

NEWLY DEVELOPED FOUR METHODS FOR DIVISIBILITY OF 7

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ABSTRACT

Today every student is getting problem to check whether a given number is divisible by 7 or not. Actual division takes a lot of time to check out divisibility result of 7. Through these methods we can check whether the given number is divisible by 7 without performing actual division.

Keywords: Divisibility, actual division and seven.

INTRODUCTION

There are simpler methods for test of divisibility by 2,3,4,5,6,8,9,11,13 and we are well-known about these divisibility rules. My invention provide lesson to middle school educator, undergraduate students and teacher in checking the divisibility result of 7. So many people present paper on test of divisibility of seven like 6-9 method which is designed to verify whether a given number is divisible by 7 [1]. Literature survey reveals that Nahir [2] calculators and Computers have changed the school mathematics curriculum to the point where even experienced teachers are pondering the merits and demerits of drill and practice (D and P), a notion that used to stand at the heart of curriculum for “cementing in” ideas. Mathematics educators have been debating the role of D and P for years, with each side giving passionate arguments as to why their thinking on the subject should be adopted curriculum decision makers, and by those who are still uncertain as to which side of polemic to endorse. Do D and P really help students develop a deeper understanding and appreciation for the notion under study, or is it as the adversaries claim, “Boring to the Student” and a major contributing factor as to why student hate mathematics? An answer to this will never be definitive within our profession as a whole, but each teacher must answer this question for them self. Where does one draw the line these day with respect to D and P, taking into account the existence of sophisticated computer an algebra system where most of problem encountered school mathematics can be solved nano seconds, if one know how to set up the computer to solve them? Every has listed to essential topics and skills they believe all children should know, but unfortunately, topics on these lists and the depth of knowledge we wish to impart to the children concerning these topics are not standard, even with respect to simplest of notions, should children be expected to know how to multiply a three digit whole number by a two digit whole number? I, and thousands of other teachers, say “yes” but I am certain that just as many teachers can be found supporting a negative answer on this topic-and they hold to this negative stance even when the issue is phrased in personal way: do you want your children (or grandchildren) to be able to correctly carry out the work long-hand to compute $(538)(79)$ To many dismay, I have colleagues who claim that they do not care whether or not their own children can carry out such a multiplication. They validate this stance by saying that we are living in 21st century, where calculators and computers are everywhere; they can be found even on one’s wristwatch. And $2+3$ (They seem not to care about this either). My developed paper presents the divisibility rule of 7 without single challenge. Reported most of an early tests result from the genius of the Islamic mathematicians. Ibn Sina (980-1037 AD), known as Avicenna in the western world, is said to have discovered the method of “casting out 9” to check arithmetic operations. Al-Karkhi (c.1015), who had studied Diophantus and is famous for his work *Fakhri* on Algebra, had a test for 9 and for 11. The "Father of Algebra," al-Khowarizimi (9th century), had a test for 9. The Arab mathematician al-Banna (1256-1321 AD) had tests for 7, 8 and 9. In the 15th century, another Arab mathematician, Sibte el-Maridini, checked addition by "casting out multiples of 7 or 8". The Renaissance mathematicians were not far behind. Leonardo Fibonacci of Pis, in his famous book *Liber Abaci* (1202 AD), had a proof of the test for 9, and indicated tests for 7 and 11. For this paper we did not find it appropriate to classify the tests in chronological fashion.

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Instead, we have grouped the tests according to the mathematical concepts involved. Of course, any test could involve more than one concept. The tests within each group have a common thread, a common concept. We now begin with divisibility tests arranged in different groups.

I. A number N can be written as $N = 10t + u$, For example, in $N = 2536$, then $N = 10(253) + 6$, thus $t = 253$ and $u=6$. The tests [7, 12, 13, 15, 16, 17] in this group *add a multiple of the unit digit* to the rest of the number (or to its multiple) and check divisibility of the number thus obtained. For example to check for divisibility by 7, we may proceed as follows:

$$N = 10t + u \rightarrow 3t + 6u \pmod{7}.$$

Since 3 is relatively prime to 7, we can factor out 3 and get $10t + u \equiv 0 \pmod{7}$ iff $t + 2u \equiv 0 \pmod{7}$. This gives us a test for 7.

Test Ia for 7. A number $N = 10t + u$ is divisible by 7 iff $t + 2u$ is divisible 7.

Again, $N = 10t + u \equiv 10t + 8u \pmod{7}$. We factor out 2 and conclude that

$$N = 10t + u \equiv 0 \pmod{7} \text{ iff } 5t + 4u \equiv 0 \pmod{7}.$$

Test IIa for 7:- $N = 100h + 10t + u$ is divisible by 7 iff $2h + 3t + u$ is divisible by 7 [3]

We may also think of a number N as $N = 100h + x$. For example, if $N = 24539$, it can be written as $N = 100(245) + 39$, so that $h= 245$ and $x = 39$. Here the digits of N are divided into two groups, 245 and 39. The tests in this group add a multiple of h to x and test divisibility of the new number thus obtained [4, 5]

Test Ia for 7:- $N = 100h + x$ is divisible by 7 iff $2h + x$ is divisible by 7

Similar tests can be designed [6] when we consider $N=100a + b$, where b is the number consisting of the last three digits of N . Since $1000 \equiv 1 \pmod{7}$

Test III (general) for any divisor d. If $10^k \equiv d \pmod{d}$, then a number $N = P(10)$ is divisible by d iff $P(k)$ is divisible by d .

Since $10 \equiv 3 \pmod{7}$, we have

$$1967 \equiv 1(3^3) + 9(3^2) + 7 \pmod{7} \equiv 1(27) + 9(9) + 6(3) + 7 \pmod{7}$$

$$\equiv 27 + 81 + 18 + 7 \pmod{7} \equiv 133 \pmod{7} \equiv 0 \pmod{7}$$

This gives us a test for 7

Test IIIa for 7. A number $N = P(10)$ is divisible by 7 iff $P(3)$ is divisible by 7.

Since $10 \equiv 1 \pmod{3}$, $10 \equiv 1 \pmod{9}$ and $10 \equiv 1 \pmod{11}$, $P(1)$ amounts to the sum of the digits, whereas $P(1)$ equals the difference of the two sums of the odd and even numbered digits, thus we have the following tests for 9 and 11:

A. L. Crelle [7] used the fact that $1000 \equiv 1 \pmod{7}$. Here is an example. Suppose we want to test if 7 divides the number $N = 235, 689, 436, 773$. Considering N as a polynomial in 1000, we may write.

$$N = P(1000) = 235(1000^3) + 689(1000^2) + 436(1000) + 773.$$

Using $1000 \equiv 1 \pmod{7}$, we see that N is divisible by 7 if $N_2 = 773 + 436 + 689 + 235 = 2127$ is divisible by 7, so is N .

IV. We may as well call this a group of Miscellaneous tests, because there is no central idea connecting them. Each test in this group uses a different concept.

Test IVa. In this test, to check the divisibility of a number $N = abcdef$ by a prime p , we add or subtract a suitable multiple of p to N so that the result ends in 0. This is possible if the prime p is relative prime to 10.

In [8], Bezuska showed the divisibility of a number N by a prime p , say $p = 7$, as follows. We add a suitable multiple of 7 to N so that the sum ends in 0. Since 10 is relatively prime to 7, we can delete the 0 and test the new number N_2 thus obtained.

Here is an example of how the test works. To test 2366 for 7, we add to it a multiple of 7 that ends in 4. Since $2 \cdot 7 = 14$, adding 2366 and 14 gives us 2380. We drop the 0 and look at $N_2 = 238$. To repeat the test we need a multiple of 7 that ends in 2. Since $6 \cdot 7 = 42$, we add 42 to 238 which results in 280. Dropping the 0, we see that 28 is divisible by 7, hence so is N

Test IVb: Let N be a number written as $N = abcdefg$. To check divisibility by p , we replace the number ab by $ab \pmod{p}$. Suppose that is x . Now we look at the new number $N_2 = xcdefg$.

Example: We will test divisibility of 2366 by 7. We replace 23 by 2 because $23 \equiv 2 \pmod{7}$. We look at the new number 266. Repeating the test, we replace 26 by $26 \pmod{7}$ which is 5. The new number is 56 which is divisible by 7, hence so is N .

We may use more than two digits to apply this test. In other words, if $N = abcdefg$ and we are testing divisibility by p , we may replace the number abc by a number $x = abc \pmod{p}$, and then look at the number $xdefg$. Or we may replace $abcd$ by $y = abcd \pmod{p}$, and look at the new number $yefg$ [9]. Similar attempts were done by few mathematicians regarding to such type of divisibility [10-26].

Test IVc. There are various tests when the digits of a number N have a certain pattern. Our number N may be of the type $aabbcc$, or $ababab$, or $abcabc$. In each case we make use of the pattern and devise suitable tests.

Example: If $N = 234234$, then $N = 234(1001)$, and any divisor of 1001 or 234 will divide N .

Example: If $N = ababab$, then $N = ab(10101)$, and we look at the divisors of 10101 as well as ab

We have developed four methods without performing actual division. The number N can be tested by following methods:

1. Leave only one number on RHS of given number and triple the remaining LHS number. Then add derived result of LHS in RHS.
2. Leave two digits on RHS and Double the remaining number on LHS. Then add derived result of LHS to RHS.
3. Keep three digits of given number on RHS and other on LHS. After this subtract smaller number from LHS or RHS.

Every published paper rule takes so much time to check divisibility of 7 for more than 7 digit number. Among these methods one method clears this problem.

Method description:- We have introduced mainly four methods by taking number of example

RECENTLY NEWLY DEVELOPED THREE METHODS AND METHODOLOGY

Method 1:

Test of divisibility by 7 for 789

9 is the R.H.S. digit

78 is the L.H.S. digit

Multiply L.H.S. two digit number by 3

$$78 \times 3 = 234$$

Add 234 to R.H.S. digit 9

$$234 + 9 = 242$$

2 is the R.H.S. digit

24 is the L.H.S. digit

Multiply L.H.S. two digit number by 3

$$24 \times 3 = 72$$

Add 72 to R.H.S. digit 2

$$72 + 2 = 74$$

Result: 74 is not divisible by 7

Hence the given number 789 is not divisible by 7

Method 2:

Test of divisibility by 7 for 4687

46 is the L.H.S. digit

87 is the R.H.S. digit

First keep R.H.S. two digit number 87 aside

Multiply L.H.S. two digit number by 2

$$46 \times 2 = 92$$

Add 92 to R.H.S. digit 87

$$87 + 92 = 179$$

Keep R.H.S. two digit number 79 aside

Multiply L.H.S. one digit number by 2

$$1 \times 2 = 2$$

Add 2 to R.H.S. digit 79

$$79 + 2 = 81$$

81 which not divisible by 7

Hence 4687 is not divisible by 7

Method 3:-

Part-I

Test of divisibility by 7 for 321342

321 is LHS digit

342 is RHS digit

Firstly decide which number is greater than other one

342 is greater than 321

Subtract smaller number 321 from larger number 342

$342-321=21$

21 which is divisible by 7

Hence 321342 is divisible by 7.

In third method there are few limitations for six digit containing only nine numbers as, 111111

To overcome this problem for these numbers we developed another additional one step as shown in Part II

Part-II

Test of divisibility for 666666

6 is RHS number

66666 is LHS number

Multiply LHS five digit number by three

$$66666 \times 3 = 199998$$

Add 199998 to RHS digit 6

$$199998 + 6 = 200004$$

Now proceeds as method 3

200 is LHS digit

004 is RHS digit

Firstly decide which number is greater than other one

200 is greater than 004

Subtract smaller number 004 from larger number 200

$200-004=196$

196 which is divisible by 7

Hence 666666 is divisible by 7.

CONCLUSION

Now we are in 21st century so every student or people are using electronic equipment for simple calculation. All these electronic equipment makes mathematics subject boring. Our paper provides a lesson to change their mind from boring mathematics to favourite mathematics. Our paper presents the test of divisibility by 7 without any limitation. Some people or student cannot use calculator, so their test of divisibility of 7 by actual division takes so much time, but this paper has completely cleared out time consuming problem. All these four methods test of divisibility is so simple without an actual division. Fourth method solves limitation in third method.

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