

**CRYPTOGRAPHY BY APPLYING SUMUDU TRANSFORM  
TO TRIGONOMETRIC SINE FUNCTION**

**H. K. UNDEGAONKAR\***

**Assistant Professor, Department of Mathematics,  
Bahirji Smarak Mahavidyalaya, Basmathnagar, India.**

**R. N. INGLE**

**Principal & Associate Professor, Department of Mathematics,  
Bahirji Smarak Mahavidyalaya, Basmathnagar, India.**

*(Received On: 04-05-17; Revised & Accepted On: 24-05-17)*

---

**ABSTRACT**

*In this paper we introduce the application of Sumudu transform & inverse Sumudu transform in the process of encryption & decryption (i.e Cryptography) respectively. In the first part of the paper we consider one plain text and convert it to cipher text by applying Sumudu transform to trigonometric sine function and in the second part we convert cipher text to plain text by applying inverse sumudu transform. Finally we generalize some results for encryption and decryption.*

**Keywords:** *Sumudu transform, encryption, decryption, Cryptography, trigonometric sine function.*

---

**1. INTRODUCTION**

In Mathematical Sciences transformation is such a device which is used to convert an integrable function  $f(x)$  to function of new variable like  $f(s), f(p)$ , etc. There are several integral transforms as Laplace transform, Sumudu transform, Elzaki transform etc.

In 1990 Gamage K. Watugala has introduced new integral transform which is similar to Laplace transform [1] which is defined by

$$G(w) = \int_0^{\infty} \frac{1}{w} e^{-\frac{x}{w}} f(x) dx$$

He introduced this transform to solve differential equations and control engineering problems [1]. Some fundamental properties of Sumudu transform were established [7].

The word Cryptography comes from the Greek word kryptos which means hidden and graphein means ‘to write’. After demonetization in 2016 people prefer cashless transactions like A.T.M., Paytm, Mobile banking, internet banking etc. To operate these facilities password is required. Cryptography is the branch in which we study some techniques to secure communication between two persons or to keep confidentiality of their communication. Some integral transforms play an important role in the process of cryptography. A new cryptographic scheme was developed by applying Laplace transform in [3], [4] and obtained key which is the number of multiples of mod n therefore it is not easy for eavesdropper to trace the key by any attack

In this work we present cryptography by applying Sumudu transform and inverse Sumudu transform to trigonometric sine function.

Generally scientists, software engineers work in various projects and password is required for communication between two scientists confidentially. As Engineers and scientists have knowledge of transforms theory so they can use process of cryptography applying integral transforms easily.

---

***Corresponding Author: H. K. Undegaonkar\*,  
Assistant Professor, Department of Mathematics,  
Bahirji Smarak Mahavidyalaya, Basmathnagar, India.***

Some definitions and theorems are given below

**Theorem 1.1:** Let  $H_0, H_1, H_2, H_3, H_4, \dots$  be coefficients of  $t^2 \sinh 2t$  then given plaintext in terms of  $H_i$   $i=0, 1, 2, 3, 4, \dots$  under Laplace transform of  $Ht^2 \sinh 2t$  can be converted to cipher text  $H_i' = r_i - 26k_i$  for  $i=0, 1, 2, 3, \dots$  where

$$r_i = 2^{2i+1}(2i+3)(2i+2) H_i \quad \text{for } i=1, 2, 3, 4, \dots \text{ and a key is given by } k_i = \frac{r_i - H_i'}{26} \text{ for } i=0, 1, 2, 3, 4, \dots$$

**Theorem 1.2:** The given cipher text in terms of  $H_i'$ ,  $i=0,1,2,3,4, \dots$  With a given key  $k_i$  for  $i=0,1,2,3,4, \dots$  can be converted to plain text  $H_i$  under the inverse Laplace transform of

$$H \frac{d^2}{dp^2} \frac{2}{p^2 - 2^2} = \sum_{i=0}^{\infty} \frac{r_i}{p^{2i+4}} \quad \text{where } H_i = \frac{26k_i + H_i'}{2^{2i+1}(2i+2)(2i+3)} \text{ for } i=1, 2, 3, 4, \dots \text{ and } r_i = 26k_i + H_i'$$

The above theorems are proved in [3] by using Laplace transform and inverse Laplace transform of hyperbolic sine and cosine functions. Here we will prove the above theorems by applying Sumudu transform to trigonometric sine function.

**Definition 1.1:** Cryptology: It is the study of secrecy systems which can be traced back to the early Egyptians.

**Definition 1.2:** Plain text: The original message which is to be transmitted in such a form having secrecy.

**Definition 1.3:** Cipher text: when we convert the original message in the form having secrecy then this new form is said to be cipher text.

**Definition 1.4:** cipher: The method of converting plain text to cipher text is called cipher.

**Definition 1.5:** Encrypting: The process of converting plain text to cipher text is known as encrypting.

**Decrypting:** The reverse process by the beneficiary who knows key is known as decrypting and is accomplished by a decrypt.

The encrypt and decrypt or may be algorithms executed by people or computers.

## 2. CONVERSION OF PLAINTEXT TO CLIPHERTEXT BY APPLYING SUMUDU TRANSFORM TO TRIGONOMETRIC SINE FUNCTION

Consider the trigonometric sine series given by

$$\sin nx = nx - \frac{n^3 x^3}{3!} + \frac{n^5 x^5}{5!} - \frac{n^7 x^7}{7!} + \frac{n^9 x^9}{9!} + \dots \text{ then we have}$$

$$x^m \sin nx = nx^{m+1} - \frac{n^3 x^{m+3}}{3!} + \frac{n^5 x^{m+5}}{5!} - \frac{n^7 x^{m+7}}{7!} + \dots \tag{2.1}$$

First we consider the case when  $m = 1$  &  $n = 1$  then we have

$$x \sin x = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{8x^{10}}{7! \cdot 9!} - \frac{x^{12}}{11!} + \dots \tag{2.2}$$

In the process to convert given plaintext to cipher text we will allocate 0 to A, 1 to B, 2 to C, 3 to D ... .. 25 to Z

**Example 4. 1:** Let the given original message (plain text) be 'B A H I R J I' and suppose that this plain text be equivalent to

$$1 \quad 0 \quad 7 \quad 8 \quad 17 \quad 9 \quad 8$$

Let us assume that  $H_0=1, H_1=0, H_2=7, H_3=8, H_4=17, H_5=9, H_6=8$

writing  $H_0, H_1, H_2, H_3, H_4, H_5, H_6$  as the coefficients of  $x \sin x$  equation (5.3.2) becomes

$$Hx \sin x = Hx \sin x = H_0 x^2 - H_1 \frac{x^4}{3!} + H_2 \frac{x^6}{5!} - H_3 \frac{8x^{10}}{7!} + H_4 \frac{x^{10}}{9!} - H_5 \frac{x^{12}}{11!} + H_6 \frac{x^{14}}{13!} = \sum_{i=0}^6 (-1)^i \frac{x^{2i+2}}{2i+1} H_i$$

Therefore we have

$$Hx \sin x = 1x^2 - 0 \frac{x^4}{3!} + 7 \frac{x^6}{5!} - 8 \frac{x^{10}}{7!} + 17 \frac{x^{10}}{9!} - 9 \frac{x^{12}}{11!} + 8 \frac{x^{14}}{13!} \tag{2.3}$$

Applying Sumudu transform to equation (2.3) we have

$$S[Hx \sin x] = S[x^2] - 0S[x^4] + \frac{7}{5!} S[x^6] - \frac{8}{7!} S[x^8] + \frac{17}{9!} S[x^{10}] - \frac{9}{11!} S[x^{12}] + \frac{8}{13!} S[x^{14}]$$

$$S[Hx \sin x] = 2! w^2 - 0w^4 + 42w^6 - 64w^8 + 170w^{10} - 108w^{12} + 112w^{14} \tag{2.4}$$

Adjusting the resulting values 2, 0, 42,-64, 170,-108,112 modulo 26 we have  
 $2 \equiv 2(\text{mod } 26)$ ,  $0 \equiv 0(\text{mod } 26)$ ,  $42 \equiv -10(\text{mod } 26)$ ,  $-64 \equiv -12(\text{mod } 26)$ ,  $170 \equiv 14(\text{mod } 26)$ ,  $-108 \equiv -4(\text{mod } 26)$ ,  
 $112 \equiv 8(\text{mod } 26)$  we have

$$\begin{aligned} H_0' &= 2, & H_1' &= 0, & H_2' &= -10, & H_3' &= -12, & H_4' &= 14 \\ H_5' &= 4, & H_6' &= 8 & H_\alpha' &= 0 \text{ for } \alpha \geq 7 \end{aligned}$$

Thus the given plaintext is converted in to      2      0      10      12      14      4      8

i.e. The clipher text is                              C      A      K      M      O      E      I

Assuming  $r_0=2$ ,  $r_1=0$ ,  $r_2=42$ ,  $r_3=-64$ ,  $r_4=170$ ,  $r_5=-108$ ,  $r_6=112$  and by using the formula  
 $k_i = \frac{r_i - H_i'}{26}$  for  $i=0, 1, 2, \dots$  We obtained the key  
 0      0      2      -2      6      -4      4

**Table-2.1**

i	$H_i$	$k_i = \frac{r_i - H_i'}{26}$	$r_i = (-1)^i (2i + 2)H_i$	$H_i' = r_i - 26k_i$
0	1	0	2	2
1	0	0	0	0
2	7	2	42	-10
3	8	-2	-64	-12
4	17	6	170	14
5	9	-4	-108	4
6	8	4	112	8

From the above table (2.1) we have

**Theorem 2.1:** The given plain text in terms of  $H_i, i = 0,1,2,3,4, \dots$  . Under the Sumudu transform of  $Hx \sin x$  assuming  $H_0, H_1, H_2, \dots$  as non negative coefficients of  $x \sin x$  can be converted to clipher text  $H_i' = r_i - 26k_i, i = 0,1,2,3,4, \dots$  . Where  $r_i = (-1)^i (2i + 2)H_i$  and a key is given by  $k_i = \frac{r_i - H_i'}{26}$  for  $i=0, 1, 2, 3, \dots$

### 3. CONVERSION OF CLIPHERTEXT TO PLAINTEXT BY APPLYING INVERSE SUMUDU RANSFORMS

Applying inverse Sumudu transform to equation (2.4) we get again equation (2.3) i.e.

$$Hx \sin x = 1x^2 - 0 \frac{x^4}{3!} + 7 \frac{x^6}{5!} - 8 \frac{x^8}{7!} + 17 \frac{x^{10}}{9!} - 9 \frac{x^{12}}{11!} + 8 \frac{x^{14}}{13!}$$

Assuming coefficients of the above equation we get the plaintext B A H I R J I

**Table-3.1**

i	$H_i$	$(-1)^i \left[ \frac{26k_i + H_i'}{2i + 2} \right]^i$
0	1	1
1	0	0
2	7	7
3	8	8
4	17	17
5	9	9
6	8	8

From the above table (3.1) we have

**Theorem 3.1:** The given clipher text  $H_i'$  for  $i = 0,1,2,3,4, \dots$  . with a given key  $k_i$  can be converted to plain text  $H_i, i = 0,1,2,3,4, \dots$  .under the inverse Sumudu transform of  $S[Hx \sin x] = \sum_{i=0}^{\infty} (-1)^i r_i w^{2i+2}$  where

$$H_i = (-1)^i \left[ \frac{26k_i + H_i'}{2i + 2} \right]^i$$

4. In this case we consider  $m = 1$  &  $n = 2$  then equation (2.1) becomes

$$Hx \sin 2x = 2x^2 - 0x^4 + \frac{224}{5!}x^6 - \frac{1024}{7!}x^8 + \frac{8704}{9!}x^{10} - \frac{18432}{11!}x^{12} + \frac{65536}{13!}x^{14} \quad (4.1)$$

Applying Sumudu transform to equation (4.1) we have

$$G(w) = 4w^2 - 0w^4 + 1344w^6 - 8192w^8 + 87040w^{10} - 221184w^{12} + 917504w^{14} \quad (4.2)$$

Adjusting the resulting values 4, 0, 1344, -8192, 87040, -221184, 917504 modulo 26 we have  $4 \equiv 4 \pmod{26}$ ,  $0 \equiv 0 \pmod{26}$ ,  $1344 \equiv -8 \pmod{26}$ ,  $-8192 \equiv -2 \pmod{26}$ ,  $87040 \equiv -8 \pmod{26}$ ,  $-221184 \equiv -2 \pmod{26}$ ,  $917504 \equiv -10 \pmod{26}$  we have

$$H_0' = 4, \quad H_1' = 0, \quad H_2' = -8, \quad H_3' = -2, \quad H_4' = -8 \\ H_5' = -2, \quad H_6' = -10 \quad H_\alpha' = 0 \text{ for } \alpha \geq 7$$

Thus the plaintext is converted to clipher text 4 0 8 2 8 2 10 i.e., E A I C I C K

Assuming  $r_0 = 4$ ,  $r_1 = 0$ ,  $r_2 = 1344$ ,  $r_3 = -8192$ ,  $r_4 = 87040$ ,

$r_5 = -221184$ ,  $r_6 = 917504$  and by using the formula  $k_i = \frac{r_i - H_i'}{26}$  for  $i=0, 1, 2, \dots$  We obtained the key

$$0 \quad 0 \quad 52 \quad -315 \quad 334 \quad -8507 \quad 35289$$

**Table-4.1**

i	$H_i$	$k_i = \frac{r_i - H_i'}{26}$	$r_i = (-1)^i n^{2i+1} (2i + 2) H_i$	$H_i' = r_i - 26k_i$
0	1	0	4	4
1	0	0	0	0
2	7	52	1344	-8
3	8	-315	-8192	-2
4	17	3348	87040	-8
5	9	-8507	-221184	-2
6	8	35289	917504	-10

From the above table (4.1) we have

**Theorem 4.1:** The given plain text in terms of  $H_i$ ,  $i = 0, 1, 2, 3, 4, \dots$  under the Sumudu transform of  $Hx \sin 2x$  assuming  $H_0, H_1, H_2, \dots$  as non negative coefficients of  $x^2 \sin x$  can be converted to clipher text  $H_i' = r_i - 26k_i$  where  $r_i = (-1)^i n^{2i+1} (2i + 2) H_i$ . And a key is given by  $k_i = \frac{r_i - H_i'}{26}$  for,  $i = 0, 1, 2, 3, 4, \dots$ .

**Table-4.2**

i	$H_i$	$(-1)^i \left[ \frac{26k_i + H_i'}{2^{2i+1}(2i+2)} \right]^i$
0	1	1
1	0	0
2	7	7
3	8	8
4	17	17
5	9	9
6	8	8

From the above table (4.2) we have

**Theorem 4.2:** The given clipher text  $H_i'$  for  $i = 0, 1, 2, 3, 4, \dots$  With a given key  $k_i$  can be converted to plain text  $H_i$ ,  $i = 0, 1, 2, 3, 4, \dots$  under the inverse Sumudu transform of  $S[Hx \sin 2x] = \sum_{i=0}^{\infty} (-1)^i r_i (2i + 2)$  where

$$H_i = (-1)^i \left[ \frac{26k_i + H_i'}{2^{2i+1}(2i+2)} \right]^i \text{ for, } i = 0, 1, 2, 3, 4, \dots$$

From theorems (2.1) & (4.1) we can state more generally

**Theorem 4.3:** The given plain text in terms of  $H_i$ ,  $i = 0, 1, 2, 3, 4, \dots$  under the Sumudu transform of  $Hx^m \sin nx$  assuming  $H_0, H_1, H_2, \dots$  as non negative coefficients of  $x^m \sin nx$  can be converted to clipher text  $H_i' = r_i - 26k_i$  where

$r_i = (-1)^i n^{(2i+1)} (2i + 2)(2i + 3) \dots (2i + m + 1) H_i$ . And a key is given by  $k_i = \frac{r_i - H_i'}{26}$  for,  $i = 0, 1, 2, 3, 4, \dots$ .

This is the more generalization for encryption process in cryptography.

From theorem (2.2) & (4.2) we can state more generally

**Theorem 4.4:** The given cipher text  $H_i'$  for  $i = 0, 1, 2, 3, 4, \dots$ . With a given key  $k_i$  can be converted to plain text  $H_i$ ,  $i = 0, 1, 2, 3, 4, \dots$  under the inverse Sumudu transform of  $S[Hx^m \sin nx] = \sum_{i=0}^{\infty} (-1)^i r_i w^{2i+m+1}$

where  $H_i = (-1)^i \left[ \frac{26k_i + H_i'}{n^{2i+1}(2i+2)(2i+3)\dots(2i+m+1)} \right]$  for,  $i = 0, 1, 2, 3, 4, \dots$ .

## 5. CONCLUSION

In this paper we have applied Sumudu transform to trigonometric sine function and converted plaintext to cipher text successfully. In the second part we have obtained plaintext from ciphertext by applying inverse Sumudu transform successfully. We have also derived some generalizations which will be applicable in the process of encryption and decryption. Finally we may conclude that Sumudu transform has an important role in cryptography.

## 6. ACKNOWLEDGEMENTS

Authors are thankful to the coordinator of Research Centre & Principal of N.E.S. Science College Nanded to provide facilities of library, internet, and infibnet to complete this research work

## REFERENCES

1. G. K. Watugala, Sumudu transforms: a new integral transform to solve differential equations and control engineering problems, International Journal of Mathematical Education in Science and Technology 24(1993), vol. no 1, 35-43.
2. T.H.Barr, Invitation to Cryptography, Prentice Hall, (2002)
3. A.P. Hiwarekar: A new method of Cryptography using Laplace transform of Hyperbolic function. International Journal of Mathematical archive, 2011
4. G. Naga Lakshmi, B. Ravikuar and A. Chandra Sekher, A Cryptographic Scheme of Laplace transforms, International Journal of Mathematical archive, 2011, pp. 65-70
5. Sumeerai Nidhi Tyagi and Pradeep Kumar: Secure Communication for mobile Advoc network using Lagrange Polynomial and Integral transform (LPIT) with exponential function, International Journal of Innovative Research in Advanced Engineering (IJIRAE), 1(16),(2014).
6. G.A.Dhanorkar and A.P.Hiwarekar, A generalized Hill cipher using Matrix transformation international Journal of Math.Sci and Engineering applications, Vol.5.no. 4(July 2011). pp. 19-23.
7. Asiru M.A., Further properties and its applications, Int.Journal of Mathematical education in science and Technology, 33(3), pp. 441-449.

**Source of support: Nil, Conflict of interest: None Declared.**

**[Copy right © 2017. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]**