

ON COC-LIGHT FUNCTIONS

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ABSTRACT

In this paper we study coc-open sets which used to introduce some concepts namely coc-disconnected, coc-totally disconnected spaces, coc*-continuous, Inversely coc (coc*, coc**) totally disconnected and coc-light functions. Some facts, Examples and propositions have been given to support our work.

Key words: coc-open sets, coc-compact and coc- T_2 spaces.coc-continuous, coc-homeomorphism functions.

1-INTRODUCTION

In [5] S.AIGHour and S.Samarah introduced the concept coc-open sets in topological spaces and in [3] a space X is said to be totally disconnected if for every pair of distinct points $a, b \in X$ has a disconnection $A \cup B = X$ such that $a \in A$ and $b \in B$ and the author in [6] introduced a concept namely light mapping (= A surjective mapping $f: X \rightarrow Y$ is called light mapping if for every $y \in Y$, $f^{-1}(y)$ is totally disconnected set) we used the concept coc-open set to define some types of spaces and functions like coc-disconnected, coc-totally disconnected spaces and coc-light, coc*-continuous, Inversely coc (coc*, coc**) totally disconnected and coc-homeomorphism functions. Throw our work X is mean a topological space.

Definition (1), [5]: A sub set A of a space (X, \mathcal{T}) is called coc-compact open set (briefly coc-open) if for every $x \in A$, there exists an open set $U \subseteq X$ and a compact subset K such that $x \in U - K \subseteq A$. The complement of coc-open subset is called coc-closed.

Remarks 1:

1. The set of all coc-open sets forms a topology on X denoted by \mathcal{T}^k [5]
2. Every open set is coc-open set but the converse may be not true in general as in the following:

Example 1: $(\mathbb{R}, \mathcal{T}_{ind})$, then Q and Q^c are coc-open sets which they not open, where \mathcal{T}_{ind} is the indiscrete topology.

Definition 2, [2]: Let f be a function from a space (X, \mathcal{T}) into an space (Y, \mathcal{T}) , then f is said to be coc-continuous function if the inverse image of every open set U in Y is coc-open in X .

Clearly, every continuous function is coc-continuous function but the converse may be not true.

Example 2: Let $f: (\mathbb{R}, \mathcal{T}_{ind}) \rightarrow (\mathbb{R}, \mathcal{T}_D)$ such that $f(x) = x$ for each $x \in \mathbb{R}$

Since any subset of $(\mathbb{R}, \mathcal{T}_D)$ is open.

If $\{1\} \in \mathcal{T}_D$, $f^{-1}(\{1\}) = \{1\}$ is not open in $(\mathbb{R}, \mathcal{T}_{ind})$, since the only open sets in $(\mathbb{R}, \mathcal{T}_{ind})$ are \emptyset and \mathbb{R} .

Now if we take $A = \{1\}$

For each $x \in A \exists$ just $1 \in A$ such that $1 \in \mathbb{R} - (\mathbb{R}, \{1\}) = \{1\} \subseteq A = \{1\}$ clearly $\mathbb{R} - \{1\}$ is compact in $(\mathbb{R}, \mathcal{T}_{ind})$. So $f^{-1}(A) = A$ is coc-open set.

Therefore f is coc-continuous but not continuous.

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Definition 3 [1]: A space X is said to be coc-compact if every coc-open cover of X has finite sub cover.

Remark: Every coc-compact space is compact.

Example 3: $(\mathbb{R}, \tau_{\text{ind}})$ is compact but not coc-compact

Since the space is compact

To prove its not coc-compact.

Let $\{\{x\}\}_{x \in \mathbb{R}}$ be coc –open cover to \mathbb{R} , Where $\{x\}$ is coc-open set for each $x \in \mathbb{R}$, there exists th only open set R which contain x and $R-\{x\}$ is compact subset of \mathbb{R} such that $x \in \{x\} = R - (R - \{x\}) \subseteq \{x\}$.

But we cannot reduce this cover to a finite sub cover. Since if we remove one coc-open set $\{x_0\}$ so the remind coc-open cover is cover $\mathbb{R} - \{x_0\}$.which is not finite sub cover to \mathbb{R} .

So $(\mathbb{R}, \tau_{\text{ind}})$ is not coc-compact space.

Definition 4[2]:A space X is said to be coc-hausdorff space if for each $x \neq y$ in X there exist two distinct points an coc-open sets U, V such that $x \in U, y \in V$.

Remark 2, [2]: It is clear that every hausdorff space is coc-hausdorff space. But the converse is not true in general as in the following.

Example 2-4: Let $(\mathbb{R}, \tau_{\text{ind}})$ be indiscrete space.

Then $(\mathbb{R}, \tau_{\text{ind}})$ is not T_2 -space, but $(\mathbb{R}, \tau_{\text{ind}})$ is coc T_2 -space, since for each $x, y \in \mathbb{R}$ with $x \neq y$ we have $\{x\}$ and $\{y\}$ are disjoint coc-open sets containing x and y respectively.

Definitions 5: Let $f: X \rightarrow Y$ be function of a space X into a space Y then:

- i: f is called an coc-closed function if $f(A)$ is an coc-closed set in Y for every closed set A in X [2].
- ii f is called an coc*-closed function if $f(A)$ is an closed set in Y for every coc-closed set A in X .
- iii f is called an coc**-closed function if $f(A)$ is an coc-closed set in Y for every coc-closed set A in X [2].

Definition 6 [2]: Let X and Y be spaces and let f be a function from X into Y then f is said to be coc-homeomorphism if:

- 1. f is bijective.
- 2. f is coc-continuous.
- 3. f is coc-closed (coc-open).

It is clear that every homeomorphism is an coc-homeomorphism.

Example 5: $I_{\mathbb{R}}: (\mathbb{R}, \tau_{\text{ind}}) \rightarrow (\mathbb{R}, \tau_{\text{D}})$ where $I_{\mathbb{R}}$ be the identity function .Then $I_{\mathbb{R}}$ is bijective, coc-open and coc-continuous.

So it is coc-homeomorphism but not homeomorphism since if it is not continuous.

Definition 7: Let (X, τ) be a space and let A, B be nonempty coc -open sets on X , then $A \cup B$ is said to be coc-disconnection to X if and only if $A \cup B = X$ and $A \cap B = \emptyset$.

Definition 8: A space X is said to be coc-disconnected if there is coc-disconnection $A \cup B$ to X .

So a space X is coc-connected if it is not coc-disconnected.

Example 6: $(\mathbb{R}, \tau_{\text{D}})$ is coc-disconnected space.

Since $\mathbb{R} = Q \cup Q^c$, $Q \cap Q^c = \emptyset$ where Q and Q^c are coc-open sets.

Clearly the space is also disconnected.

Remark 3: If the set is both coc-open and coc-closed, then we say that its coc-clopen as in discrete space.

Lemma 1: A space X is said to be coc-disconnected if it has nonempty proper subset which is coc-clopen set.

Proof: Let A be a nonempty proper subset of X which is coc-clopen.

To prove X is coc-disconnected. let $B=A^c$, then B is nonempty (since A is proper subset of X) moreover, $A \cup B = X$ and $A \cap B = \emptyset$.

Since A is coc-clopen that is, A is coc-closed so B is coc-open but A is coc-open

So X is coc-disconnected.

Conversely:

Let X is coc-disconnected. Then there exist nonempty subsets A and B are coc-open sets in X such that $A \cap B = \emptyset$, $A \cup B = X$. Since B is coc-open in $X \Rightarrow B^c$ is coc-closed in X but $A \cap B = \emptyset$, so $B^c = A$ (A, B are nonempty sets) $\Rightarrow A$ is coc-clopen in X . So B is similarly.

Then A and B are coc-clopen sets.

Definition 9: A space X is said to be coc-totally disconnected if for every pair of distinct points $a, b \in X$ has coc-disconnection $A \cup B$ to X such that $a \in A$ and $b \in B$.

Example 7: (\mathbb{R}, τ_D) is coc-totally disconnected.

let \mathbb{R} be the set of all real number with discrete space, then for every two distinct points s, n we have $\{s\}, \mathbb{R} - \{s\}$ are two coc-open sets containing s, n respectively.

Remark 4: Every coc-totally disconnected is coc-disconnected but the converse may be not true in general.

Example 8: Let $X = (m, n) \cup (r, s)$ such that $m < n < r < s$, m, n, r, s distinct point in \mathbb{R} (the set of all real numbers). Let τ be topological space define to X .

But $X = (m, n) \cup (r, s)$, $(m, n) \cap (r, s) = \emptyset$, so $(m, n) \cup (r, s)$ is coc-disconnection to X .

Then X is coc-disconnected but X may be not coc-totally disconnected.

Proposition 1: Let X be a space and let $Y \subseteq X$ if X is coc-totally disconnected, then Y is also coc-totally disconnected space.

Proof: Let m, n are different points in Y but $Y \subseteq X$, then $m, n \in X$, which is coc-totally disconnected, then there exists coc-disconnection $M \cup N$ to X such that $m \in M$ and $n \in N$, so $m \in M \cap Y$ and $n \in N \cap Y$.

$$\begin{aligned} \text{But } (M \cap Y) \cup (N \cap Y) &= (M \cup N) \cap Y = X \cap Y = Y \\ (M \cap Y) \cap (N \cap Y) &= (M \cap N) \cap Y = \emptyset \cap Y = \emptyset. \end{aligned}$$

Then there exists coc-disconnection $M \cup N$ to Y , such that $M \cap Y \neq \emptyset$ and $N \cap Y \neq \emptyset$ are coc-open sets in Y .

Then Y is also coc-totally disconnected sub space.

Now we introduce the following definition:

Definition 10: A function $f: X \rightarrow Y$ is said to be coc*-continuous if the inverse image of coc-open set in Y is open set in X .

Proposition 2: Let $f: X \rightarrow Y$ be bijective coc*-continuous function, a space Y is coc-compact if X is compact space.

Proof: Let $\{U_\alpha\}_{\alpha \in \Omega}$ be a coc-open cover to Y then $f^{-1}\{U_\alpha\}_{\alpha \in \Omega}$ be an open cover to X (since f is coc*-continuous function).

$$\text{But } Y \subseteq \bigcup_{\alpha \in \Omega} U_\alpha \Rightarrow X = f^{-1}(Y) \subseteq f^{-1}\left(\bigcup_{\alpha \in \Omega} U_\alpha\right) = f^{-1}\{U_\alpha\}$$

Proposition 3: Every coc-compact sub set of coc- T_2 space is coc-closed.

Proposition 4: Let $f: X \rightarrow Y$ be coc*- continuous function where X and Y are spaces.

If X is coc-compact and Y is coc- T_2 space then f is coc-closed.

Proof: Let F be Closed Set in X which coc-compact Then X is Compact (by remark 2)

So F compact, but f is coc^{*}-continuous. Then $f(F)$ is coc-compact in Y , Which coc- T_2 then by (proposition (3))

$f(F)$ is coc-closed set in Y .

Then f is coc -closed function.

Proposition 5: Let $f: X \rightarrow Y$ be continuous function where X and Y are spaces.

If X is coc-compact and Y is T_2 space then f is coc*-closed.

Proof: Since F is coc-closed set in X which is X is coc-compact so by proposition (every coc-closed subset of coc-compact is coc-compact)

Then F is coc-compact in X (by remark 2) $\Rightarrow F$ is compact in X

But f is continuous then $f(F)$ is compact in Y by proposition(the continuous image of compact set is also compact)

But Y is T_2 -space then by proposition: (every compact subset of T_2 space is closed).

Then $f(F)$ is closed in Y .

Therefore f is coc*-closed function.

Proposition 6: Let $f: X \rightarrow Y$ be coc** - continuous function where X and Y are spaces.

If X is coc-compact and Y is coc- T_2 space then f is coc**-closed.

Proof: Since F is coc-closed set in X which is X is coc-compact so by proposition (every coc-closed subset of coc-compact is coc-compact)

Then F is coc-compact in X

But f is coc**-continuous then $f(F)$ is compact in Y by proposition(the coc**-continuous image of compact set is coc-compact)

But Y is coc- T_2 space then by proposition: (every coc-compact subset of coc- T_2 space is coc-closed)

Then $f(F)$ is coc-closed in Y

Therefore f is coc*-closed function.

Definition 11: A surjective function $f: X \rightarrow Y$ is said to be coc-totally disconnected function if and only if for every coc-totally disconnected $G \subseteq X$, $f(G)$ is coc-totally disconnected in Y .

Remark 5: The coc-continuous image of coc-totally disconnected set not necessary coc-totally disconnected set for example:

Example 9: Let $X = \{a, b, c\}$ where τ_D is discrete topology to X .

And $Y = \{p, q\}$ where τ_{ind} is indiscrete topology to Y .

A coc-continuous $f: X \rightarrow Y$ define the following:

$$f(a)=f(b) =p, f(c)=q$$

Note that f is coc-continuous and X is coc-totally disconnected but Y is indiscrete space which is not coc-totally disconnected.

Remark 6: Every totally disconnected is coc-totally disconnected but the converse may be not true in general:

Example 10: Let (\mathbb{R}, τ_{ind}) is coc-totally but not totally disconnected.

To prove it is coc-totally disconnected.

If $x, y \in Q \subseteq \mathbb{R}$ with $x \neq y$, there exist $q \in Q^c$ such that $x < q < y$ then $U = \{r \in \mathbb{R} : r < q\}$ and $V = \{r \in \mathbb{R} : r \geq q\}$ so $U \cup V = \mathbb{R}$, $U \cap V = \emptyset$, we claim U is coc-open set in \mathbb{R} , since there exist only \mathbb{R} open in \mathbb{R} with $x \in \mathbb{R}$ and $U_1 = \{r \in \mathbb{R} : r < x\}$ so $U_1 \cup V$ is compact in \mathbb{R} .

$\Rightarrow x \in \mathbb{R} - (U_1 \cup V) \subseteq U$!!!!

Proposition 7: Let X and Y be spaces and let $f: X \rightarrow Y$ be bijective and coc-open function.

If X be totally disconnected, then Y is coc-totally disconnected.

Proof: Let y_1 and $y_2 \in Y$ with $y_1 \neq y_2$, but f is surjective function, then there exist only two points $x_1, x_2 \in X$ such that $f(x_1) = y_1, f(x_2) = y_2$, also X is totally disconnected

Then there exists disconnection $G \cup H$ to X such that $x_1 \in G$ and $x_2 \in H$, f is coc-homeomorphism. Where G and H are two open sets.

Then $f(G)$ and $f(H)$ are coc-open sets in Y and $f(G) \cup f(H) = f(G \cup H) = f(X) = Y$, but f is bijective function, $f(G) \cap f(H) = f(G \cap H) = f(\emptyset) = \emptyset$ such that $y_1 \in f(G), y_2 \in f(H)$

Then $f(G) \cup f(H)$ is coc-disconnection to Y .

That mean Y is coc-totally disconnected.

Corollary 1: Let X and Y are spaces and let $f: X \rightarrow Y$ be coc-homeomorphism.

If X be totally disconnected, then Y is coc-totally disconnected.

Proposition 8: Let X and Y are spaces and let $f: X \rightarrow Y$ be coc-homeomorphism.

If X be coc-totally disconnected, then Y is also coc-totally disconnected.

Proof: Let y_1 and $y_2 \in Y$ with $y_1 \neq y_2$, but f is surjective function, then there exist only two points $x_1, x_2 \in X$ such that $f(x_1) = y_1, f(x_2) = y_2$, also X is coc-totally disconnected

Then there exists coc-disconnection $G \cup H$ to X such that $x_1 \in G$ and $x_2 \in H$, f is coc-homeomorphism. so G and H are coc-open sets in X .

Then $f(G)$ and $f(H)$ are coc-open sets in Y and $f(G) \cup f(H) = f(G \cup H) = f(X) = Y$, but f is bijective function, $f(G) \cap f(H) = f(G \cap H) = f(\emptyset) = \emptyset$ such that $y_1 \in f(G), y_2 \in f(H)$

Then $f(G) \cup f(H)$ is coc-disconnection to Y .

That mean Y is coc-totally disconnected

Definition 12: $f: X \rightarrow Y$ is said to be coc*-open if $f(A)$ is open in Y whenever A is coc-open in X .

Proposition 9: Let $f: X \rightarrow Y$ be coc*-open function and bijective, if X is coc-totally disconnected then Y is totally disconnected.

Proof: Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$ but f is bijective function, then there exist only two point $x_1, x_2 \in X$ such that $f(x_1) = y_1, f(x_2) = y_2$.

Also X is coc-totally disconnected, then there exist coc-disconnected $G \cup H$ to X such that $x_1 \in G$ and $x_2 \in H$. f is coc*-open function, then $f(G)$ and $f(H)$ are open in Y . $f(G) \cup f(H) = f(G \cup H) = f(X) = Y$

But f is bijective function, $f(G) \cap f(H) = f(G \cap H) = f(\emptyset) = \emptyset$

Such that $y_1 \in f(G)$ and $y_2 \in f(H)$ then $f(G) \cup f(H)$ is disconnected to Y .

That mean Y is totally disconnected.

Now we introduce the following definitions:

Definition 13: A surjective function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be inversely coc -totally disconnected mapping if and only if for every totally disconnected set G in Y , $f^{-1}(G)$ is coc-totally disconnected set in X .

Definition 14: A surjective function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be inversely coc* -totally disconnected mapping if and only if for every coc- totally disconnected set G in Y , $f^{-1}(G)$ is totally disconnected set in X .

Definition 15: A surjective function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be inversely coc** -totally disconnected mapping if and only if for every coc-totally disconnected set G in Y , $f^{-1}(G)$ is coc-totally disconnected set in X .

Propositions 10:

1. Every coc*-totally disconnected is coc-totally disconnected.
2. Every coc**-totally disconnected is coc-totally disconnected.
3. Every coc*-totally disconnected is coc**-totally disconnected.

Proof:

1. Let G is totally disconnected in $Y \Rightarrow G$ is coc-totally disconnected (remark 6).
 $f^{-1}(G)$ is totally disconnected by (definition 14) $\Rightarrow f^{-1}(G)$ is coc-totally disconnected (by remark 6).
 So f is coc-totally disconnected.
2. Let G is totally disconnected in $Y \Rightarrow G$ is coc-totally disconnected (remark 6).
 $f^{-1}(G)$ is coc-totally disconnected by (definition 15).
 So f is coc-totally disconnected
3. Let G is coc-totally disconnected in Y .
 $f^{-1}(G)$ is totally disconnected by (definition 14) $\Rightarrow f^{-1}(G)$ is coc-totally disconnected (by remark 6).
 So f is coc**-totally disconnected.

Definition 16: Let X and Y be two spaces. A surjective function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be coc-light function if $f(y)$ is coc-totally disconnected for each $y \in Y$.

Example 11: Let $f: (R, \tau_D) \rightarrow (R, \tau_U)$ be coc-continuous define the following: $f(x)=5$ for each $x \in R$ so f is coc-light mapping.

Remarks 7:

1. Every light mapping is coc-light mapping but the converse may be not true in general for example:
2. Every coc-homeomorphism is coc-light mapping but the converse may be not true in general as in example7:

Theorem 1: Let $f: X \rightarrow Y$ and $g: Y \rightarrow K$ are surjective functions then a surjective function $h: X \rightarrow K$ such that $h=g \circ f$ is light function if f is inversely coc*-totally disconnected and g is coc-light function.

Proof: To prove h is light function, let $k \in K$ but g is coc-light function, then $g^{-1}(k)$ is coc-totally disconnected.

Also f is inversely coc*-totally disconnected mapping, then $f^{-1}(g^{-1}(k))$ is totally disconnected.

But $f^{-1}(g^{-1}(k)) = (g \circ f)^{-1}(k) = h^{-1}(k)$.

So h is light mapping.

Theorem 2: Let $f: X \rightarrow Y$ and $g: Y \rightarrow K$ are surjective functions then a surjective function $h: X \rightarrow K$ such that $h=g \circ f$ is coc-light function if f is inversely coc-totally disconnected and g is light function.

Proof: To prove h is coc-light function, let $k \in K$ but g is light function, then $g^{-1}(k)$ is totally disconnected

Also f is inversely coc-totally disconnected function, then $f^{-1}(g^{-1}(k))$ is coc-totally disconnected.

But $f^{-1}(g^{-1}(k)) = (g \circ f)^{-1}(k) = h^{-1}(k)$.

So h is coc-light mapping.

Theorem 3: Let $f: X \rightarrow Y$ and $g: Y \rightarrow K$ are surjective functions then a surjective function $h: X \rightarrow K$ such that $h=g \circ f$ is coc-light function if f is inversely coc**-totally disconnected and g is coc-light function.

Proof: To prove h is coc-light mapping, let $k \in K$ but g is coc-light function, then $g^{-1}(k)$ is inversely coc** -totally disconnected.

Also f is inversely coc-totally disconnected function, then $f^{-1}(g^{-1}(k))$ is coc-totally disconnected.

But $f^{-1}(g^{-1}(k)) = (g \circ f)^{-1}(k) = h^{-1}(k)$. So h is coc-light mapping.

Theorem (4): Let $h: X \rightarrow Y$ be surjective mapping, where $h = g \circ f$, $f: X \rightarrow K$ and $g: K \rightarrow Y$ are mappings then:

1. If g bijective mapping and f is coc-light mapping, then h is coc-light mapping.
2. If h coc-light mapping and g is injective mapping, then f is coc-light mapping.
3. If h is coc-light mapping and f is surjective coc-totally disconnected then g is coc-light mapping again.

Proof:

1. let $y \in Y$

As g is bijective mapping then there exists one and only one point $k \in K$ such that $g(k) = y$.

$$\text{But } h^{-1}(y) = (g \circ f)^{-1}(y) = f^{-1}(g^{-1}(y)) = f^{-1}(g^{-1}(g(k))) = f^{-1}(k).$$

But f is coc-light mapping, then $f^{-1}(k)$ is coc-totally disconnected in X .

but $h^{-1}(y) = f^{-1}(k)$, then $h^{-1}(y)$ is coc-totally disconnected.

So we get h is coc-light mapping.

2. let $k \in K$, then $g(k) \in Y$ but h is coc-light mapping, then $h^{-1}(g(k))$ is coc-totally disconnected in X .
Let $h^{-1}(g(k)) = (g \circ f)^{-1}(g(k)) = f^{-1}(g^{-1}(g(k))) = f^{-1}(k)$ (since g is injective mapping).

So $f^{-1}(k)$ is coc-totally disconnected in X , then f is coc-light mapping.

3. let $y \in Y$

But h is coc-light mapping, so $h^{-1}(y)$ is coc-totally disconnected in X .

But f is coc-totally disconnected, then $f(h^{-1}(y))$ is coc-totally disconnected set in K .

$$\text{But } f(h^{-1}(y)) = f((g \circ f)^{-1}(y)) = f(f^{-1}(g^{-1}(y))) = g^{-1}(y) \text{ (since } f \text{ is surjective mapping).}$$

So $g^{-1}(y)$ is coc-totally disconnected in K .

Then g is coc-light mapping.

Theorem 5: Let $f: X_1 \rightarrow Y_1$ and $g: X_2 \rightarrow Y_2$ be surjective mappings so a mapping $f \times g: X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is coc-light mapping if f is coc-homeomorphism and g is coc-light mapping.

Proof: Let $(y_1, y_2) \in Y_1 \times Y_2$

$$\text{So } (f \times g)^{-1}(y_1, y_2) = (f^{-1} \times g^{-1})(y_1, y_2) = f^{-1}(y_1) \times g^{-1}(y_2)$$

But f is homeomorphism, so there exist $x_1 \in X_1$ such that $f^{-1}(y_1) = f^{-1}(f(x_1)) = x_1$

$$\text{I mean } (f \times g)^{-1}(y_1, y_2) = x_1 \times g^{-1}(y_2)$$

But g is coc-light mapping, so $f_2^{-1}(y_2)$ is coc-totally disconnected.

But $\{x_1\} \times g^{-1}(y_2)$ is homeomorphic to $g^{-1}(y_2)$, then $(f \times g)^{-1}(y_1, y_2)$ is coc-totally disconnected. So $f \times g$ is coc-light mapping.

Lemma 2: Let $f: X_1 \rightarrow Y_1$ and $g: X_2 \rightarrow Y_2$ are surjective mappings.

If a mapping $f \times g: X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is coc-light mapping then:

1. If f is coc-homeomorphism, then g is coc-light mapping.
2. If g is coc-homeomorphism, then f is coc-light mapping.

Proof:

1. let $(y_1, y_2) \in Y_1 \times Y_2$, whereas $(f \times g)^{-1}(y_1, y_2) = (f^{-1} \times g^{-1})(y_1, y_2) = f^{-1}(y_1) \times g^{-1}(y_2)$.

But f is coc-homeomorphism, then there exists x_1 in X_1 such that $f^{-1}(y_1) = x_1$

$(f \times g)^{-1}(y_1, y_2) = \{x_1\} \times g^{-1}(y_2)$. But $f \times g$ is coc-light mapping, then $(f \times g)^{-1}(y_1, y_2)$ is coc-totally disconnected

So we get $\{x_1\} \times g^{-1}(y_2)$ is coc-totally disconnected.

But $\{x_1\} \times g^{-1}(y_2)$ is coc-homeomorphic to $g^{-1}(y_2)$, then $g^{-1}(y_2)$ is coc-totally disconnected. Hence g is coc-light mapping.

2. let $(y_1, y_2) \in Y_1 \times Y_2$.
 $(f \times g)^{-1}(y_1, y_2) = f^{-1}(y_1) \times g^{-1}(y_2)$.

But g is coc-homeomorphism, then there exists $x_2 \in X$ such that $g^{-1}(y_2) = x_2$

So $(f \times g)^{-1}(y_1, y_2) = f^{-1}(y_1) \times \{x_2\}$

But $f \times g$ is coc-light mapping, then $(f \times g)^{-1}(y_1, y_2)$ is coc-totally disconnected.

So we get $f^{-1} \times \{x_2\}$ is coc-totally disconnected.

But $f^{-1}(y_1) \times \{x_2\}$ is coc-homeomorphic to $f^{-1}(y_1)$, then $f^{-1}(y_1)$ is coc-totally disconnected. So f is coc-light mapping.

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