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# Generalized b-strongly b*-closed sets in Topological Spaces

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***ABSTRACT***

*In this paper a new class of generalized closed sets in topological spaces, namely generalized b-strongly b*-closed (briefly, gbsb*-closed) set is introduced. We give some basic properties and various characterizations of gbsb*-closed sets. Also we introduce gbsb*-neighbourhood in a topological spaces and investigate some basic properties.*

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***Keywords:*** *gbsb*-closed, gbsb*-open, gbsb*-nbhd.*

## 1. INTRODUCTION

In 1970, Levine [8] introduced the class of generalized closed sets. The notion of generalized closed sets has been extended and studied exclusively in recent years by many topologists. In 1996, Andrjivic [16] gave a new type of generalized closed sets in topological spaces called b-closed sets. Later in 2012 A.Poongothai and P.Parimelazhagan [21] introduced sb*-closed sets and investigated some of their properties.

In this paper, a new class of generalized closed set called generalized b-strongly b*-closed set is introduced. The notion of generalized b-strongly b*-closed set and its different characterizations are given in this paper.

## 2. PRELIMINARIES

Throughout this paper (X, $\tau$) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. (X, $\tau$) will be replaced by X if there is no changes of confusion. For a subset A of a topological space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. We recall the following definitions and results.

**Definition 2.1:.** Let (X, $\tau$) be a topological space. A subset A of X is said to be

1. semi-open [9] if A $\subseteq$ cl(int(A)) and semi-closed if int(cl(A)) $\subseteq$ A.
2. $\alpha$-open [13] if A $\subseteq$ int(cl(int(A))) and $\alpha$-closed if cl(int(cl(A))) $\subseteq$ A.
3. pre-open [14] if A $\subseteq$ int(cl(A)) and pre-closed if cl(int(A)) $\subseteq$ A.
4. b-open [16] if A$\subseteq$int(cl(A))$\cup$cl(int(A)) and b-closed if int(cl(A))$\cap$cl(int(A)) $\subseteq$A.
5. regular open [1] if int(cl(A))=A and regular closed if cl(int(A))=A.
6. $\pi$-open [4] if A is the union of regular open sets and $\pi$-closed if A is the intersection of regular closed sets.

**Definition 2.2:** Let (X, $\tau$) be a topological space and A $\subseteq$ X. The b-closure (resp.pre-closure, semi-closure, $\alpha$-closure) of A, denoted by bcl(A) (resp.pcl(A), scl(A), $\alpha$cl(A)) and is defined by the intersection of all b-closed (resp. pre-closed, semi-closed, $\alpha$-closed) sets containing A.

**Definition 2.3:** Let (X, $\tau$) be a topological space and A $\subseteq$ X. The b-interior of A, denoted by bint(A) and is defined by the union of all b-open sets contained in A.

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**Definition 2.4:** Let (X, $\tau$) be a topological space. A subset Aof X is said to be

1. generalized closed [8](briefly g-closed) if cl(A)$\subseteq$U whenever A$\subseteq$U and U is open in (X, $\tau$).
2. generalizedb-closed [2] (briefly gb-closed) if bcl(A)$\subseteq$U whenever A$\subseteq$U and U is open in (X, $\tau$).
3. regular generalized closed [7] (briefly rg-closed) if cl(A) $\subseteq$U whenever A$\subseteq$U and U is regular open in (X,$\tau$).
4. regular generalized b-closed [17](briefly rgb-closed) if bcl(A)$\subseteq$U whenever A$\subseteq$U and U is regular open in (X, $\tau$).
5. generalized $\alpha$b-closed [15](briefly g$\alpha$b-closed) if bcl(A)$\subseteq$U whenever A$\subseteq$U and U is $\alpha$-open in (X, $\tau$)
6. generalized pre-regular closed [20] (briefly gpr-closed) if pcl(A) $\subseteq$U whenever A$\subseteq$U and U is rg-open in (X,$\tau$).
7. generalized p-closed [11](briefly gp-closed) if pcl(A)$\subseteq$U whenever A$\subseteq$U and U is open in (X, $\tau$).
8. $\alpha$-generalized closed [10] (briefly $\alpha$g-closed) if $\alpha$cl(A)$\subseteq$U whenever A$\subseteq$U and U is an open in (X, $\tau$).
9. $\pi$-generalized b-closed [6](briefly $\pi$gb-closed) if bcl(A)$\subseteq$U whenever A$\subseteq$U and U is $\pi$-open in (X, $\tau$).
10. weakly closed [19](briefly w-closed) if cl(A) ⊆ U whenever A $\subseteq$U and U is a semi-open in (X, $\tau$).
11. weakly generalized closed [18] (briefly wg-closed) if cl(int(A))$\subseteq$U whenever A$\subseteq$U and U is an open in (X,$\tau$).
12. semi weakly generalized closed [5](briefly swg-closed) if scl(A)$\subseteq$U whenever A$\subseteq$U and U is an wg-open in (X, $\tau$).
13. w$\alpha$-closed [3] if $\alpha$cl(A)⊆U whenever A⊆U and U is a w-open in (X, $\tau$).
14. strongly b*-closed [21](briefly sb*-closed) if cl(int(A)))$\subseteq$U whenever A$\subseteq$U and U is b-open in (X, $\tau$).

The complements of the above mentioned closed sets are their respective open sets.

**Theorem 2.5 [21]:** For a topological space (X, $\tau$),

(i) Every open set is sb*-open.
(ii) Every $\alpha$-open set is sb*-open.
(iii) Every sb*-open set is b-open.

**Theorem 2.6 [22]:** For any subset A of a topological space (X, $\tau$),

(i) sint(A)=A∩cl(int(A))
(ii) pin(A)=A∩int(cl(A))
(iii) scl(A)=A∪int(cl(A))
(iv) pcl(A)=A∪cl(int(A)).

**Remark 2.7 [12]:** Jankovic and Reilly pointed out that every singleton {x} of a space X is either nowhere dense or pre-open. This provides another decomposition X=$X_1$∪$X_2$, where $X_1$={x∈X/ {x} is nowhere dense} and $X_2$={x∈X/ {x} is pre-open}.

**Definition 2.8 [12]:** The intersection of all gb-open sets containing A is called the gb-kernel of A and it is denoted by gb-ker(A).

**Lemma 2.9 [12]:** For any subset A of X, $X_2$∩cl(A)$\subseteq$gb-ker(A).

**Remark 2.10:** For any subset A of a topological space (X, $\tau$),

(i) X\bcl(A)=bint(X\A)
(ii) X\bint(A)=bcl(X\A).

### 3. Generalized b-strongly b*-closed set

**Definition 3.1:** A subset A of a topological space (X, $\tau$) is called a generalized b-strongly b*-closed set (briefly,gbsb*-closed) if bcl(A)$\subseteq$U whenever A$\subseteq$U and U is sb*-open in (X, $\tau$). The collection of all gbsb*-closed sets of X is denoted by gbsb*-C(X, $\tau$).

**Theorem 3.2:**

(i) Every closed set is gbsb*-closed.
(ii) Every semi-closed set is gbsb*-closed.
(iii) Every $\alpha$-closed set is gbsb*-closed.
(iv) Every pre-closed set is gbsb*-closed.
(v) Every b-closed set is gbsb*-closed.
(vi) Every regular closed set is gbsb*-closed.
(vii) Every $\pi$-closed set is gbsb*-closed.

**Proof:**

(i) Let A be a closed set. Let $A \subseteq U$, U is sb*-open in X. Since A is closed, then $cl(A)=A \subseteq U$. But $bcl(A) \subseteq cl(A)$. Thus we have $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is sb*-open and therefore, A is a gbsb*-closed set.

(ii) (ii), (iii), (iv), (v), (vi) and (vii) are similar to (i).

The reverse implications need not be true which is shown in the following examples.

**Example 3.3:** Let X={a, b, c} and $\tau$={$\phi$, {a}, X}.

(i) Then the set {b} is gbsb*-closed but not a closed set.

(ii) The set {a, c}is a gbsb*-closed set but not a b-closed set

**Example 3.4:** Let X={a, b, c, d} with$\tau$={$\phi$, {a, b}, {a, b, c}, X}.

(i) The set {a, c, d}is gbsb*-closed set but not a semi-closed set.

(ii) The set {a}is gbsb*-closed set but not a $\alpha$-closed set.

(iii) The set {b, c} is gbsb*-closed set but not a pre-closed set.

**Example 3.5:** Let X = {a, b, c, d} with$\tau$= {$\phi$, {a},{b},{a, b},{a, b, c}, X}. The set {d} is gbsb*-closed set but not a regular-closed set.

**Example 3.6.** Let X= {a, b, c, d} with $\tau$={$\phi$, {a},{c},{a, c},{c, d},{a, c, d},{a, b, c}, X}. The sets {a, b} and {b, c, d} are gbsb*-closed but not a $\pi$-closed set.

**Theorem 3.7:**

(i) Every gbsb*-closed set is gb-closed.

(ii) Every gbsb*-closed set is rgb-closed set.

(iii) Every gbsb*-closed set is g$\alpha$b-closed set.

(iv) Every gbsb*-closed set is $\pi$gb-closed set.

**Proof:**

(i) Let A be a gbsb*-closed set. Let $A \subseteq U$, U is open. Since open set is sb*-open, then U is sb*-open. Since A is gbsb*-closed, $bcl(A) \subseteq U$. Thus, we have $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open and therefore, A is gb-closed set.

(ii) (ii), (iii) and (iv) are similar to (i).

The reverse implications need not be true which is shown in the following examples.

**Example 3.8:** Let X = {a, b, c, d} with$\tau$= {$\phi$, {a},{b},{a, b},{a, b, c}, X}.

(i) The set {a, b, d} is gb-closed but not a gbsb*-closed set.

(ii) The sets {a, b, d} is g$\alpha$b-closed but not a gbsb*-closed set.

**Example 3.9:** Let X = {a, b, c, d} with $\tau$= {$\phi$, {a, b}, {a, b, c}, X}. The set {a, b} is rgb-closed but not a gbsb*-closed set.

**Example 3.10:** Let X = {a, b, c, d} with $\tau$ = {$\phi$, {a},{c},{a, c},{c, d},{a, c, d},{a, b, c}, X}. The set {a,c}is$\pi$gb-clos ed set but not a gbsb*-closed set.

**Remark 3.11:** The following examples shows that gbsb*-closed sets are independent from $\alpha$g-closed set, g-closed set, rg-closed set, gpr-closed set, wg-closed set, swg-closed set and gp-closed set.

**Example 3.12:** Let X= {a, b, c, d} with $\tau$= {$\phi$, {a}, {b},{a, b},{b,c},{b, c, d},{a, b, c}, X}.

(i) The sets {a, b},{a, b, d} are rg-closed sets but not a gbsb*-closed in (X, $\tau$) and the set {c} is gbsb*-closed but not rg-closed.

(ii) The sets {a, b, d}, {b, d} are $\alpha$g-closed sets but not gbsb*-closed.

(iii) The set {a, b, d} is wg-closed but not gbsb*-closed.

**Example 3.13:** Let X = {a, b, c, d} with $\tau$= {$\phi$, {a}, {b},{a, b},{a, b, c}, X}.

(i) The sets {a, b},{a, b, d} are gpr-closed sets but not gbsb*-closed in (X, $\tau$) and the sets {a},{b} are gbsb*-closed but not a gpr-closed.

(ii) The set {a, b, d} is a g-closed set but not gbsb*-closed in (X,$\tau$) and the sets {a},{b},{c} are gbsb*-closed but not g-closed.

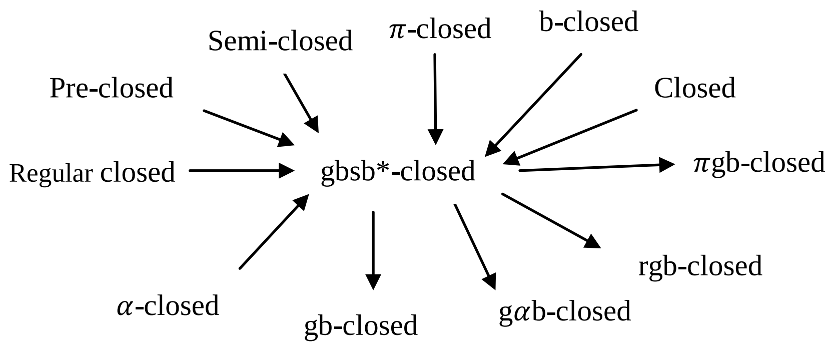
 

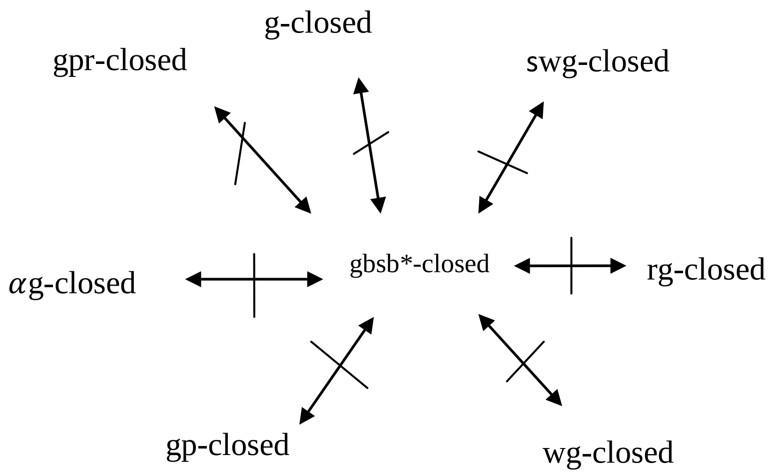
(iii) The set {a, b, d} is as wg-closed set but not gbsb*-closed in (X, $\tau$) and the sets {a},{b},{c} are gbsb*-closed but not swg-closed.
(iv) The set {a, b, d} isgp-closed but not gbsb*-closed in (X,$\tau$) and the sets {a}, {b}, {c} are gbsb*-closed but not gp-closed.
(v) The sets {a, b, d}, {b, d} are gbsb*-closed sets but not $\alpha$g-closed.
(vi) The set {a, b, d} is wg-closed but not gbsb*-closed.

**Theorem 3.14:** Let A be a subset of a space (X,$\tau$). Then
(i) If A is both open and g-closed then A is gbsb*-closed.
(ii) If A is both regular-open and rg-closed then A is gbsb*-closed.
(iii) If A is both w-open and w$\alpha$-closed then A is gbsb*-closed.
(iv) If A is both open and gp-closed then A is gbsb*-closed.
(v) If A is both regular-open and gpr-closed then A is gbsb*-closed.
(vi) If A is both open and $\alpha$g-closed then A is gbsb*-closed.

**Proof:** Straight forward.

**Remark 3.15:** From the above results, we have the following implication diagrams.




The above discussion we can see that the gbsb*-closed set is properly lies between b-closed and gb-closed sets.

## 4. CHARACTERIZATION

**Theorem 4.1:** If a set A is gbsb*-closed in (X,$\tau$), then bcl(A)\A contains no non empty sb*-closed sets in (X, $\tau$).

**Proof:** Let F be a sb*-closed subset of X such that F$\subseteq$bcl(A)\A. Then F$\subseteq$bcl(A)∩(X\A).That implies, F$\subseteq$bcl(A) and F$\subseteq$(X\A). Then A$\subseteq$X\F and X\F is sb*-open in (X, $\tau$). Since A is gbsb*-closed in X, bcl(A)$\subseteq$X\F, F$\subseteq$X\bcl(A). Thus F$\subseteq$bcl(A)∩(X\bcl(A))= $\phi$. Hence bcl(A)\A does not contain any non-empty sb*-closed sets.

**Theorem 4.2:** If a subset A is gbsb*-closed set in (X, $\tau$) and A$\subseteq$B$\subseteq$bcl(A), then B is also a gbsb*-closed set.
**Proof:** Let A be a gbsb*-closed set and B be any subset of X such that A$\subseteq$B$\subseteq$bcl(A). Let U be sb*-open in (X, $\tau$) such that B$\subseteq$U. Then A$\subseteq$U. Also since A is gbsb*-closed, bcl(A)$\subseteq$U. Since B$\subseteq$bcl(A), bcl(B)$\subseteq$bcl(bcl(A)) = bcl(A) $\subseteq$U. This implies, bcl(B) $\subseteq$U. Thus B is a gbsb*-closed set.

**Definition 4.3:** Let X be a topological spaces and Y be a subspace of X. Then the subset A of Y is sb*-open in Y if A=G∩Y, where G is sb*-open in X.

**Theorem 4.4:** Let A⊆Y⊆X and suppose that A is gbsb*-closed in X then A is gbsb*-closed relative to Y.

**Proof:** Given that A ⊆ Y ⊆ X and A is a gbsb*-closed set in X. To prove that A is gbsb*-closed set relative to Y. Let us assume that A ⊆ Y∩U, where U is sb*-open in X. Since A is gbsb*-closed set in X, then bcl(A)⊆U. That implies Y∩bcl(A)⊆Y∩U, where Y∩bcl(A) is the b-closure of A in Y and Y∩U is sb*-open in Y. Therefore bcl(A)⊆Y∩U in Y. Hence, A is gbsb*-closed set relative to Y.

**Theorem 4.5:** Let A be any gbsb*-closed set in (X, $\tau$). Then A is b-closed in (X, $\tau$) iff bcl(A)\A is sb*-closed.

**Proof:** Necessity: Since A is b-closed, bcl(A)=A. Then bcl(A)\A= $\phi$, which is asb*-closed set in (X,$\tau$). Sufficiency: Since A is gbsb*-closed, by Theorem 4.1, bcl(A)\A does not contains any non-empty sb*-closed set. Therefore, bcl(A)\A= $\phi$. Hence bcl(A)=A. Thus A is b-closed set in (X, $\tau$).

**Theorem 4.6:** For every element x in a space X, X–{x} is gbsb*-closed or sb*-open.

**Proof:**
**Case-(i):** Suppose X–{x} is not sb*-open. Then X is the only sb*-open set containing X–{x}. This implies bcl(X–{x})⊆X. Hence X–{x} isgbsb*-closed.

**Case-(ii):** Suppose X–{x} is not gbsb*-closed. Then there exists a sb*-open set U containing X–{x} such that bcl(X–{x}) does not contained in U. Now bcl(X–{x}) is either X–{x} or X. If bcl(X–{x})=X–{x}, then X–{x} is b-closed. Since every b-closed set is gbsb*-closed, X–{x} is gbsb*-closed, which is a contradiction. Therefore bcl(X–{x})=X. To prove that X–{x} is sb*-open. suppose not. Then by case (i), X–{x} is gbsb*-closed. There is a contradiction to our assumption. Hence X–{x} is sb*-open.

**Theorem 4.7:** If A is both sb*-open and gbsb*-closed set in X, then A is b-closed set.

**Proof:** Since A is sb*-open and gbsb*-closed in X, bcl(A)⊆A. But always A⊆bcl(A). Therefore, A=bcl(A). Hence A is a b-closed set.

**Definition 4.8:** The intersection of all sb*-open sets containing A is called the sb*-kernel of A and it is denoted by sb*-ker(A).

**Theorem 4.9:** A subset A of X is gbsb*-closed iffbcl(A)⊆sb*-ker(A).

**Proof:**
**Necessity:** Let A be a gbsb*-closed subset of X and x∈bcl(A). Suppose x∉sb*-ker(A). Then there exists a sb*-open set U containing A such that x∉U. Since A is gbsb*-closed set, then bcl(A)⊆U. This implies that, x∉bcl(A), which is a contradiction to x∈bcl(A). Therefore bcl(A)⊆sb*-ker(A).

**Sufficiency:** Suppose bcl(A)⊆sb*-ker(A). If U is any sb*-open set containing A, then sb*-ker(A)⊆U. That implies, bcl(A)⊆U. Hence A is gbsb*-closed in X.

**Remark 4.10:** For any subset A of X, gb-ker(A)⊆sb*-ker(A).

**Theorem 4.11:** For any subset A of X, $X_2$∩bcl(A)⊆sb*-ker(A).

**Proof:** Since bcl(A)⊆cl(A), then $X_2$∩bcl(A)⊆$X_2$∩cl(A). Then by Lemma 2.9 and Remark 4.10, $X_2$∩bcl(A)⊆sb*-ker(A).

**Theorem 4.12:** A subset A of X is gbsb*-closed if and only if $X_1$∩bcl(A)⊆A.

**Proof:**
**Necessity:** Suppose that A is gbsb*-closed and x∈$X_1$∩bcl(A). Then x∈$X_1$ and x∈bcl(A). Since x∈$X_1$, then int(cl({x}))=∅. That implies, cl(int(cl({x})))=∅. Therefore {x} is $\alpha$-closed. By Theorem 2.5, {x} is sb*-closed. If x does not belongs to A, then U=X–{x} is a sb*-open set containing A and so bcl(A)⊆U. Since x∈bcl(A), x∈U. This is a contradiction to x not in U. Hence $X_1$∩bcl(A)⊆A.

**Sufficiency:** Let $X_1$∩bcl(A)⊆A. Then $X_1$∩bcl(A)⊆sb*-ker(A). Now, bcl(A)= X∩bcl(A)=($X_1$∪$X_2$) ∩bcl(A) = ($X_1$∩bcl(A)) ∪ ($X_2$∩bcl(A)) ⊆sb*-ker(A). Then by Theorem 4.9, A is gbsb*-closed.

**Remark 4.13:** Union of any two gbsb*-closed sets in (X, $\tau$) need not be agbsb*-closed set which is shown in the following example.

**Example 4.14:** Let X= {a, b, c, d} with $\tau$={$\phi$, {a}, {b}, {a, b}, {a, b, c}, X}. The sets {a} and {b} are gbsb*-closed sets but their union {a, b} is not a gbsb*-closed set in X.

**Theorem 4.15:** Arbitrary intersection of gbsb*-closed sets is gbsb*-closed.

**Proof:** Let {$A_i$} be the collection of gbsb*-closed sets of X. Let A=∩$A_i$. Since A⊆$A_i$, for each i, then bcl(A)⊆bcl($A_i$). That implies, $X_1$∩bcl(A)⊆$X_1$∩bcl($A_i$). Since each $A_i$ is gbsb*-closed, then by Theorem 4.12, $X_1$∩bcl($A_i$)⊆$A_i$, for each i. Now, $X_1$∩bcl(A)=$X_1$∩bcl(∩$A_i$)⊆∩($X_1$∩bcl($A_i$))⊆∩$A_i$=A. Again by Theorem 4.12, A is gbsb*-closed.

**Theorem 4.16:** Let A be a gbsb*-closed in X. Then
- (i) sint(A) is gbsb*-closed.
- (ii) If A is regular open, then pint(A) and scl(A) are also gbsb*-closed.
- (iii) If A is regular closed, then pcl(A) is also gbsb*-closed.

**Proof:** Let A be a gbsb*-closed set of X.
- (i) Since cl(int(A)) is closed, then by Theorem 3.2, cl(int(A)) is gbsb*-closed. By Theorem 4.15 and Lemma 2.6, sint(A) is gbsb*-closed.
- (ii) Suppose A is regular open. Then int(cl(A))=A. By Lemma 2.6, scl(A)=A. Since A is gbsb*-closed, then scl(A) is gbsb*-closed. Similarly pint(A) is gbsb*-closed.
- (iii) If A is regular closed, then cl(int(A))=A. By Lemma 2.6, pcl(A)=A and hence gbsb*-closed.

## 5. Generalized b-strongly b*-open

**Definition 5.1:** A subset A of (X, τ) is said be generalized b-strongly b*-open (briefly gbsb*-open) set if its complement X\A is gbsb*-closed in X. The family of all gbsb*-open sets in X is denoted by gbsb*-O(X).

**Theorem 5.2:** Let (X, τ) be a topological space and A⊆X. Then A is a gbsb*-open if and only if F⊆bint(A), whenever F⊆A and F is sb*-closed.

**Proof:**
**Necessity:** Let A be a gbsb*-open set in (X, τ). Let F⊆A and F is sb*-closed. Then X\A is gbsb*-closed and it is contained in the sb*-open set X\F. Therefore bcl(X\A)⊆X \ F. This implies that X \ bint(A) ⊆ X \ F. Hence F ⊆bint(A).

**Sufficiency:** If F is sb*-closed set such that F⊆bint(A) whenever F⊆A. It follows that X\A⊆X\ F and X\bint(A)⊆X \ F. Therefore bcl(X\ A) ⊆ X\ F. Hence X \ A is gbsb*-closed and hence A is gbsb*-open.

**Theorem 5.3:** If a set A is gbsb*-open and B⊆X such that bint(A)⊆B⊆ A, then B is gbsb*-open.

**Proof:** If bint(A)⊆B⊆A then, X\A⊆ X\B ⊆ X \ bint(A). That is, X\A ⊆ X \ B ⊆ bcl(X \ A). Since X \ A is gbsb*-closed, then by Theorem 4.2, X \ B is gbsb*-closed and hence B is gbsb*-open.

**Theorem 5.4:** If a subset A is gbsb*-open in X and G is sb*-open in X with bint(A)∪(X\G)⊆ G then X= G.

**Proof:** Suppose that G is sb*-open and bint(A)∪(X\G)⊆ G. This implies, X\G⊆(X\bint(A))∩A=bcl(X\A)\(X\A). Since X\A is gbsb*-closed and X\G is sb*-closed, then by Theorem 4.1, X\G=$\phi$. Hence X=G.

**Remark 5.5:** Every union of gbsb*-open sets is gbsb*-open but the intersection of gbsb*-open sets need not be a gbsb*-open in X which is shown in the following example.

**Example 5.6:** Let X = {a, b, c, d} with topology $\tau$ = {$\phi$, {a,b},{a,b,c}, X}. In this topological space (X, $\tau$), gbsb*-O(X) = {$\phi$, {a}, {b}, {a, b}, {a,c},{a, d}, {b, c}, {b, d}, {a, b,c}, {a, b, d},{a,c,d}, {b, c d}, X}. The sets {a, c, d} and {b, c, d} are gbsb*-open but their intersection {c, d} is not gbsb*-open in X.

**Theorem 5.7:** If B is gbsb*-open and bint(B) ⊆A, then A∩B is gbsb*-open.

**Proof:** Suppose B is gbsb*-open and bint(B)⊆A. Then bint(A∩B) ⊆A∩B⊆B. By Theorem 5.3, A∩B is gbsb*-open.

**Theorem 5.8:** If a topological space (X, $\tau$), let $\tau_{gbsb*}$={U∈gbsb*-O(X, $\tau$)/ U∩A∈gbsb*-O(X, $\tau$) for all A∈gbsb*-O(X, $\tau$)}. Then $\tau_{gbsb*}$ is a topology on X.

**Proof:** Clearly $\phi$, X∈ $\tau_{gbsb*}$. Let $U_\beta \in \tau_{gbsb*}$. and U=∪ $U_\beta$. Since each $U_\beta \in \tau_{gbsb*}$, then by Remark 5.5, U∈gbsb*-O(X, $\tau$). Let A∈gbsb*-O(X, $\tau$). Then $U_\beta$ ∩A∈gbsb*-O(X, $\tau$) for each β. Hence U∩A=(∪ $U_\beta$) ∩A=∪ ($U_\beta$ ∩A) ∈gbsb*-O(X, $\tau$). Therefore U∈ $\tau_{gbsb*}$. Let $U_1$,$U_2$∈ $\tau_{gbsb*}$. Then $U_1$,$U_2$∈gbsb*-O(X, $\tau$) and from definition of $\tau_{gbsb*}$, $U_1$∩$U_2$∈gbsb*-O(X, $\tau$). If A∈gbsb*-O(X, $\tau$),and from definition of $\tau_{gbsb*}$, $U_1$∩$U_2$∩A ∈gbsb*-O(X, $\tau$). Hence $U_1$∩$U_2$∈ $\tau_{gbsb*}$. This shows that $\tau_{gbsb*}$ is closed under finite intersection. Hence $\tau_{gbsb*}$ is a topology on X.

## 6. gbsb*-neighbourhood

**Definition 6.1:** Let X be a topological space and let x∈X. A subset N of X is said to be a gbsb*-neighbourhood (shortly, gbsb*-nbhd) of x if there exsits a gbsb*-open set U such that x∈U⊆N.

**Definition 6.2:** A subset N of a space X, is called a gbsb*-nbhd of A⊆X if there exists an gbsb*-open set U such that A⊆U⊆N.

**Theorem 6.3:** Every nbhd N of x∈X is a gbsb*-nbhd of x.

**Proof:** Let N be an bhd of point x∈ X. Then there exists an open set U such that x∈U⊆N. Since every open set is gbsb*-open, U is a gbsb*-open set such that x∈U⊆N. This implies, N is a gbsb*-nbhd of x.

**Remark 6.4:** The converse of the above theorem need not be true which is shown in the following example.

**Example 6.5:** Let X = {a, b, c, d} with topology $\tau$ = {$\phi$, {a},{b}, {a,b},{b,c},{a,b,c},{b,c,d}, X}. In this topological space (X, $\tau$), gbsb*-O(X) = {$\phi$, {a}, {b}, {a, b}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c d}, X}. The set {b, d} is the gbsb*-nbhd of d, since {b,d} is gbsb*-open set such that d ∈ {b,d}⊆ {b, d}. However, the set {b, d} is not a nbhd of the point d.

**Remark 6.6:** Every gbsb*-open set is a gbsb*-nbhd of each of its points.

**Theorem 6.7:** If F is a gbsb*-closed subset of X and x∈X\F, then there exists a gbsb*-nbhd N of x such that N∩F=$\phi$.

**Proof:** Let F be gbsb*-closed subset of X and x∈ X\F. Then X\F is gbsb*-open set of X. By Remark 6.6, X\F contains a gbsb*-nbhd of each of its points. Hence there exists a gbsb*-nbhd N of x such that N⊆ X\F. Hence N∩F=$\phi$.

**Definition 6.8:** The collection of all gbsb*-neighborhoods of x∈X is called the gbsb*-neighborhood system of x and is denoted by gbsb*-N(x).

**Theorem 6.9:** Let (X, $\tau$) be a topological space and x ∈ X. Then
(i) gbsb*-N(x)≠ $\phi$ and x∈each member of gbsb*-N(x)
(ii) If N ∈ gbsb*-N(x) and N ⊆ M, then M ∈ gbsb*-N(x).
(iii) Each member N ∈gbsb*-N(x) is a superset of a member G∈gbsb*-N(x) where G is a gbsb*-open set.

**Proof:**
(i) Since X is gbsb*-open set containing x, it is a gbsb*-nbhd of every x∈X. Thus for each x∈X, there exists atleast one gbsb*-nbhd, namely X. Therefore, gbsb*-N(x)≠ $\phi$. Let N∈gbsb*-N(x). Then N is a gbsb*-nbhd of x. Hence there exists a gbsb*-open set G such that x∈G ⊆N, so x ∈ N. Therefore x∈every member N of gbsb*-N(x).
(ii) If N ∈gbsb*-N(x), then there is a gbsb*-open set G such that x∈G⊆N. Since N⊆M, M is gbsb*-nbhd of x. Hence M∈gbsb*-N(x).
(iii) Let N∈gbsb*-N(x). Then there is a gbsb*-open set G, such that x ∈ G⊆N. Since G is gbsb*-open and x∈G, G is gbsb*-nbhd of x. Therefore G∈gbsb*-N(x) and also G ⊆ N.

## REFERENCES

1. Willard S., General Topology, Addison Wesley, 1970.
2. Ahmad Al-Omari and Mohd.Salmi Md. Noorani, On Generalized b-closed sets. Bull. Malays.math. Sci. Soc (2) 32(1) (2009), 19-30.
3. S. S. Benchalli, P.G. Patil and T. D. Rayangaudar, w$\alpha$-closed sets in topological spaces, The Global. J. Appl. Math. Sci., 2, 2009, 53-63.
4. V.Zaitsav, On certain classes of topological spaces and their bicompactifications, Dokl. Akad.Nauk SSSR, 178 (1968), 778-779
5. Sanjay Mishra, NitinBhardawaj, Varun Joshi, On regular generalized weakly closed sets in topological spaces, Int. Journal of math. Analysis, Vol.6, 2012, No 39, 1939-1952.

6. A.K.Al-Obiadi, $\pi$-generalized b-closed sets in topolosical spaces, Ibn Al-Haitham J. for pure and appl. Sci., 24(3) (2001).
7. N.Palaniappan and K.C. Rao, Regular generalized closed sets, Kyungpook Math.J., 33(2) (1993), 211-219.
8. N. Levine, Generalized closed sets in topology, Rand. Circ. Mat. Palermo, 19(2) (1970), 89- 96.
9. N. Levine, Semi-open sets and semi-continuity in Topological Spaces, Amer. Mat. Monthly 70(1) (1963), 36- 41.
10. H. Maki, R. Devi and K. Balachandran, Associated Topologies of Generalized $\alpha$ – closed sets and $\alpha$ – Generalized closed sets Mem. Sci. Kochi. Univ. Ser. A. Math., 15(1994), 51- 63.
11. H.Maki, J.Umehara and T.Noiri, Generalized preclosed sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math, 17(1996), 33-42.
12. Adiya.K.Hussein, New type of generalized closed sets, Mathematical Theory and Modeling, Vol.3 (4), 2013.
13. O. Njastad, Some classes of nearly open sets, Pacific J. Math., 15(1965), 961- 970.
14. A.S.Mashhour, M.E.Abd. El-.Monsef, Ei-Deeb.S.N., On Pre continuous and weak Pre-continuous mapping, Proc. Math., Phys. Soc. Egypt, 53 (1982), 47-83.
15. L.Vinayagamoorthi, N.Nagaveni, On Generalized $\alpha$ b-closed sets, proceeding ICMD-Allahabad, Puspha Publication Vol.1, 2010-11.
16. D.Andrijevic, Onb-open sets, Mat.Vesnik 48(1996), 59-64.
17. K.Mariappa and S.Sekar, On Regular generalized b-closed sets, Int. Journal of Math. Analysis, Vol.7,(2013), 613-624.
18. P. Sundaram, and N. Navalagi, On weakly Generalized continuous maps, weakly Generalized closed maps andweaklyIrrosolute maps, Far East. J. Math Sci., 6, 1998.
19. P. Sundaram and M. Shrik John, On w- closed sets in topology, ActaCienciaIndica, 4(2000), 389- 392.
20. Y.Gnanambal, On generalized pre-regular closeds in topological spaces, Indian J. Pure Appl. Math 28 (1997), 351-360.
21. A.Poongothai and P.Parimelazhagan, sb*-closed sets in a topological spaces, Int. Journal of Math. Analysis 6(47), (2012), 2325-2333.
22. M.Anitha and P.Thangavelu, Generalized closed sets in submaximal spaces, Antarctica J. Math., 4(1) (2007), 99-108.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