34.08, 35.92]

(c) **High level 0.7 to 1.0**
Lower bound lies between the interval [26.43, 29.17]
Upper bound lies in the interval [30.77, 33.23]

Case-(iii): The Certainty levels are splitted into 2 types viz, (a) 0.5 and below (b) 0.5 and above.

(a) 0.5 and below
The lower bound lies between [20.24,23.26]
The upper bound lies between [36.13,38.87]

(b) 0.5 and above

The lower bound lies between [26.43, 29.17]

The upper bound lies between [30.77, 33.23]

CONCLUSION

In this paper, we proposed a new algorithm to obtain the Fuzzy assignment cost by using One's Termination algorithm and the fuzzy optimal solutions are analyzed in RSA method and to find the various ranges are analyzed using t-test. It is most useful to the industrialists, quickly obtained the blocking range and it is very helpful to take optimal to the decision maker. This method is easy to understand for all and quickly obtained the interval of Fuzzy Assignment Cost.

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Source of support: Nil, Conflict of interest: None Declared.