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Generalized b-strongly b*-closed sets in Topological Spaces

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ABSTRACT

In this paper a new class of generalized closed sets in topological spaces, namely generalized b-strongly b-closed (briefly, gbsb*-closed) set is introduced. We give some basic properties and various characterizations of gbsb*-closed sets. Also we introduce gbsb*-neighbourhood in a topological spaces and investigate some basic properties.*

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1. INTRODUCTION

In 1970, Levine [8] introduced the class of generalized closed sets. The notion of generalized closed sets has been extended and studied exclusively in recent years by many topologists. In 1996, Andrjivic [16] gave a new type of generalized closed sets in topological spaces called b-closed sets. Later in 2012 A.Poongothai and P.Parimelazhagan [21] introduced sb*-closed sets and investigated some of their properties.

In this paper, a new class of generalized closed set called generalized b-strongly b*-closed set is introduced. The notion of generalized b-strongly b*-closed set and its different characterizations are given in this paper.

2. PRELIMINARIES

Throughout this paper (X, τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. (X, τ) will be replaced by X if there is no changes of confusion. For a subset A of a topological space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. We recall the following definitions and results.

Definition 2.1:. Let (X, τ) be a topological space. A subset A of X is said to be

1. semi-open [9] if A $\subseteq$ cl(int(A)) and semi-closed if int(cl(A)) $\subseteq$ A.
2. α-open [13] if A $\subseteq$ int(cl(int(A))) and α-closed if cl(int(cl(A))) $\subseteq$ A.
3. pre-open [14] if A $\subseteq$ int(cl(A)) and pre-closed if cl(int(A)) $\subseteq$ A.
4. b-open [16] if A$\subseteq$int(cl(A))$\cup$cl(int(A)) and b-closed if int(cl(A))$\cap$cl(int(A)) $\subseteq$A.
5. regular open [1] if int(cl(A))=A and regular closed if cl(int(A))=A.
6. π-open [4] if A is the union of regular open sets and π-closed if A is the intersection of regular closed sets.

Definition 2.2: Let (X, τ) be a topological space and A $\subseteq$ X. The b-closure (resp.pre-closure, semi-closure, α-closure) of A, denoted by bcl(A) (resp.pcl(A), scl(A), αcl(A)) and is defined by the intersection of all b-closed (resp. pre-closed, semi-closed, α-closed) sets containing A.

Definition 2.3: Let (X, τ) be a topological space and A $\subseteq$ X. The b-interior of A, denoted by bint(A) and is defined by the union of all b-open sets contained in A.

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Definition 2.4: Let (X, τ) be a topological space. A subset Aof X is said to be

1. generalized closed [8](briefly g-closed) if cl(A)$\subseteq$U whenever A$\subseteq$U and U is open in (X, τ).
2. generalizedb-closed [2] (briefly gb-closed) if bcl(A)$\subseteq$U whenever A$\subseteq$U and U is open in (X, τ).
3. regular generalized closed [7] (briefly rg-closed) if cl(A) $\subseteq$U whenever A$\subseteq$U and U is regular open in (X,τ).
4. regular generalized b-closed [17](briefly rgb-closed) if bcl(A)$\subseteq$U whenever A$\subseteq$U and U is regular open in (X, τ).
5. generalized αb-closed [15](briefly gαb-closed) if bcl(A)$\subseteq$U whenever A$\subseteq$U and U is α-open in (X, τ)
6. generalized pre-regular closed [20] (briefly gpr-closed) if pcl(A) $\subseteq$U whenever A$\subseteq$U and U is rg-open in (X,τ).
7. generalized p-closed [11](briefly gp-closed) if pcl(A)$\subseteq$U whenever A$\subseteq$U and U is open in (X, τ).
8. α-generalized closed [10] (briefly αg-closed) if αcl(A)$\subseteq$U whenever A$\subseteq$U and U is an open in (X, τ).
9. π-generalized b-closed [6](briefly πgb-closed) if bcl(A)$\subseteq$U whenever A$\subseteq$U and U is π-open in (X, τ).
10. weakly closed [19](briefly w-closed) if cl(A) ⊆ U whenever A $\subseteq$U and U is a semi-open in (X, τ).
11. weakly generalized closed [18] (briefly wg-closed) if cl(int(A))$\subseteq$U whenever A$\subseteq$U and U is an open in (X,τ).
12. semi weakly generalized closed [5](briefly swg-closed) if scl(A)$\subseteq$U whenever A$\subseteq$U and U is an wg-open in (X, τ).
13. wα-closed [3] if αcl(A)⊆U whenever A⊆U and U is a w-open in (X, τ).
14. strongly b*-closed [21](briefly sb*-closed) if cl(int(A)))$\subseteq$U whenever A$\subseteq$U and U is b-open in (X, τ).

The complements of the above mentioned closed sets are their respective open sets.

Theorem 2.5 [21]: For a topological space (X, τ),
(i) Every open set is sb*-open.
(ii) Every α-open set is sb*-open.
(iii) Every sb*-open set is b-open.

Theorem 2.6 [22]: For any subset A of a topological space (X, τ),
(i) sint(A)=A∩cl(int(A))
(ii) pin(A)=A∩int(cl(A))
(iii) scl(A)=A∪int(cl(A))
(iv) pcl(A)=A∪cl(int(A)).

Remark 2.7 [12]: Jankovic and Reilly pointed out that every singleton {x} of a space X is either nowhere dense or pre-open. This provides another decomposition $X=X_1 \cup X_2$, where X_1={x∈X/ {x} is nowhere dense} and X_2={x∈X/ {x} is pre-open}.

Definition 2.8 [12]: The intersection of all gb-open sets containing A is called the gb-kernel of A and it is denoted by gb-ker(A).

Lemma 2.9 [12]: For any subset A of X, X_2∩cl(A)$\subseteq$gb-ker(A).

Remark 2.10: For any subset A of a topological space (X, τ),
(i) X\bcl(A)=bint(X\A)
(ii) X\bint(A)=bcl(X\A).

3. Generalized b-strongly b*-closed set

Definition 3.1: A subset A of a topological space (X, τ) is called a generalized b-strongly b*-closed set (briefly,gbsb*-closed) if bcl(A)$\subseteq$U whenever A$\subseteq$U and U is sb*-open in (X, τ). The collection of all gbsb*-closed sets of X is denoted by gbsb*-C(X, τ).

Theorem 3.2:
(i) Every closed set is gbsb*-closed.
(ii) Every semi-closed set is gbsb*-closed.
(iii) Every α-closed set is gbsb*-closed.
(iv) Every pre-closed set is gbsb*-closed.
(v) Every b-closed set is gbsb*-closed.
(vi) Every regular closed set is gbsb*-closed.
(vii) Every π-closed set is gbsb*-closed.

Proof:

(i) Let A be a closed set. Let $A \subseteq U$, U is sb*-open in X. Since A is closed, then $cl(A)=A \subseteq U$. But $bcl(A) \subseteq cl(A)$. Thus we have $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is sb*-open and therefore, A is a gbsb*-closed set.

(ii) (ii), (iii), (iv), (v), (vi) and (vii) are similar to (i).

The reverse implications need not be true which is shown in the following examples.

Example 3.3: Let X={a, b, c} and τ={ϕ, {a}, X}.

(i) Then the set {b} is gbsb*-closed but not a closed set.

(ii) The set {a, c}is a gbsb*-closed set but not a b-closed set

Example 3.4: Let X={a, b, c, d} withτ={ϕ, {a, b}, {a, b, c}, X}.

(i) The set {a, c, d}is gbsb*-closed set but not a semi-closed set.

(ii) The set {a}is gbsb*-closed set but not a α-closed set.

(iii) The set {b, c} is gbsb*-closed set but not a pre-closed set.

Example 3.5: Let X = {a, b, c, d} withτ= {ϕ, {a},{b},{a, b},{a, b, c}, X}. The set {d} is gbsb*-closed set but not a regular-closed set.

Example 3.6. Let X= {a, b, c, d} with τ={ϕ, {a},{c},{a, c},{c, d},{a, c, d},{a, b, c}, X}. The sets {a, b} and {b, c, d} are gbsb*-closed but not a π-closed set.

Theorem 3.7:

(i) Every gbsb*-closed set is gb-closed.

(ii) Every gbsb*-closed set is rgb-closed set.

(iii) Every gbsb*-closed set is gαb-closed set.

(iv) Every gbsb*-closed set is πgb-closed set.

Proof:

(i) Let A be a gbsb*-closed set. Let $A \subseteq U$, U is open. Since open set is sb*-open, then U is sb*-open. Since A is gbsb*-closed, $bcl(A) \subseteq U$. Thus, we have $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open and therefore, A is gb-closed set.

(ii) (ii), (iii) and (iv) are similar to (i).

The reverse implications need not be true which is shown in the following examples.

Example 3.8: Let X = {a, b, c, d} withτ= {ϕ, {a},{b},{a, b},{a, b, c}, X}.

(i) The set {a, b, d} is gb-closed but not a gbsb*-closed set.

(ii) The sets {a, b, d} is gαb-closed but not a gbsb*-closed set.

Example 3.9: Let X = {a, b, c, d} with τ= {ϕ, {a, b}, {a, b, c}, X}. The set {a, b} is rgb-closed but not a gbsb*-closed set.

Example 3.10: Let X = {a, b, c, d} with τ = {ϕ, {a},{c},{a, c},{c, d},{a, c, d},{a, b, c}, X}. The set {a,c}isπgb-clos ed set but not a gbsb*-closed set.

Remark 3.11: The following examples shows that gbsb*-closed sets are independent from αg-closed set, g-closed set, rg-closed set, gpr-closed set, wg-closed set, swg-closed set and gp-closed set.

Example 3.12: Let X= {a, b, c, d} with τ= {ϕ, {a}, {b},{a, b},{b,c},{b, c, d},{a, b, c}, X}.

(i) The sets {a, b},{a, b, d} are rg-closed sets but not a gbsb*-closed in (X, τ) and the set {c} is gbsb*-closed but not rg-closed.

(ii) The sets {a, b, d}, {b, d} are αg-closed sets but not gbsb*-closed.

(iii) The set {a, b, d} is wg-closed but not gbsb*-closed.

Example 3.13: Let X = {a, b, c, d} with τ= {ϕ, {a}, {b},{a, b},{a, b, c}, X}.

(i) The sets {a, b},{a, b, d} are gpr-closed sets but not gbsb*-closed in (X, τ) and the sets {a},{b} are gbsb*-closed but not a gpr-closed.

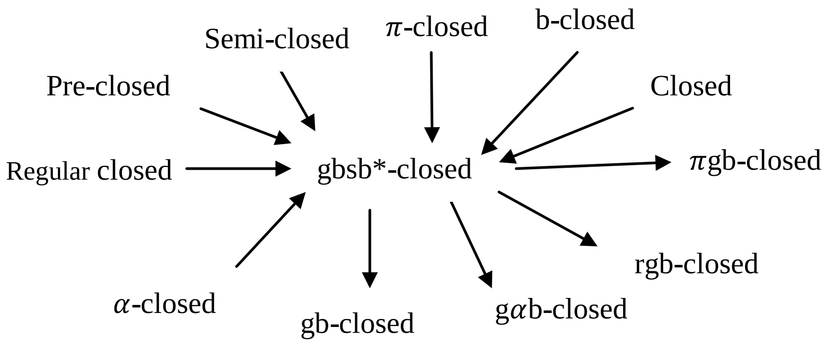
(ii) The set {a, b, d} is a g-closed set but not gbsb*-closed in (X,τ) and the sets {a},{b},{c} are gbsb*-closed but not g-closed.

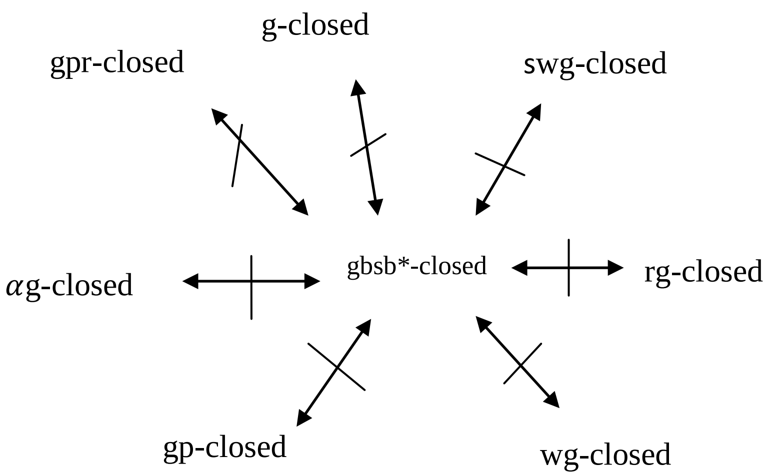
(iii) The set {a, b, d} is as wg-closed set but not gbsb*-closed in (X, τ) and the sets {a},{b},{c} are gbsb*-closed but not swg-closed.
(iv) The set {a, b, d} isgp-closed but not gbsb*-closed in (X,τ) and the sets {a}, {b}, {c} are gbsb*-closed but not gp-closed.
(v) The sets {a, b, d}, {b, d} are gbsb*-closed sets but not αg-closed.
(vi) The set {a, b, d} is wg-closed but not gbsb*-closed.

Theorem 3.14: Let A be a subset of a space (X,τ). Then
(i) If A is both open and g-closed then A is gbsb*-closed.
(ii) If A is both regular-open and rg-closed then A is gbsb*-closed.
(iii) If A is both w-open and wα-closed then A is gbsb*-closed.
(iv) If A is both open and gp-closed then A is gbsb*-closed.
(v) If A is both regular-open and gpr-closed then A is gbsb*-closed.
(vi) If A is both open and αg-closed then A is gbsb*-closed.

Proof: Straight forward.

Remark 3.15: From the above results, we have the following implication diagrams.





The above discussion we can see that the gbsb*-closed set is properly lies between b-closed and gb-closed sets.

4. CHARACTERIZATION

Theorem 4.1: If a set A is gbsb*-closed in (X,τ), then bcl(A)\A contains no non empty sb*-closed sets in (X, τ).

Proof: Let F be a sb*-closed subset of X such that F$\subseteq$bcl(A)\A. Then F$\subseteq$bcl(A)∩(X\A).That implies, F$\subseteq$bcl(A) and F$\subseteq$(X\A). Then A$\subseteq$X\F and X\F is sb*-open in (X, τ). Since A is gbsb*-closed in X, bcl(A)$\subseteq$X\F, F$\subseteq$X\bcl(A). Thus F$\subseteq$bcl(A)∩(X\bcl(A))= ϕ. Hence bcl(A)\A does not contain any non-empty sb*-closed sets.

Theorem 4.2: If a subset A is gbsb*-closed set in (X, τ) and A$\subseteq$B$\subseteq$bcl(A), then B is also a gbsb*-closed set.
Proof: Let A be a gbsb*-closed set and B be any subset of X such that A$\subseteq$B$\subseteq$bcl(A). Let U be sb*-open in (X, τ) such that B$\subseteq$U. Then A$\subseteq$U. Also since A is gbsb*-closed, bcl(A)$\subseteq$U. Since B$\subseteq$bcl(A), bcl(B)$\subseteq$bcl(bcl(A)) = bcl(A) $\subseteq$U. This implies, bcl(B) $\subseteq$U. Thus B is a gbsb*-closed set.

Definition 4.3: Let X be a topological spaces and Y be a subspace of X. Then the subset A of Y is sb*-open in Y if A=G∩Y, where G is sb*-open in X.

Theorem 4.4: Let A⊆Y⊆X and suppose that A is gbsb*-closed in X then A is gbsb*-closed relative to Y.

Proof: Given that A ⊆ Y ⊆ X and A is a gbsb*-closed set in X. To prove that A is gbsb*-closed set relative to Y. Let us assume that A ⊆ Y∩U, where U is sb*-open in X. Since A is gbsb*-closed set in X, then bcl(A)⊆U. That implies Y∩bcl(A)⊆Y∩U, where Y∩bcl(A) is the b-closure of A in Y and Y∩U is sb*-open in Y. Therefore bcl(A)⊆Y∩U in Y. Hence, A is gbsb*-closed set relative to Y.

Theorem 4.5: Let A be any gbsb*-closed set in (X, τ). Then A is b-closed in (X, τ) iff bcl(A)\A is sb*-closed.

Proof: Necessity: Since A is b-closed, bcl(A)=A. Then bcl(A)\A= ϕ, which is asb*-closed set in (X,τ). Sufficiency: Since A is gbsb*-closed, by Theorem 4.1, bcl(A)\A does not contains any non-empty sb*-closed set. Therefore, bcl(A)\A= ϕ. Hence bcl(A)=A. Thus A is b-closed set in (X, τ).

Theorem 4.6: For every element x in a space X, X–{x} is gbsb*-closed or sb*-open.

Proof:
Case-(i): Suppose X–{x} is not sb*-open. Then X is the only sb*-open set containing X–{x}. This implies bcl(X–{x})⊆X. Hence X–{x} isgbsb*-closed.

Case-(ii): Suppose X–{x} is not gbsb*-closed. Then there exists a sb*-open set U containing X–{x} such that bcl(X–{x}) does not contained in U. Now bcl(X–{x}) is either X–{x} or X. If bcl(X–{x})=X–{x}, then X–{x} is b-closed. Since every b-closed set is gbsb*-closed, X–{x} is gbsb*-closed, which is a contradiction. Therefore bcl(X–{x})=X. To prove that X–{x} is sb*-open. suppose not. Then by case (i), X–{x} is gbsb*-closed. There is a contradiction to our assumption. Hence X–{x} is sb*-open.

Theorem 4.7: If A is both sb*-open and gbsb*-closed set in X, then A is b-closed set.

Proof: Since A is sb*-open and gbsb*-closed in X, bcl(A)⊆A. But always A⊆bcl(A). Therefore, A=bcl(A). Hence A is a b-closed set.

Definition 4.8: The intersection of all sb*-open sets containing A is called the sb*-kernel of A and it is denoted by sb*-ker(A).

Theorem 4.9: A subset A of X is gbsb*-closed iffbcl(A)⊆sb*-ker(A).

Proof:
Necessity: Let A be a gbsb*-closed subset of X and x∈bcl(A). Suppose x∉sb*-ker(A). Then there exists a sb*-open set U containing A such that x∉U. Since A is gbsb*-closed set, then bcl(A)⊆U. This implies that, x∉bcl(A), which is a contradiction to x∈bcl(A). Therefore bcl(A)⊆sb*-ker(A).

Sufficiency: Suppose bcl(A)⊆sb*-ker(A). If U is any sb*-open set containing A, then sb*-ker(A)⊆U. That implies, bcl(A)⊆U. Hence A is gbsb*-closed in X.

Remark 4.10: For any subset A of X, gb-ker(A)⊆sb*-ker(A).

Theorem 4.11: For any subset A of X, X_2∩bcl(A)⊆sb*-ker(A).

Proof: Since bcl(A)⊆cl(A), then X_2∩bcl(A)⊆X_2∩cl(A). Then by Lemma 2.9 and Remark 4.10, X_2∩bcl(A)⊆sb*-ker(A).

Theorem 4.12: A subset A of X is gbsb*-closed if and only if X_1∩bcl(A)⊆A.

Proof:
Necessity: Suppose that A is gbsb*-closed and x∈X_1∩bcl(A). Then x∈X_1 and x∈bcl(A). Since x∈X_1, then int(cl({x}))=∅. That implies, cl(int(cl({x})))=∅. Therefore {x} is α-closed. By Theorem 2.5, {x} is sb*-closed. If x does not belongs to A, then U=X–{x} is a sb*-open set containing A and so bcl(A)⊆U. Since x∈bcl(A), x∈U. This is a contradiction to x not in U. Hence X_1∩bcl(A)⊆A.

Sufficiency: Let X_1∩bcl(A)⊆A. Then X_1∩bcl(A)⊆sb*-ker(A). Now, bcl(A)= X∩bcl(A)=(X_1∪X_2) ∩bcl(A) = (X_1∩bcl(A)) ∪ (X_2∩bcl(A)) ⊆sb*-ker(A). Then by Theorem 4.9, A is gbsb*-closed.

Remark 4.13: Union of any two gbsb*-closed sets in (X, τ) need not be agbsb*-closed set which is shown in the following example.

Example 4.14: Let X= {a, b, c, d} with τ={ϕ, {a}, {b}, {a, b}, {a, b, c}, X}. The sets {a} and {b} are gbsb*-closed sets but their union {a, b} is not a gbsb*-closed set in X.

Theorem 4.15: Arbitrary intersection of gbsb*-closed sets is gbsb*-closed.

Proof: Let {A_i} be the collection of gbsb*-closed sets of X. Let A=$\cap A_i$. Since A⊆A_i, for each i, then bcl(A)⊆bcl(A_i). That implies, X_1∩bcl(A)⊆X_1∩bcl(A_i). Since each A_i is gbsb*-closed, then by Theorem 4.12, X_1∩bcl(A_i)⊆A_i, for each i. Now, X_1∩bcl(A)=X_1∩bcl(∩A_i)⊆∩(X_1∩bcl(A_i))⊆∩A_i=A. Again by Theorem 4.12, A is gbsb*-closed.

Theorem 4.16: Let A be a gbsb*-closed in X. Then

(i) sint(A) is gbsb*-closed.
(ii) If A is regular open, then pint(A) and scl(A) are also gbsb*-closed.
(iii) If A is regular closed, then pcl(A) is also gbsb*-closed.

Proof: Let A be a gbsb*-closed set of X.

(i) Since cl(int(A)) is closed, then by Theorem 3.2, cl(int(A)) is gbsb*-closed. By Theorem 4.15 and Lemma 2.6, sint(A) is gbsb*-closed.
(ii) Suppose A is regular open. Then int(cl(A))=A. By Lemma 2.6, scl(A)=A. Since A is gbsb*-closed, then scl(A) is gbsb*-closed. Similarly pint(A) is gbsb*-closed.
(iii) If A is regular closed, then cl(int(A))=A. By Lemma 2.6, pcl(A)=A and hence gbsb*-closed.

5. Generalized b-strongly b*-open

Definition 5.1: A subset A of (X, τ) is said be generalized b-strongly b*-open (briefly gbsb*-open) set if its complement X\A is gbsb*-closed in X. The family of all gbsb*-open sets in X is denoted by gbsb*-O(X).

Theorem 5.2: Let (X, τ) be a topological space and A⊆X. Then A is a gbsb*-open if and only if F⊆bint(A), whenever F⊆A and F is sb*-closed.

Proof:
Necessity: Let A be a gbsb*-open set in (X, τ). Let F⊆A and F is sb*-closed. Then X\A is gbsb*-closed and it is contained in the sb*-open set X\F. Therefore bcl(X\A)⊆X \ F. This implies that X \ bint(A) ⊆ X \ F. Hence F ⊆bint(A).

Sufficiency: If F is sb*-closed set such that F⊆bint(A) whenever F⊆A. It follows that X\A⊆X\ F and X\bint(A)⊆X \ F. Therefore bcl(X\ A) ⊆ X\ F. Hence X \ A is gbsb*-closed and hence A is gbsb*-open.

Theorem 5.3: If a set A is gbsb*-open and B⊆X such that bint(A)⊆B⊆ A, then B is gbsb*-open.

Proof: If bint(A)⊆B⊆A then, X\A⊆ X\B ⊆ X \ bint(A). That is, X\A ⊆ X \ B ⊆ bcl(X \ A). Since X \ A is gbsb*-closed, then by Theorem 4.2, X \ B is gbsb*-closed and hence B is gbsb*-open.

Theorem 5.4: If a subset A is gbsb*-open in X and G is sb*-open in X with bint(A)∪(X\G)⊆ G then X= G.

Proof: Suppose that G is sb*-open and bint(A)∪(X\G)⊆ G. This implies, X\G⊆(X\bint(A))∩A=bcl(X\A)\(X\A). Since X\A is gbsb*-closed and X\G is sb*-closed, then by Theorem 4.1, X\G=ϕ. Hence X=G.

Remark 5.5: Every union of gbsb*-open sets is gbsb*-open but the intersection of gbsb*-open sets need not be a gbsb*-open in X which is shown in the following example.

Example 5.6: Let X = {a, b, c, d} with topology τ = {ϕ, {a,b},{a,b,c}, X}. In this topological space (X, τ), gbsb*-O(X) = {ϕ, {a}, {b}, {a, b}, {a,c},{a, d}, {b, c}, {b, d}, {a, b,c}, {a, b, d},{a,c,d}, {b, c d}, X}. The sets {a, c, d} and {b, c, d} are gbsb*-open but their intersection {c, d} is not gbsb*-open in X.

Theorem 5.7: If B is gbsb*-open and bint(B) ⊆A, then A∩B is gbsb*-open.

Proof: Suppose B is gbsb*-open and bint(B)⊆A. Then bint(A∩B) ⊆A∩B⊆B. By Theorem 5.3, A∩B is gbsb*-open.

Theorem 5.8: If a topological space (X, τ), let τ_{gbsb*}={U∈gbsb*-O(X, τ)/ U∩A∈gbsb*-O(X, τ) for all A∈gbsb*-O(X, τ)}. Then τ_{gbsb*} is a topology on X.

Proof: Clearly ϕ, X∈ τ_{gbsb*}. Let $U_\beta \in \tau_{gbsb*}$. and U=∪ U_β. Since each $U_\beta \in \tau_{gbsb*}$, then by Remark 5.5, U∈gbsb*-O(X, τ). Let A∈gbsb*-O(X, τ). Then U_β ∩A∈gbsb*-O(X, τ) for each β. Hence U∩A=(∪ U_β) ∩A=∪ (U_β ∩A) ∈gbsb*-O(X, τ). Therefore U∈ τ_{gbsb*}. Let U_1,U_2∈ τ_{gbsb*}. Then U_1,U_2∈gbsb*-O(X, τ) and from definition of τ_{gbsb*}, U_1∩U_2∈gbsb*-O(X, τ). If A∈gbsb*-O(X, τ),and from definition of τ_{gbsb*}, U_1∩U_2∩A ∈gbsb*-O(X, τ). Hence U_1∩U_2∈ τ_{gbsb*}. This shows that τ_{gbsb*} is closed under finite intersection. Hence τ_{gbsb*} is a topology on X.

6. gbsb*-neighbourhood

Definition 6.1: Let X be a topological space and let x∈X. A subset N of X is said to be a gbsb*-neighbourhood (shortly, gbsb*-nbhd) of x if there exsits a gbsb*-open set U such that x∈U⊆N.

Definition 6.2: A subset N of a space X, is called a gbsb*-nbhd of A⊆X if there exists an gbsb*-open set U such that A⊆U⊆N.

Theorem 6.3: Every nbhd N of x∈X is a gbsb*-nbhd of x.

Proof: Let N be an bhd of point x∈ X. Then there exists an open set U such that x∈U⊆N. Since every open set is gbsb*-open, U is a gbsb*-open set such that x∈U⊆N. This implies, N is a gbsb*-nbhd of x.

Remark 6.4: The converse of the above theorem need not be true which is shown in the following example.

Example 6.5: Let X = {a, b, c, d} with topology τ = {ϕ, {a},{b}, {a,b},{b,c},{a,b,c},{b,c,d}, X}. In this topological space (X, τ), gbsb*-O(X) = {ϕ, {a}, {b}, {a, b}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c d}, X}. The set {b, d} is the gbsb*-nbhd of d, since {b,d} is gbsb*-open set such that d ∈ {b,d}⊆ {b, d}. However, the set {b, d} is not a nbhd of the point d.

Remark 6.6: Every gbsb*-open set is a gbsb*-nbhd of each of its points.

Theorem 6.7: If F is a gbsb*-closed subset of X and x∈X\F, then there exists a gbsb*-nbhd N of x such that N∩F=ϕ.

Proof: Let F be gbsb*-closed subset of X and x∈ X\F. Then X\F is gbsb*-open set of X. By Remark 6.6, X\F contains a gbsb*-nbhd of each of its points. Hence there exists a gbsb*-nbhd N of x such that N⊆ X\F. Hence N∩F=ϕ.

Definition 6.8: The collection of all gbsb*-neighborhoods of x∈X is called the gbsb*-neighborhood system of x and is denoted by gbsb*-N(x).

Theorem 6.9: Let (X, τ) be a topological space and x ∈ X. Then
(i) gbsb*-N(x)≠ ϕ and x∈each member of gbsb*-N(x)
(ii) If N ∈ gbsb*-N(x) and N ⊆ M, then M ∈ gbsb*-N(x).
(iii) Each member N ∈gbsb*-N(x) is a superset of a member G∈gbsb*-N(x) where G is a gbsb*-open set.

Proof:
(i) Since X is gbsb*-open set containing x, it is a gbsb*-nbhd of every x∈X. Thus for each x∈X, there exists atleast one gbsb*-nbhd, namely X. Therefore, gbsb*-N(x)≠ ϕ. Let N∈gbsb*-N(x). Then N is a gbsb*-nbhd of x. Hence there exists a gbsb*-open set G such that x∈G ⊆N, so x ∈ N. Therefore x∈every member N of gbsb*-N(x).
(ii) If N ∈gbsb*-N(x), then there is a gbsb*-open set G such that x∈G⊆N. Since N⊆M, M is gbsb*-nbhd of x. Hence M∈gbsb*-N(x).
(iii) Let N∈gbsb*-N(x). Then there is a gbsb*-open set G, such that x ∈ G⊆N. Since G is gbsb*-open and x∈G, G is gbsb*-nbhd of x. Therefore G∈gbsb*-N(x) and also G ⊆ N.

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