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On $b^{\#}$ generalized Closed Sets in Topological Spaces

K. ABSANA BANU*[1], Dr. S. PASUNKILIPANDIAN[2]

[1]M.phil scholar, Aditanar College of Arts and Science, Tiruchendur - (T.N), India.

[2]Associate professor Department of Mathematics,
Aditanar College of Arts and Science, Tiruchendur - (T.N), India.

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ABSTRACT

In this paper a new class of generalized closed sets, namely $b^{\#}$g-closed sets is introduced in topological spaces. We prove that this class lies between the class of $b^{\#}$-closed sets and the class of bg- closed sets. Also we find some basic properties and characterizations of $b^{\#}$ g –closed sets.

Keywords: *g-closed, gb –closed sets, $b^{\#}$-closed sets, $b^{\#}$g-closed set.*

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1. INTRODUCTION

In the year 1996, Andrijivic.D, introduced [4] and studied b-open sets. Later in 1998, Maki H., Noiri T. [10] gave a new type of generalized closed sets in topological space called gp- closed sets. OmariA.*et.al* [11] introduced and studied the concept of generalized b-closed sets (briefly gb-closed) in topological spaces. Recently Usha Parameswari R.*et. al*. [19] introduced the notions of $b^{\#}$-open sets and $b^{\#}$-closed sets by taking equality in the definitions of b-open sets and b-closed sets respectively. In this paper the notion of generalized $b^{\#}$ generalized-closed set is introduced and their basic properties are discussed.

2. PRELIMINARIES

Throughout this paper X denotes a topological space on which no separation axiom is assumed. For any subset A of X, *cl*(A) denotes the closure of A and *int*(A) denotes the interior of A in the topological space X. Further X \ A denotes the complement of A in X.

The following definitions and results are very useful in the subsequent sections.

Definition 2.1.A subset A of a space X is called
- (i) α–open [4] if A ⊆int(cl(int(A))) and α-closed if cl(int(cl(A)))⊆A ,
- (ii) semi-open [8] if A ⊆cl(int(A)) and semi-closed if int(cl(A)) ⊆A,
- (iii) pre-open [4] if A ⊆int(cl(A)) and pre-closed if cl(int((A)) ⊆A,
- (iv) semi-pre-open [5] or β -open [1] if A ⊆cl(int(cl(A))) and semi-pre-closed or β-closed if int(cl(int(A))) ⊆A,
- **(v)** regular open [7] if A= int(cl(A)) and regular closed if A = cl(int(A)).

Definition 2.2: Let (X,τ) be a topological space and $A \subseteq X$. The $b^{\#}$-closure of A, denoted by $b^{\#}$cl(A) and is defined by the intersection of all $b^{\#}$-closed sets containing A.

Definition 2.3: Let (X,τ) be a topological space and $A \subseteq X$. The $b^{\#}$-interior of A, denoted by $b^{\#}$int(A) and is defined by the union of all b-open sets contained in A.

Corresponding Author: K. Absana banu[1],*
[1]M.phil scholar, Aditanar College of Arts and Science, Tiruchendur - (T.N), India.

Definition 2.4: A subset A of space X is said to be

(i) b-open [4] if A ⊆cl(int(A)) ∪int(cl(A)) and b-closed if cl(int(A)) ∩int(cl(A)) ⊆A,
(ii) $b^{\#}$-open [19] if A = cl(int(A))∪int(cl(A)) and $b^{\#}$-closed if A = cl(int(A))∩int(cl(A)),
(iii) a p-set [17] if cl(int(A)) ⊆int(cl(A)),
(iv) a q-set [18] if int(cl(A)) ⊆cl(int(A)),
(v) π-open [20] if A is a finite union of regular open sets.

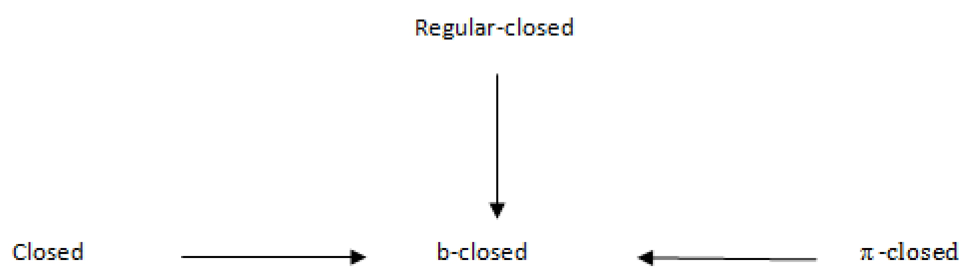
Lemma 2.5 [5]: Let A be a subset of a space X. Then (i) scl(A) = A ∪int(cl(A)),
(ii) pcl(A) = A∪cl(int(A)), (iii) spcl(A) = A∪int(cl(int(A))).

Definition 2.6: A subset A of a space X is called

(i) generalized closed [9](briefly g-closed) if cl(A) ⊆U whenever A ⊆U and U is open in X,
(ii) generalized semi-pre-closed [6] (briefly gsp-closed) if spcl(A) ⊆U whenever A ⊆U and U is open in X,
(iii) π-generalized pre-closed [15] (briefly π gp-closed) if pcl(A) ⊆U whenever A ⊆U and U is π-open in X,
(iv) regular weakly generalized closed [13] (briefly rwg-closed) if cl(int(A)) ⊆U whenever A ⊆U and U is regular open in X,
(v) generalized b-closed set [2] (briefly gb-closed) if bcl(A) ⊆U whenever A ⊆U and U is open in X,
(vi) regular generalized b-closed set [11] (briefly rgb-closed) if bcl(A) ⊆U whenever A ⊆U and U is regular open in X,
(vii) π-generalized b-closed set [3] (briefly πgb- closed) if bcl(A) ⊆U wheneverA ⊆U and U is π-open in X,
(viii) generalizedα-closed set [10] (briefly gα-closed) if αcl(A) ⊆U whenever A ⊆U and U is α-open in X,
(ix) regular generalized closed [14] (briefly rg-closed) if *cl(*A) ⊆U whenever A⊆U and U is regular open in X,
(x) π-generalized b^{*}-closed set [3] (briefly πgb^{*}- closed) if int (bcl(A))⊆U whenever A ⊆ U and U is π-open in X.

The complements of the above mentioned closed sets are their respective open sets.

Remark: 2.7:



Lemma 2.8[4]: Let A be a sub set of a space X. Then bcl(A) = scl(A) ∪pcl(A).

3. $b^{\#}$generalized closed set:

Definition 3.1: Let X be a space. A subset A of X is called $b^{\#}$-generalized closed (briefly $b^{\#}$g-closed) if $b^{\#}$ cl(A) ⊆U whenever A ⊆U, and U is b-open .

Theorem 3.2: Every $b^{\#}$-closed set is $b^{\#}$g-closed.

Proof: Let A be a $b^{\#}$-closed set in X. Let A⊆U where U is b-open. Since A is $b^{\#}$-closed, $b^{\#}$cl(A) = A ⊆ U. Thus we have $b^{\#}$cl(A) ⊆U. Therefore A is $b^{\#}$g-closed set.

Remark 3.3: The converse of the above Theorem need not be true .

Example 3.4: Let X = {a, b, c, d} with τ ={ Φ, {a}, {d}, {a, d}, {a, b}, {a, b, d}, {a, c, d}, X}. Consider A = {b}. A is not a $b^{\#}$-closed, However A is a $b^{\#}$g-closed.

Theorem 3.5: Every $b^{\#}$g-closed set is gb-closed.

Proof: Let A be $b^{\#}$g-closed set in X. Let A⊆U where U is open. Thus U is b-open. Since A is $b^{\#}$g-closed, $b^{\#}$cl(A) ⊆U. But bcl(A) ⊆$b^{\#}$cl(A).Thus we have bcl(A) ⊆U whenever A ⊆U and U is b-open.. Therefore A is gb-closed set.

Remark 3.6: The converse of the above Theorem need not be true.

Example 3.7: Let X = {a, b, c, d} with = {Φ, {a}, {d}, {a, d}, {a, b}, {a, b, d}, {a, c, d}, X}. Consider A = {c}. A is not a $b^{\#}$g-closed, However A is a gb-closed.

Theorem 3.8: Every $b^{\#}$g-closed set is πgb-closed.

Proof: proof is straight forward

Remark 3.9: The converse of the above theorem need not be true .

Example 3.10: Let X = {a, b, c, d} with = {Φ, {a}, {d}, {a, d}, {a, b}, {a, b, d}, {a, c, d}, X}.Consider A = {a}. A is not a $b^{\#}$g-closed, However A is a πgb-closed.

Theorem 3.11: Every $b^{\#}$g-closed set is rgb-closed.

Proof: proof is straight forward

Remark 3.12: The converse of the above theorem need not be true.

Example 3.13: Let X = {a, b, c, d} with = {Φ, {a}, {d}, {a, d}, {a, b}, {a, b, d}, {a, c, d}, X}.Consider A = {a, c}. A is not a $b^{\#}$g-closed, However A is a rgb-closed.

Remark 3.14: The following example shows that $b^{\#}$g-closed sets independent from α –closed set, gα-closed set, g-closed set, rg-closed set, rwg-closed set.

Example 3.15: Let X = {a, b, c, d} and Y = {a, b, c, d}be the topological spaces.
(i) consider ={Φ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, Y}. Then the set {c} is an α-closed set but not $b^{\#}$g-closed, and also the set {a} is an $b^{\#}$g-closed but not α-closed.

(ii) consider τ ={Φ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, Y}. Then the set {d} is an gα-closed set but not $b^{\#}$g-closed set in X, and also the set {b, c} is an $b^{\#}$g-closed but not gα-closed.

(iii) consider ={Φ, {a}, {d}, {a, d}, {a, b}, {a, b, d}, {a, c, d}, X}. Then the set {c, d} is an g-closed set but not $b^{\#}$g-closed set in X, and also the set {d} is an $b^{\#}$g-closed but not g-closed.

(iv) consider ={Φ, {a}, {d}, {a, d}, {a, b}, {a, b, d}, {a, c, d}, X}. Then the set {a, d} is an rg-closed set but not $b^{\#}$g-closed set in X, and also the set {a, b} is an $b^{\#}$g-closed but not rg-closed.

(v) consider ={Φ, {a}, {d}, {a, d}, {a, b}, {a, b, d}, {a, c, d}, X}. Then the set {b, c} is an rwg-closed set but not $b^{\#}$g-closed set in X, and also the set {d} is an $b^{\#}$g-closed but not rwg -closed.

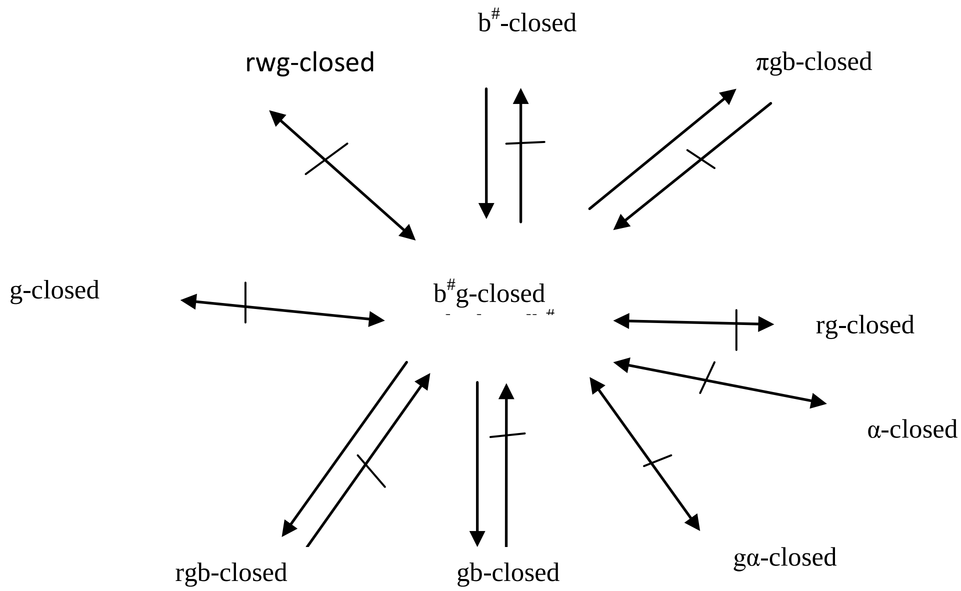
Theorem 3.16: Let A be a subset of a topological space X. Then $cl(int(A)) \cap int(cl(A)) \subseteq bcl(A) \subseteq b^{\#}cl(A)$.

Proof: Obvious.

Theorem 3.17:
(i) If A is a p-set, then $cl(int(A)) \subseteq b^{\#}cl(A)$,
(ii) If A is a q-set, then $int(cl(A)) \subseteq b^{\#}cl(A)$,
(iii) If A is a t-set, then $int(A) \subseteq b^{\#}cl(A)$.

Proof: Let A be a p-set. Then $cl(int(A)) \subseteq int(cl(A))$. That is $cl(int(A)) = cl(int(A)) \cap int(cl(A))$. Therefore by Theorem 3.16, $cl(int(A)) \subseteq bcl(A)$. This proves (i). Similarly the proof of (ii),(iii) .

Remark 3.18:



A $\longrightarrow$ B means A imply B. A $\not\longrightarrow$ B means A does not imply B. A $\longleftrightarrow$ B means A and B are independent.

4. CHARACTERIZATION

Theorem 4.1. Suppose A is a p-set and $b^{\#}$g-closed. Then

(i) A is πgp-closed,
(ii) A is πgb*-closed,
(iii) A is gsp-closed.

Proof: Let A be a p-set and $b^{\#}$g-closed in X. Then by using Theorem 3.16 (i) cl(int(A)) $\subseteq b^{\#}$cl(A). Let A $\subseteq$U and U is π-open. Then $b^{\#}$cl(A) $\subseteq$U. This implies cl(int(A)) $\subseteq$U. That is A $\cup$cl(int(A)) $\subseteq$U. Hence pcl(A) $\subseteq$U. Hence A is πgp-closed. This proves (i).Similarly the Proof of (ii) and (iii).

Theorem 4.2: Suppose A is a q-set and $b^{\#}$g-closed. Then A is πgs-closed.

Proof: Let A be a q-set and $b^{\#}$g-closed in X. Then by using Theorem 3.16 (ii) int(cl(A)) $\subseteq b^{\#}$cl(A). Let A $\subseteq$U and U is π-open. Then $b^{\#}$cl(A) $\subseteq$U. This implies int(cl(A)) $\subseteq$U. That is A $\cup$int (cl(A)) $\subseteq$U. Hence scl(A) $\subseteq$U. Hence A is πgs-closed.

Remark 4.3: union and intersection of any two $b^{\#}$g-closed need not be $b^{\#}$g-closed.

Example 4.4: Let X = {a, b, c, d} with τ ={Φ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, X}. Then the sets {a} and {b, c} is $b^{\#}$g-closed but {a, b, c} is not $b^{\#}$g-closed. And also {b, c} and {a, c, d} is $b^{\#}$g-closed. but{c}is not $b^{\#}$g-closed.

Theorem 4.5: If A and B are two $b^{\#}$g-closed set in X such that either A$\subseteq$B=B or B$\subseteq$A both intersection and union of two $b^{\#}$g closed set is $b^{\#}$g closed.

Proof: Let A and B are two $b^{\#}$g closed set in a topological space X. since, A$\subseteq$B or B$\subseteq$A, then A$\cup$B=A or A$\cup$B=B. Since A and B are $b^{\#}$g closed sets then A$\cup$B is $b^{\#}$g closed. Similarly A$\cap$B=A or A$\cap$B then A$\cap$B is $b^{\#}$g closed.

Theorem 4.6: A set A is $b^{\#}$g-closed set if and only if $b^{\#}$cl(A) $\subseteq$A contains no non-empty b- closed sets.

Proof:
Necessity: Suppose that F is a non- empty b- closed subset of X such that F $\subseteq b^{\#}$cl(A) \A. Then F $\subseteq$ b $^{\#}$cl(A) and X\F is b-open in X. since A is $b^{\#}$g-closed in X, $b^{\#}$cl(A) $\subseteq$ X\F, F $\subseteq$X\$b^{\#}$cl(A).Thus F $\subseteq b^{\#}$cl(A) $\cap$ (X\$b^{\#}$cl(A)) = Φ.

Sufficiency: A ⊆U and U is b-open. Suppose $b^{\#}cl(A)$ is not contain U, then $b^{\#}cl(A) \cap U^c$ is a non - empty b- closed set of $b^{\#}cl(A)\backslash A$, which is a contradiction. Therefore $b^{\#}cl(A) \subseteq U$ and hence A is $b^{\#}$g-closed.

Theorem 4.7: If A is $b^{\#}$g closed. set and $A \subseteq B \subseteq b^{\#}cl(A)$ then B is $b^{\#}$g closed. subset of X.

Proof: Let A be any $b^{\#}$g-closed. Set and B be any subset of X such that $A \subseteq B \subseteq b^{\#}cl(A)$

Let U be any b-open such that B⊆U. Since A⊆B, then A⊆U.Since A is $b^{\#}$g closed.

Then $b^{\#}cl(A) \subseteq U$. Since $B \subseteq b^{\#}cl(A)$, then $b^{\#}cl(B) \subseteq b^{\#}cl(A) \subseteq U$.Therefore $b^{\#}cl(A) \subseteq U$. Hence B is $b^{\#}$g-closed.

Theorem 4.8: Let A be $b^{\#}$g-closed. Then A is $b^{\#}$-closed if and only if $b^{\#}cl(A)\backslash A$ is b^{-}closed.

Proof: Let A be a topological space (X, τ). Suppose A is $b^{\#}$-closed. Then $b^{\#}cl(A)=A$. This implies $b^{\#}cl(A)\backslash A=\Phi$, which is b-closed. Conversely suppose that $b^{\#}cl(A)\backslash A$ is b-closed. Since A is $b^{\#}$g-closed, by above theorem 4.6, $b^{\#}cl\backslash A$ does not contains any non-empty b-closed set. Therefore $b^{\#}cl(A)\backslash A=\Phi$. Hence $b^{\#}cl(A)=A$. Thus A is $b^{\#}$- closed.

Theorem 4.9: If a subset A of X is $b^{\#}$g-closed set in X then $b^{\#}cl(A)\backslash A$ contains no non-empty Closed set.

Proof: using 4.6, we get the proof

Theorem 4.10: For every element x in a space X, X-{x} is a $b^{\#}$g- closed or b-open.

Proof: Suppose X-{x**}** is not b-open. Then X is the only b-open set containing X-{x}. This implies $b^{\#}cl(X-\{x\}) \subseteq X$. Hence X-{x} is $b^{\#}$g closed.

Theorem4.11: If A is both b-open and $b^{\#}$g-closed set in X, then A is $b^{\#}$-closed set.

Proof: Since A is b-open and $b^{\#}$g-closed in X, $b^{\#}cl(A) \subseteq A$. But always $A \subseteq b^{\#}cl(A)$. Therefore A= $b^{\#}cl(A)$. Hence A is $b^{\#}$-closed.

Theorem4.12: Every subset is $b^{\#}$g-closed in X if and only if every b-open set is $b^{\#}$-closed.

Proof: Let A be a b-open in X, by hypothesis A is $b^{\#}$g-closed in X, By theorem 4.11, A is a $b^{\#}$-closed set conversely Let A be a subset of X and U a b-open set such that A⊆U. Then by hypothesis U is $b^{\#}$-closed. This implies that $b^{\#}cl(A) \subseteq b^{\#}cl(U)=U$. Hence A is $b^{\#}$g-closed.

CONCLUSION

The present chapter has introduced a new concept called $b^{\#}$ g-closed set in a topological spaces. It also analyzed some of properties. The implication shows the relationship between $b^{\#}$g-closed sets and the other existing sets.

REFERENCES

1. Abd El-Monsef.M.E, El-Deeb.S.N and Mahmoud.R.A, *β*-open sets and *β*–continuous map-pings, Bull. Fac. Sci. Assiut Univ. 12 (1983), 77-90.
2. Ahmad Al-Omari and Mohd.SalmiMd.Noorani, Generalized b-closed sets, Bull. Malays.Math. Sci. Soc., 32(1) (2009), 19-30.
3. Al-Obiadi .A.K, π -generalized b-closed sets in topological spaces, Ibn Al-Haitham J. for Pure and Appl.Sci., 24(3) (2011), 1-9.
4. Andrijevic .D, On b-open sets, Mat. Vesnik 48 (1996), 59-64.
5. Andrijevic .D, Semi-preopen sets, ibid. 38 (1986), 24-32.
6. Dontchev.D, On generalizing semi-pre-open sets, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math., 16(1995), 35-48.
7. Dugundji.J, Topology, Allyn and Bacon, Boston 1966.
8. Levine.N, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly 70 (1963), 36-41.
9. Levine.N, Generalized Closed sets in topology, Rendi.Circ. Mat.,Palermo., 19(2)(1970), 89-96.
10. Maki .H, Umehara.J and Noiri .T, Every topological space is Pre-T1/2, Mem.Fac.Sci.Kochi. Univ.Ser.A Math., 17 (1996), 33-42.

11. Mariappa.K and Sekar.S, On Regular Generalized b-Closed Set, Int. Journal of Math.Analysis., 7 (13) (2013), 613-624
12. Mashhour .A.S, Abd El-Monsef.M.E, and El-Deeb.S.N, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt 53 (1982), 47-53.
13. Nagaveni.N, Studies on Generalizations of Homeomorphisms in Topological Spaces, Ph.D.Thesis, Bharathiar University, Coimbatore,
14. Palaniappan. N., and Rao. K. C., Regular generalized closed sets, KyungpookMath.J. 33(1993), 211-219.
15. Park.J.H, On πgp-closed sets in topological spaces, Indian J. Pure Appl. Math., 112 (4) (2006), 257-283.
16. Sarsak.M.S, and Rajesh.N, On Generalized semi-preclosed sets, Int. Math. Forum., 5(12) (2010), 573-578.
17. Thangavelu .P, Rao K. C., p–sets in topological spaces, Bull. Pure and Appl. Sci. 21(E) (2) (2002), 341-345.
18. Thangavelu .P, Rao K. C., q–sets in topological spaces, Prog of Maths 36(1&2) (2002), 159-165.
19. UshaParameswari.R and Thangavelu.P, On $b^{\#}$-open sets, International Journal of Mathematics Trends and Technology., 3 (5) (2014), 202-218.
20. Zaitsav.V, On certain classes of topological spaces and their bicompactifications, DoklAkadSSSR., 178 (1968), 778-779.

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