

Characterization of Contra $sg\alpha$ -Continuous Functions In Topological Spaces

A. DEVIKA*

Associate Professor, P.S.G. College of Arts and Science, Coimbatore, India.

S. SATHYAPRIYA

Assistant Professor, Sri Krishna College of Arts and Science, Coimbatore, India.

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ABSTRACT

In this paper, we introduce and investigate the notion of Characterization of Contra $sg\alpha$ -Continuous Functions in Topological Spaces. We obtain separation axiom of contra $sg\alpha$ -continuous functions and discuss the relationships between contra- $sg\alpha$ -continuity and other related functions.

Subject Classification: 54C05, 54C08, 54C10.

Keywords: contra $sg\alpha$ -continuous functions, $sg\alpha$ -graph, $sg\alpha$ -dense, $sg\alpha$ -clopen, $sg\alpha$ - T-spaces, $sg\alpha$ -Normal, $sg\alpha$ -lindeloff.

1. INTRODUCTION

N. Levine [16] introduced generalized closed sets (briefly g-closed set) in 1970. N. Levine [15] introduced the concepts of semi-open sets in 1963. Bhattacharya and Lahiri [6] introduced and investigated semi-generalized closed (briefly sg- closed) sets in 1987. Arya and Nour [3] defined generalized semi-closed (briefly gs-closed) sets for obtaining some characterization of s-normal spaces in 1990. O.Njastad in 1965 defined α -open sets [23].

In 1996, Dontchev [11] introduced a new class of functions called contra- continuous functions. A new weaker form of this class of functions called contra semi-continuous function is introduced and investigated by Dontchev and Noiri [12].

In this paper, the notion of $sg\alpha$ -closed sets [9] and contra $sg\alpha$ - continuous Space in topological spaces [8] is applied to introduce and study a new class of functions called characterization of contra $sg\alpha$ -continuous functions, as a new generalization of contra continuity, separation axioms, also the relationships with some other functions are discussed.

2. PRELIMINARIES

Through this paper (X, τ) , (Y, σ) and (Z, η) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X . The closure of A and the interior of A are denoted by $cl(A)$ and $int(A)$ respectively. (X, τ) will be replaced by X if there is no chance of confusion. Let us recall the following definitions as pre requests.

A subset A of a topological space X is said to be open if $A \in \tau$. A subset A of a topological space X is said to be closed if the set $X-A$ is open. The interior of a subset A of a topological space X is defined as the union of all open sets contained in A . It is denoted by $int(A)$. The closure of a subset A of a topological space X is defined as the intersection of all closed sets containing A . It is denoted by $cl(A)$.

Corresponding Author: A. Devika*

Associate Professor, P.S.G. College of Arts and Science, Coimbatore, India.

Definitions 2.1: A subset A of a space (X, τ) is said to be

1. semi open [15] if $A \subseteq \text{cl}(\text{int}(A))$ and semi closed if $\text{int}(\text{cl}(A)) \subseteq A$.
2. α -open [23] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and α -closed if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.
3. β -open or semi pre-open [1] if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ and β -closed or semi pre-closed if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.
4. pre-open [11] if $A \subseteq \text{int}(\text{cl}(A))$ and pre-closed if $\text{cl}(\text{int}(A)) \subseteq A$.

The complement of a semi-open (resp.pre-open, α -open, β -open) set is called semi-closed (resp.pre-closed, α -closed, β -closed). The intersection of all semi-closed (resp.pre-closed, α -closed, β -closed) sets containing A is called the semi-closure (resp.pre-closure, α -closure, β -closure) of A and is denoted by $\text{scl}(A)$ (resp. $\text{pcl}(A)$, $\alpha\text{-cl}(A)$, $\beta\text{-cl}(A)$). The union of all semi-open (resp.pre-open, α -open, β -open) sets contained in A is called the semi-interior (resp.pre-interior, α -interior, β -interior) of A and is denoted by $\text{sint}(A)$ (resp. $\text{pint}(A)$, $\alpha\text{-int}(A)$, $\beta\text{-int}(A)$). The family of all semi-open (resp.pre-open, α -open, β -open) sets is denoted by $SO(X)$ (resp. $PO(X)$, $\alpha - O(X)$, $\beta - O(X)$). The family of all semi-closed (resp.pre-closed, α -closed, β -closed) sets is denoted by $SCl(X)$ (resp. $PCl(X)$, $\alpha\text{-Cl}(X)$, $\beta\text{-Cl}(X)$).

Definitions 2.2: A subset A of a space (X, τ) is called

1. g -closed[16] if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ) . The complement of a g -closed set is called g -open set.
2. gs -closed set[7] if $\text{scl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ) .
3. sg -closed set[6] if $\text{scl}(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in (X, τ) .
4. αg -closed[17] if $\alpha(\text{cl}(A)) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ) .
5. $g\alpha$ -closed [18] if $\alpha(\text{cl}(A)) \subseteq U$, whenever $A \subseteq U$ and U is α -open in (X, τ) .
6. gp -closed [19] if $\text{pcl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ) .

Definition 2.3: Let X and Y be topological spaces. A function $f: X \rightarrow Y$ is said to be

1. continuous [14] if for each open set V of Y the set $f^{-1}(V)$ is an open subset of X .
2. α -continuous [23] if $f^{-1}(V)$ is a α -closed set of (X, τ) for every closed set V of (Y, σ) .
3. β -continuous [1] if $f^{-1}(V)$ is a β -closed set of (X, τ) for every closed set V of (Y, σ) .
4. pre-continuous [21] if $f^{-1}(V)$ is a pre-closed set of (X, τ) for every closed set V of (Y, σ) .
5. semi-continuous [15] if $f^{-1}(V)$ is a semi-closed set of (X, τ) for every closed set V of (Y, σ) .

Definition 2.4: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be

1. g -continuous [16] if $f^{-1}(V)$ is a g -closed set of (X, τ) for every closed set V of (Y, σ) .
2. gs -continuous[7] if $f^{-1}(V)$ is a gs -closed set of (X, τ) for every closed set V of (Y, σ) .
3. sg -continuous [6] if $f^{-1}(V)$ is a sg -closed set of (X, τ) for every closed set V of (Y, σ) .
4. αg -continuous [17] if $f^{-1}(V)$ is a αg -closed set of (X, τ) for every closed set V of (Y, σ) .
5. $g\alpha$ -continuous [18] if $f^{-1}(V)$ is a $g\alpha$ -closed set of (X, τ) for every closed set V of (Y, σ) .
6. gp -continuous [19] if $f^{-1}(V)$ is a gp -closed set of (X, τ) for every closed set V of (Y, σ) .

Definitions 2.5[22]: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be almost continuous if for every open set V of Y , $f^{-1}(V)$ is regular open in X .

Definitions 2.6[9]: A subset A of space (X, τ) is called $sg\alpha$ -closed if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is α -open in X . The family of all $sg\alpha$ -closed subsets of the space X is denoted by $SG\alpha C(X)$.

Definitions 2.7[9]: The intersection of all $sg\alpha$ -closed sets containing a set A is called $sg\alpha$ -closure of A and is denoted by $sg\alpha-cl(A)$. A set A is $sg\alpha$ -closed set if and only if $sg\alpha Cl(A) = A$.

Definitions 2.8[9]: A subset A in X is called $sg\alpha$ -open in X if A^c is $sg\alpha$ -closed in X . The family of a $sg\alpha$ -open sets is denoted by $SG\alpha O(X)$.

Definitions 2.9[9]: The union of all $sg\alpha$ -open sets containing a set A is called $sg\alpha$ -interior of A and is denoted by $sg\alpha-Int(A)$. A set A is $sg\alpha$ -open set if and only if $sg\alpha Int(A) = A$.

Definition 2.10[8]: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called $sg\alpha$ -continuous if $f^{-1}(V)$ is $sg\alpha$ -closed in (X, τ) for every closed set V of (Y, σ) .

Definition 2.11[8]: A function $f: X \rightarrow Y$ is said to be Contra $sg\alpha$ -Continuous if $f^{-1}(V)$ is $sg\alpha$ -closed in X for each open set V of Y .

Definition 2.12[8]: A space X is called locally $sg\alpha$ -indiscrete if every $sg\alpha$ -open set is closed in X .

Definition 2.13 [8]: If a function $f: X \rightarrow Y$ is called almost $sg\alpha$ -continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in SG\alpha O(X, x)$ such that $f(U) \subset Int(cl(V))$.

Definition 2.14[8]: If a function $f: X \rightarrow Y$ is called quasi $sg\alpha$ -open if image of every $sg\alpha$ -open set of X is open set in Y .

Definition 2.15[8]: If a function $f: X \rightarrow Y$ is called weakly $sg\alpha$ -continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in SG\alpha O(X, x)$ such that $f(U) \subset scl(V)$.

Definition 2.16 [8]: Let A be a subset of X . Then $sg\alpha-C l(A)-sg\alpha-Int(A)$ is called $sg\alpha$ -frontier of A and is denoted by $sg\alpha-F r(A)$.

Lemma 2.17[13]: The following properties hold for subsets A and B of a space X .

1. $x \in \ker(A)$ if and only if $A \cap F = \emptyset$ for any closed set F of X containing x .
2. $A \subset \ker(A)$ and $A = \ker(A)$ if A is open in X .
3. if $A \subset B$, then $\ker(A) \subset \ker(B)$

3. Characterization of Contra – $sg\alpha$ Continuous Functions in Topological Spaces

Definition 3.1: The graph $G(f)$ of a function $f: X \rightarrow Y$ is said to be contra $sg\alpha$ -graph if for each $(x, y) \in (X \times Y) \cap G(f)$, there exists a $sg\alpha$ -open set U in X containing x and a closed set V in Y containing y such that $U \times Y \cap G(f) = \emptyset$.

Theorem 3.2: Let $f: X \rightarrow Y$ be a function and let $g: X \times X \rightarrow Y$ be the graph function of f defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra $sg\alpha$ -continuous, then f is contra $sg\alpha$ -continuous.

Proof: Let U be an open set in Y . Then $X \times U$ is an open set in $X \times Y$. Since g is contra $sg\alpha$ -continuous, it follows that $f^{-1}(U) = g^{-1}(X \times U)$ is a $sg\alpha$ -closed set in X . Therefore f is contra $sg\alpha$ -continuous.

Theorem 3.3: Assume $SG\alpha O(X)$ is closed under any intersection. If $f: X \rightarrow Y$ and $g: X \rightarrow Y$ are contra $sg\alpha$ -continuous and Y is Urysohn, then $E = \{x \in X : f(x) = g(x)\}$ is $sg\alpha$ -closed in X .

Proof: Let $x \in X - E$. Then $f(x) = g(x)$. Since Y is Urysohn, there exists open sets V and W such that $f(x) \in V$, $g(x) \in W$ and $Cl(V) \cap Cl(W) = \emptyset$. Since f and g are contra $sg\alpha$ -continuous, $f^{-1}(Cl(V))$ and $g^{-1}(Cl(W))$ are $sg\alpha$ -open sets in X . Let $U = f^{-1}(Cl(V))$ and $G = g^{-1}(Cl(W))$. Then U and G are $sg\alpha$ -open sets containing x , set $A = U \cap G$, thus A is $sg\alpha$ -open set in X . Hence $f(A) \cap g(A) = f(U \cap G) \cap g(U \cap G) \subset f(U) \cap g(G) = Cl(V) \cap Cl(W) = \emptyset$. Therefore, $A \cap E = \emptyset$. This implies $x \notin sg\alpha - Cl(E)$. Hence E is $sg\alpha$ -closed set in X .

Definition 3.4: A subset A of a topological space X is said to be $sg\alpha$ -dense in X if $sg\alpha - Cl(A) = X$.

Theorem 3.5: Assume $SG\alpha O(X)$ is closed under any intersection. If $f: X \rightarrow Y$ and $g: X \rightarrow Y$ are contra $sg\alpha$ -continuous, Y is Urysohn, and $f = g$ on $sg\alpha$ -dense set $A \subset X$, then $f = g$ on X .

Proof: Since f and g are contra $sg\alpha$ -continuous. Y is Urysohn, by the theorem 3.3, $E = \{x \in X; f(x) = g(x)\}$ is $sg\alpha$ -closed in X . By assumption, $f = g$ on $sg\alpha$ -dense set A subset of X . Since $A \subset E$ and A is $sg\alpha$ -dense set in X , then $X = sg\alpha - Cl(A) \subset sg\alpha - Cl(E) = E$. Hence $f = g$ on X .

Definition 3.6: A space X is called $sg\alpha$ -connected provided that X is not the union of two disjoint non-empty $sg\alpha$ -open sets.

Theorem 3.7: If $f: X \rightarrow Y$ is a contra $sg\alpha$ -continuous from a $sg\alpha$ -connected space X onto any space Y , then Y is not a discrete space.

Proof: Let $f: X \rightarrow Y$ is a contra $sg\alpha$ -continuous and X is $sg\alpha$ -connected space. Suppose Y is a discrete space. Let A be a proper non empty open and closed subset of Y . Then $f^{-1}(A)$ is a proper non empty $sg\alpha$ -open and $sg\alpha$ -closed subset of X , which is a contradiction to the fact that X is $sg\alpha$ -connected space. Therefore, Y is not a discrete space.

Definition 3.8: A subset A of a space (x, τ) is said to be $sg\alpha$ -clopen if A is both $sg\alpha$ -open and $sg\alpha$ -closed.

Theorem 3.9: If $f: X \rightarrow Y$ is a contra $sg\alpha$ -continuous surjection and X is $sg\alpha$ -connected space, then Y is connected.

Proof: Let $f: X \rightarrow Y$ is a contra $sg\alpha$ -continuous and X is $sg\alpha$ -connected space. Suppose Y is not connected space. Then there exists disjoint open sets U and V such that $Y = U \cup V$. Therefore U and V are clopen in Y . Since f is contra $sg\alpha$ -continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $sg\alpha$ -open sets in X . Further f is surjective implies, $f^{-1}(U)$ and $f^{-1}(V)$ are non empty disjoint and $X = f^{-1}(U) \cup f^{-1}(V)$. This is contradiction to the fact that X is $sg\alpha$ -connected space. Therefore Y is connected.

Definition 3.10: A topological space X is said to be $sg\alpha$ - T_1 -space if for any pair of disjoint points x and y , there exist disjoint $sg\alpha$ -open sets G and H such that $x \in G$ and $y \in H$.

Definition 3.11: A topological space X is said to be $sg\alpha$ - T_2 -space if for any pair of disjoint points x and y , there exist disjoint $sg\alpha$ -open sets G and H such that $x \in G$ and $y \in H$.

Theorem 3.12: Let X be a $sg\alpha$ -connected and Y be T_1 -space, if $f: X \rightarrow Y$ is a contra $sg\alpha$ -continuous, then f is constant.

Proof: Let $f: X \rightarrow Y$ is contra $sg\alpha$ -continuous, X be a $sg\alpha$ -connected and Y is T_1 . Since Y is T_1 -space, $\Delta = \{f^{-1}(y); y \in Y\}$ is a disjoint $sg\alpha$ -open partition of X . If $|\Delta| \geq 2$, then there exists a proper $sg\alpha$ -open and $sg\alpha$ -closed set W . This is contradiction to the fact that X is $sg\alpha$ -connected. Therefore $|\Delta| = 1$, and hence f is constant.

Theorem 3.13: Let X be Y be topological spaces. If

1. For each pair of distinct points x and y in X , there exists a function $f: X \rightarrow Y$ such that $f(x) = f(y)$.
2. Y is an Urysohn space.
3. f is contra $sg\alpha$ -continuous at x and y . Then X is $sg\alpha$ - T_2 .

Proof: Let x and y be any distinct points in X and $f: X \rightarrow Y$ is a function such that $f(x) = f(y)$. Let $a = f(x)$ and $b = f(y)$, then $a = b$. Since Y is an Urysohn space, there exists open sets V and W in Y containing a and b respectively, such that $Cl(V) \cap Cl(W) = \emptyset$. Since f is contra $sg\alpha$ -continuous at x and y , then there exists $sg\alpha$ -open sets A and B in X containing x and y , respectively, such that $f(A) \subset Cl(V)$ and $f(B) \subset Cl(W)$. Then $f(A) \cap f(B) \subset Cl(V) \cap Cl(W) = \emptyset$. Therefore $A \cap B = \emptyset$. Hence X is $sg\alpha$ - T_2 .

Corollary 3.14: Let $f: X \rightarrow Y$ be contra $sg\alpha$ -continuous injective function from space X into an Urysohn space Y , then X is $sg\alpha$ - T_2 .

Proof: For each pair of distinct points x and y in X , f is contra $sg\alpha$ -continuous function from a space X into a Urysohn space such that $f(x) = f(y)$ because f is injective. Hence by theorem 3.13, X is $sg\alpha$ - T_2 .

Definition 3.15 [25]: A topological space X is said to be Ultra Hausdorff space if for every each pair of disjoint points x and y in X , there exist disjoint clopen sets U and V in X containing x and y respectively.

Theorem 3.16: If $f: X \rightarrow Y$ be contra $sg\alpha$ -continuous injective function from space X into a Ultra Hausdorff space Y , then X is $sg\alpha$ - T_2 .

Proof: Let x and y be any distinct points in X . Since f is injective $f(x) \neq f(y)$ and Y is Ultra Hausdorff space, implies there exists disjoint clopen sets U and V of Y containing $f(x)$ and $f(y)$ respectively. Then $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $sg\alpha$ -open sets in X . Therefore X is $sg\alpha$ - T_2 .

Definition 3.17 [25]: A topological space X is said to be Ultra Normal space if for each pair of disjoint closed sets can be separated by disjoint clopen sets.

Definition 3.18: A topological space X is said to be $sg\alpha$ -Normal if each pair of disjoint closed sets can be separated by disjoint $sg\alpha$ -open sets.

Theorem 3.19: If $f: X \rightarrow Y$ be contra $sg\alpha$ -continuous closed injection and Y is Ultra Normal, then X is $sg\alpha$ -normal.

Proof: Let E and F be distinct closed subsets of X . Since f is closed and injective $f(E)$ and $f(F)$ are disjoint closed sets in Y . Since Y is Ultra normal there exists disjoint clopen sets U and V in Y such that $f(E) \subset U$ and $f(F) \subset V$. This implies $E \subset f^{-1}(U)$ and $F \subset f^{-1}(V)$. Since f is contra $sg\alpha$ -continuous injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $sg\alpha$ -open sets in X . This shows X is $sg\alpha$ -normal.

Theorem 3.20: If $f: X \rightarrow Y$ is contra $sg\alpha$ -continuous and $g: Y \rightarrow Z$ is continuous. Then $g \circ f: X \rightarrow Z$ is contra $sg\alpha$ -continuous.

Proof: Let V be any open set in Z . Since g is continuous $g^{-1}(V)$ is open in Y . Since f is contra $sg\alpha$ -continuous $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $sg\alpha$ -closed set in X . Therefore, $g \circ f$ is contra $sg\alpha$ -continuous.

Theorem 3.21: If $f: X \rightarrow Y$ be contra $sg\alpha$ -continuous and $g: Y \rightarrow Z$ be $sg\alpha$ -continuous. If Y is $Tsg\alpha$ -space, then $g \circ f: X \rightarrow Z$ is contra $sg\alpha$ -continuous.

Proof: Let V be any open set in Z . Since g is $sg\alpha$ -continuous, $g^{-1}(V)$ is $sg\alpha$ -open in Y and since Y is $Tsg\alpha$ -space $g^{-1}(V)$ is open in Y . Since f is contra $sg\alpha$ -continuous $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $sg\alpha$ -closed set in X . Therefore, $g \circ f$ is contra $sg\alpha$ -continuous.

Definition 3.22: A function $f: X \rightarrow Y$ is said to be strongly $sg\alpha$ -open (resp. strongly $sg\alpha$ -closed) if image of every $sg\alpha$ -open (resp. $sg\alpha$ -closed) set of X is $sg\alpha$ -open (resp. $sg\alpha$ -closed) set in Y .

Theorem 3.23: If $f: X \rightarrow Y$ is surjective strongly $sg\alpha$ -open or strongly $sg\alpha$ -closed and $g: Y \rightarrow Z$ is a function such that $g \circ f: X \rightarrow Z$ is contra $sg\alpha$ -continuous. then g is contra $sg\alpha$ -continuous.

Proof: Let V be any closed (resp. open) set in Z . Since $g \circ f$ is contra $sg\alpha$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $sg\alpha$ -open (resp. $sg\alpha$ -closed). Since f is surjective and strongly $sg\alpha$ -open or strongly $sg\alpha$ -closed, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is $sg\alpha$ -open or $sg\alpha$ -closed. Therefore g is contra $sg\alpha$ -continuous.

Definition 3.24: A space X is said to be

1. $SG\alpha$ -closed compact if every $sg\alpha$ -closed cover of X has a finite subcover.
2. Countably $SG\alpha$ -closed compact if every countable cover of X by $sg\alpha$ -closed sets has a finite subcover.
3. $SG\alpha$ -Lindeloff if every $sg\alpha$ -closed cover of X has countable subcover.

Theorem 3.25: Let $f: X \rightarrow Y$ be a contra $sg\alpha$ -continuous surjection, then the following properties hold:

1. If X is $SG\alpha$ -closed compact, then Y is compact.
2. If X is countably $SG\alpha$ -closed compact, then Y is countably compact.
3. If X is $SG\alpha$ -Lindeloff then Y is Lindeloff.

Proof:

1. Let $\{V_\alpha : \alpha \in I\}$ be an open cover of Y . Since f is contra $sg\alpha$ -continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is $sg\alpha$ -closed cover of X . Since X is $SG\alpha$ -closed compact, there exists a finite subset I_0 of I such that $X = \cup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup \{V_\alpha : \alpha \in I_0\}$, which is finite subcover for Y . Therefore, Y is compact.
2. Let $\{V_\alpha : \alpha \in I\}$ be any countable open cover of Y . Since f is contra $sg\alpha$ -continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is countable $sg\alpha$ -closed cover of X . Since X is countably $SG\alpha$ -closed compact, there exists a finite subset I_0 of I such that $X = \cup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup \{V_\alpha : \alpha \in I_0\}$ is finite subcover for Y . Therefore, Y is countably compact.
3. Let $\{V_\alpha : \alpha \in I\}$ be an open cover of Y . Since f is contra $sg\alpha$ -continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is $sg\alpha$ -closed cover of X . Since X is $SG\alpha$ -Lindeloff, there exists a finite countable subset I_0 of I such that $X = \cup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup \{V_\alpha : \alpha \in I_0\}$ is finite subcover for Y . Therefore, Y is Lindeloff.

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