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GENERALIZATION OF FUZZY SEMI OPEN SETS IN FUZZY BITOPOLOGICAL SPACES

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ABSTRACT

Focus of this paper is to introduce the concept of fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open sets $\mathfrak{C}$ - (τ_i, τ_j) – semi closed sets boundary subsets of a fuzzy bitopological space where $\mathfrak{C} : [0,1] \to [0,1]$ is a complement function. Several examples are given to illustrate the concepts introduced in this paper.

Keywords: *Fuzzy complement function $\mathfrak{C}$, fuzzy $\mathfrak{C}$ - (τ_i, τ_j) – semi closed sets, fuzzy $\mathfrak{C}$ - (τ_i, τ_j) – semi open sets, fuzzy $\mathfrak{C}$-(τ_i, τ_j)–semi closure, fuzzy $\mathfrak{C}$-(τ_i, τ_j)–semi interior subsets and fuzzy bitopological spaces.*

1. INTRODUCTION

The concept of fuzzy sets and fuzzy set operations were first introduced by L. A. Zadeh [8] in the year 1965. The theory of fuzzy topological space was introduced and developed by C. L. Chang [3]. A. Kandil [5] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological space. The concept of complement function $\mathfrak{C}$: [0, 1] $\to$ [0,1] was introduced by K. Bageerathi and P. Thangavelu in [2]. The concept of fuzzy (τ_i, τ_j) - semi open set and fuzzy (τ_i, τ_j) - semi closed set was introduced and studied by S.S.Thakur,R.Malvia in [6] In this paper the concept of fuzzy $\mathfrak{C}$-(τ_i, τ_j) -semi open sets, fuzzy $\mathfrak{C}$-(τ_i, τ_j) -semi closed sets, fuzzy $\mathfrak{C}$-(τ_i, τ_j) -semi closure and fuzzy $\mathfrak{C}$-(τ_i, τ_j) -semi interior operators in fuzzy bitopological spaces is introduced and several examples are given to illustrate the concepts introduced in this paper.

2. PRELIMINARIES

In this section we list some definitions and results that are needed. Any function $\mathfrak{C}$: [0, 1]$\to$[0, 1] defined from the interval [0, 1] to itself is called a complement function. Throughout the paper $\mathfrak{C}$ denotes an arbitrary complement function and (X, τ_i, τ_j) is a fuzzy bitopological space in the sense of A.Kandil [5].Throughout this paper, for fuzzy set λ of a fuzzy bitopological space (X, τ_i, τ_j), τ_i - intλ and τ_j - cl$_{\mathfrak{C}}\lambda$ means, respectively, the interior and closure of λ with respect to fuzzy topologies τ_i andτ_j.

Definition 2.1[2]: If λ is a fuzzy subset of X then the complement $\mathfrak{C}\lambda$ of a fuzzy set λ is a fuzzy subset with membership function defined by $\mu_{\mathfrak{C}}\lambda$ (x) = $\mathfrak{C}(\mu\lambda$ (x)) for all x$\in$X.

A subset λ of a fuzzy topological space is fuzzy closed if its standard complement λ', where λ' (x) = 1–λ(x) is fuzzy open. Several fuzzy topologists used this type of complement while extending the concepts in general topological spaces to fuzzy topological spaces. But there are other complements available in the fuzzy literature.

The properties of fuzzy complement function $\mathfrak{C}$ and $\mathfrak{C}\lambda$ are given in George Klir [4] and Bageerathi *et al.* [2]. The following lemma will be useful in sequel. Some of the complement functions are given below.

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Examples 2.2[4]:

(i) The standard complement function: $\mathfrak{C}_1(x) = 1 - x$.

(ii) The Threshold type complement function for any $t \in [0, 1)$:

$$\mathfrak{C}_t(x) = \begin{cases} 1 & \text{for } 0 \le x \le t \\ 0 & \text{for } t < x \le 1 \end{cases}.$$

(iii) Sugeno class complement function for any $\lambda \in (1, \infty)$:

$$\mathfrak{C}_{S\lambda}(x) = \frac{1-x}{1+\lambda x}, \text{ for } x \in [0, 1].$$

(iv) Yagor class of complement function for $\omega \in (0, \infty)$:

$$\mathfrak{C}_{Y\omega}(x) = (1 - x^{\omega})^{1/\omega}, \quad \text{for } x \in [0, 1].$$

The next lemma can be easily established.

Lemma 2.3[4]: The complement functions $\mathfrak{C}_1$, $\mathfrak{C}_t$, $\mathfrak{C}_{S\lambda}$ and $\mathfrak{C}_{Y\omega}$ satisfy the following conditions.

(i) Boundary condition: $\mathfrak{C}(0) = 1$ and $\mathfrak{C}(1) = 0$;

(ii) Monotonicity: for all x, y $\in [0, 1]$, $x \le y \Rightarrow \mathfrak{C}(x) \ge \mathfrak{C}(y)$;

(iii) $\mathfrak{C}$ is continuous and

(iv) Involutive: $\mathfrak{C}(\mathfrak{C}(x)) = x$ for all $x \in [0, 1]$.

Definition 2.4[1]: For a family $\{A_\alpha: \alpha\in\Delta\}$ of fuzzy sub sets of X, the union, $A = \cup \{A_\alpha: \alpha\in\Delta\}$ and the intersection, $B = \cap \{A_\alpha: \alpha\in\Delta\}$ are defined with membership functions respectively
$\mu_A(x) = \sup\{\mu_{A\alpha}(x) : \alpha\in\Delta\}$ and $\mu_B(x) = \inf\{\mu_{A\alpha}(x) : \alpha\in\Delta\}$, $x\in X$.

Lemma 2.5 [2]: Let $\mathfrak{C}: [0, 1] \to [0, 1]$ be a complement function that satisfies the involutive and monotonicity properties. Then for any family $\{A_\alpha : \alpha\in\Delta\}$ of fuzzy subsets of X we have

(i) $\mathfrak{C}(\sup\{\mu_{A\alpha}(x): \alpha\in\Delta\} = \inf\{\mathfrak{C}(\mu_{A\alpha}(x)): \alpha\in\Delta\} = \inf\{(\mu_{\mathfrak{C}A\alpha}(x)): \alpha\in\Delta\}$ and

(ii) $\mathfrak{C}(\inf\{\mu_{A\alpha}(x): \alpha\in\Delta\} = \sup\{\mathfrak{C}(\mu_{A\alpha}(x)): \alpha\in\Delta\} = \sup\{(\mu_{\mathfrak{C}A\alpha}(x)): \alpha\in\Delta\}$.

Lemma 2.6[4]: Let $\mathfrak{C}: [0, 1] \to [0, 1]$ be a complement function that satisfies involutive and monotonicity properties. Then for any family $\{A_\alpha : \alpha\in\Delta\}$ of fuzzy subsets of X. we have

(i) $\mathfrak{C}(\cup\{A_\alpha : \alpha\in\Delta\}) = \cap\{\mathfrak{C}A_\alpha : \alpha\in\Delta\}$ and (ii) $\mathfrak{C}(\cap\{A_\alpha : \alpha\in\Delta\}) = \cup\{\mathfrak{C}A_\alpha : \alpha\in\Delta\}$.

Proposition 2.7[7]: If the complement functions $\mathfrak{C}$ satisfies the monotonicity and involutive properties, then for any fuzzy subset λ of a fuzzy bitopological space ,we have

(i) $\mathfrak{C}(\tau_i\text{-int}\lambda) = \tau_i\text{-cl}_{\mathfrak{C}}(\mathfrak{C}\lambda)$ and (ii) $\mathfrak{C}(\tau_i\text{-cl}_{\mathfrak{C}}\lambda) = \tau_i\text{-int}(\mathfrak{C}\lambda)$.

Theorem 2.8 [7]: Let $\mathfrak{C}$ be a complement function that satisfies the monotonicity and involutive properties. For any two fuzzy subsets λ and μ of a fuzzy bitopological space we have

(i) $\lambda \le \tau_i\text{-cl}_{\mathfrak{C}}\lambda$;

(ii) λ is fuzzy $\mathfrak{C}$-τ_i-closed $\Leftrightarrow \tau_i\text{-cl}_{\mathfrak{C}}\lambda = \lambda$;

(iii) $\tau_i\text{-cl}_{\mathfrak{C}}(\tau_i\text{-cl}_{\mathfrak{C}}\lambda) = \tau_i\text{-cl}_{\mathfrak{C}}\lambda$;

(iv) If $\lambda \le \mu$ then $\tau_i\text{-cl}_{\mathfrak{C}}\lambda \le \tau_i\text{-cl}_{\mathfrak{C}}\mu$

(v) $\tau_i\text{-cl}_{\mathfrak{C}}(\lambda \vee \mu) = \tau_i\text{-cl}_{\mathfrak{C}}\lambda \vee \tau_i\text{-cl}_{\mathfrak{C}}\mu$ and

(vi) $\tau_i\text{-cl}_{\mathfrak{C}}(\lambda \wedge \mu) \ge \tau_i\text{-cl}_{\mathfrak{C}}\lambda \wedge \tau_i\text{-cl}_{\mathfrak{C}}\mu$

3. Fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open and Fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed sets

In this section we define the notion of fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open set and $\mathfrak{C}$ - (τ_i, τ_j) - semi closed set and discussed some of their properties.

Definition 3.1: A fuzzy set λ of a fuzzy bitopological space (X, τ_1, τ_2) is called (a) fuzzy $\mathfrak{C}$ -(τ_i,τ_j)- semi open if there exists a τ_i - fuzzy open set μ such that $\mu \le \lambda \le \tau_j\text{-cl}_C(\mu)$ $\tau_i\text{-cl}_{\mathfrak{C}}(\mu)$ (b) fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed if there exists a fuzzy $\mathfrak{C}$ - τ_i - closed μ such that $\mathfrak{C}\mu \le \mathfrak{C}\lambda \le \tau_j\text{-cl}_{\mathfrak{C}}(\mathfrak{C}\mu)$.

The concept of all fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open sets and $\mathfrak{C}$ - (τ_i, τ_j) - semi closed sets coincides with the concept of all fuzzy (τ_i, τ_j) - semi open sets and fuzzy (τ_i, τ_j) semi closed sets if the arbitrary complement coincides with the standard complement function.

Theorem 3.2: Let (X, τ_1, τ_2) be a fuzzy bitopological space and $\mathfrak{C}$ be a complement function that satisfies the involutive properties. Then λ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j)- semi closed if and only if $\mathfrak{C}\lambda$ is s fuzzy $\mathfrak{C}$ - (τ_i, τ_j)- semi open.

Proof: Let λ be a fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed. Then by using Definition 3.1 there exists a fuzzy $\mathfrak{C}$ - τ_i - closed μ such that $\mathfrak{C}\mu \le \mathfrak{C}\lambda \le \tau_j$ - $cl_{\mathfrak{C}}(\mathfrak{C}\mu)$. By replacing $\mathfrak{C}\mu = \delta$, $\delta \le \mathfrak{C}\lambda \le \tau_j$ - $cl_{\mathfrak{C}}(\delta)$. By using Definition 3.1 (b), $\mathfrak{C}\lambda$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open. Conversely let λ be a fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open. Then by using Definition 3.1 (a), there exists a $\eta \in \tau_i$ such that $\eta \le \lambda \le \tau_j$ - $cl_{\mathfrak{C}}\eta$. Let $\mu = \mathfrak{C}\eta$. Since $\mathfrak{C}$ satisfies the involutive condition, $\eta = \mathfrak{C}(\mathfrak{C}\eta) = \mathfrak{C}\mu$. That is $\mathfrak{C}\mu \le \mathfrak{C}(\mathfrak{C}\lambda) \le \tau_j$ - $cl_{\mathfrak{C}}(\mathfrak{C}\mu)$. Thus $\mathfrak{C}\lambda$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed.

The next example shows that if the complement function does not satisfy the involutive property, then the conclusion of above theorem is false.

Example 3.3: Let $X = \{a, b, c\}$, $\tau_1 = \{0, 1, \{a_{.8}, b_{.7}\}, \{a_{.8}, b_{.7}, c_{.3}\}, 1\}$ and $\tau_2 = \{0, 1, \{c_{.9}\}, \{a_{.8}, b_{.9}\}, \{a_{.8}, b_{.9}, c_{.9}\}, \{a_{.9}, b_{.9}, c_{.9}\}\}$. Let $\mathfrak{C}(x) = \frac{1}{1+3x}$, $0 \le x \le 1$, be a complement function does not satisfies involute property. Then the family of all fuzzy $\mathfrak{C}$- τ_i - closed sets are $\mathfrak{C}(\tau_1) = \{1, \{a_{.25}, b_{.25}, c_{.25}\}, \{a_{.294}, b_{.3225}, c_1\}, \{a_{.294}, b_{.3225}, c_{.526}\}\}$ and $\mathfrak{C}(\tau_2) = \{1, \{a_{.25}, b_{.25}, c_{.25}\}, \{a_1, b_1, c_{.27}\}, \{a_{.29}, b_{.27}, c_1\}, \{a_{.29}, b_{.27}, c_{.27}\}, \{a_{.27}, b_{.27}, c_{.27}\}\}$. Let $\lambda = \{a_{.2}, b_{.2}, c_{.8}\}$. Then λ is not fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi open. Now $\mathfrak{C}\lambda = \{a_{.625}, b_{.625}, c_{.29}\}$ and it is fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi closed.

Theorem 3.4: Let (X, τ_1, τ_2) be a fuzzy bitopological space and $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive properties. Then (i) fuzzy subset λ of a fuzzy bitopological space (X, τ_1, τ_2) is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open if and only if $\lambda \le \tau_j$ - $cl_{\mathfrak{C}}(\tau_i$ - Int $\lambda)$. (ii) fuzzy subset λ of a fuzzy bitopological space (X, τ_1, τ_2) is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed if and only if τ_j - Int$(\tau_i$ - $cl_{\mathfrak{C}}(\lambda)) \le \lambda$.

Proof:

(i) Let λ be a fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open. Then by using Definition 3.1 (a), there exists a $\mu \in \tau_i$ such that $\mu \le \lambda \le \tau_j$ - $cl_{\mathfrak{C}}\mu$. Since $\mathfrak{C}$ satisfies the monotonic and involutive conditions, by using Proposition 2.7, we have $\mu \le \lambda \le \mathfrak{C}(\tau_j$ - Int $\mathfrak{C}\mu)$. Since $\mathfrak{C}$ satisfies the monotonic and involutive conditions, τ_j - Int $\mathfrak{C}\mu \le \mathfrak{C}\lambda \le \mathfrak{C}\mu$. By applying Theorem [2.8] , $\mathfrak{C}\lambda \le \mathfrak{C}\mu$ implies that τ_i - $cl_{\mathfrak{C}}(\mathfrak{C}\lambda) \le \tau_i$ - $cl_{\mathfrak{C}}(\mathfrak{C}\mu) = \mathfrak{C}\mu$. This implies that τ_j - Int $(\tau_i$ - $cl_{\mathfrak{C}}(\mathfrak{C}\lambda)) \le \tau_i$ - $cl_{\mathfrak{C}}(\mathfrak{C}\lambda) \le \mathfrak{C}\mu$. Therefore τ_j - Int $(\tau_i$ - $cl_{\mathfrak{C}}(\mathfrak{C}\lambda))$ is τ_j - open contained in $\mathfrak{C}\mu$. But τ_j - Int $\mathfrak{C}\mu$ is largest τ_j - open set contained in $\mathfrak{C}\mu$. Therefore τ_j - Int $(\tau_i$ - $cl_{\mathfrak{C}}(\mathfrak{C}\lambda)) \le \tau_j$ - Int $(\mathfrak{C}\mu) \le \mathfrak{C}\lambda$. Therefore τ_j - Int $(\tau_i$ - $cl_{\mathfrak{C}}(\mathfrak{C}\lambda)) \le \mathfrak{C}\lambda$. This implies that $\lambda \le \tau_j$ - $cl_{\mathfrak{C}}(\tau_i$ - Int $\lambda)$. Conversely, we assume that $\lambda \le \tau_j$ - $cl_{\mathfrak{C}}(\tau_i$ - Int $\lambda)$. Let $\mu = \tau_i$ - Int λ. Taking complement on both sides, $\mathfrak{C}\mu = \mathfrak{C}(\tau_i$ - Int $\lambda)$. By using Proposition 2.7, $\mathfrak{C}\mu = \tau_i$ - $cl_{\mathfrak{C}}(\mathfrak{C}\lambda) \ge \mathfrak{C}\lambda$. Since $\mathfrak{C}$ satisfies the monotonic condition, $\mu \le \lambda$. By our assumption $\lambda \le \tau_j$ - $cl_{\mathfrak{C}}(\tau_i$ - Int $(\lambda)) = \tau_j$ - $cl_{\mathfrak{C}}\mu$. We have $\mu \le \lambda \le \tau_j$ - $cl_{\mathfrak{C}}(\mu)$. By using Definition 3.1, λ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open.

(ii) Let λ be a fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed. Then by using Definition 3.1, there exists $\mathfrak{C}\mu \in \tau_i$ such that $\mathfrak{C}\mu \le \mathfrak{C}\lambda \le \tau_j$ - $cl_{\mathfrak{C}}(\mathfrak{C}\mu)$. Since $\mathfrak{C}$ satisfies the monotonic and involutive conditions, by using applying proposition 2.7, we have $\mathfrak{C}\mu \le \mathfrak{C}\lambda \le \mathfrak{C}(\tau_j$ - Int $\mu)$. In particular $\mathfrak{C}$ satisfies the monotonic condition that implies τ_j - Int $\mu \le \lambda \le \mu$. Since $\lambda \le \tau_i$ - $cl_{\mathfrak{C}}\lambda$ that implies τ_j - Int $\mu \le \lambda \le \tau_i$ - $cl_{\mathfrak{C}}\lambda \le \mu$. Therefore τ_i - $cl_{\mathfrak{C}}\lambda \le \mu$. Then τ_j - Int $(\tau_i$ - $cl_{\mathfrak{C}}\lambda) \le \tau_j$ - Int $\mu \le \lambda$. Therefore τ_j - Int $(\tau_i$ - $cl_{\mathfrak{C}}\lambda) \le \lambda$. Conversely, suppose τ_j - Int $(\tau_i$ - $cl_{\mathfrak{C}}\lambda) \le \lambda$. Let $\mu = \tau_i$ - $cl_{\mathfrak{C}}\lambda$. Therefore τ_j - Int $\mu \le \lambda$. Taking complement on both sides, $\mathfrak{C}\lambda \le \mathfrak{C}(\tau_j$ - Int $\mu)$. Since $\mathfrak{C}$ satisfies the monotonic and involutive conditions, by applying proposition 2.7 $\mathfrak{C}\lambda \le \tau_j$ - $cl_{\mathfrak{C}}(\mathfrak{C}\mu)$. Since $\mu = \tau_i$ - $cl_{\mathfrak{C}}\lambda$. Taking complement on both sides, we have $\mathfrak{C}\mu = \mathfrak{C}(\tau_i$ - $cl_{\mathfrak{C}}\lambda) = \tau_i$ - Int $\mathfrak{C}\lambda \le \mathfrak{C}\lambda$. So $\mathfrak{C}\mu \le \mathfrak{C}\lambda$. Thus we have $\mathfrak{C}\mu \le \mathfrak{C}\lambda \le \tau_j$ - $cl_{\mathfrak{C}}(\mathfrak{C}\mu)$. Again by using Definition 3.1, λ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed.

The next example shows that if the complement function does not satisfy the monotonic and involutive conditions, then the conclusion of above theorem is false.

Example 3.5: Let $X = \{a, b, c\}$, $\tau_1 = \{0, \{a_{.2}, b_{.1}\}, \{a_{.5}, b_0, c_{.6}\}, \{a_{.2}, b_0, c_{.6}\}, \{a_{.5}, b_{.1}, c_{.6}\}, \{a_{.6}, b_{.4}, c_{.8}\}, 1\}$ and $\tau_2 = \{0, \{a_{.5}, c_{.1}\}, \{a_{.6}, b_{.2}, c_{.3}\}, \{a_{.5}, b_{.2}, c_{.1}\}, 1\}$. Let $\mathfrak{C}(x) = \frac{1}{1+x}$, $0 \le x \le 1$, be a complement function. We see that the complement function $\mathfrak{C}$ does not satisfy the involutive condition. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}(\tau_1) = \{1, \{a_{.8}, b_{.9}, c_1\}, \{a_{.7}, b_1, c_{.625}\}, \{a_{.8}, b_1, c_{.625}\}, \{a_{.7}, b_{.9}, c_{.625}\}, \{a_{.625}, b_{.7}, c_{.55}\}, \{a_{.5}, b_{.5}, c_{.5}\}\}$ and $\mathfrak{C}(\tau_2) = \{1, \{a_{.7}, b_1, c_{.9}\}, \{a_{.625}, b_{.8}, c_{.77}\}, \{a_{.7}, b_{.8}, c_{.9}\}, \{a_{.5}, b_{.5}, c_{.5}\}\}$. Let $\lambda = \{a_{.7}, b_{.5}, c_{.8}\}$. Then we can see that τ_2 - $cl_{\mathfrak{C}}(\tau_1$ - Int$(\lambda)) = \tau_2$ - $cl_{\mathfrak{C}}\{a_{.5}, b_{.1}, c_{.6}\} = \{a_{.625}, b_{.8}, c_{.7}\}$. Therefore $\lambda \nleq \tau_2$ - $cl_{\mathfrak{C}}(\tau_1$ - Int $\lambda)$. But λ is fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi open. Also let $\lambda = \{a_{.5}, b_{.4}, c_{.1}\}$. Then it can be computed that τ_2 - Int$(\tau_1$ - $cl_{\mathfrak{C}}(\lambda)) = \tau_2$ – Int $\{a_{.5}, b_{.5}, c_{.5}\} = \{a_{.5}, b_{.2}, c_{.1}\}$. This implies that $\lambda \ge \tau_2$ - Int$(\tau_1$ - $cl_{\mathfrak{C}}(\lambda))$.But λ is not fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi closed.

Remark 3.6: It is clear from Definition 3.1 that every τ_i - fuzzy open (respectively τ_i - fuzzy closed) set is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open (respectively fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed). The following example shows that the converse may not be true.

Example 3.7: Let X = {a, b, c}, τ_1 = { 0, {a.$_2$, b.$_1$ }, {a.$_1$, b.$_4$, c.$_8$}, {a.$_1$, b.$_1$}, {a.$_2$, b.$_4$, c.$_8$}, 1} and τ_2 = {0, {a.$_3$, c.$_6$}, {b.$_4$ }, {a.$_3$, b.$_4$, c.$_6$}, 1}. Let $\mathfrak{C}$ (x) = $\frac{1-x}{1+3x}$, 0 ≤ x ≤ 1, be a complement function. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}$ (τ_1) = **{**1, {a.$_5$, b.$_7$, c$_1$}, {a.$_7$, b.$_3$, c.$_1$}, {a.$_7$, b.$_7$, c$_1$}, {a.$_5$, b.$_3$, c.$_1$}, 0} and $\mathfrak{C}$ (τ_2) = **{**1, {a.$_4$, b$_1$, c.$_{14}$}, {a$_1$, b.$_3$, c$_1$}, {a.$_4$, b.$_3$, c.$_{14}$}, 0}. Let λ = {a.$_1$, b.$_1$, c.$_1$}. Then it can be computed that τ_1 - Int(λ) = {a.$_1$, b.$_1$} and τ_2 - cl$_\mathfrak{C}$ (τ_1 - Int(λ)) = {a.$_4$, b.$_3$, c.$_{14}$}. Then λ ≤ τ_2 - cl$_\mathfrak{C}$ (τ_1 - Int λ). Therefore λ is fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi open but λ is not fuzzy $\mathfrak{C}$ - τ_i - open. Also let λ = {a.$_2$, b.$_1$, c.$_1$}. Then it can be calculated that τ_1 - cl$_\mathfrak{C}$ (λ) = {a.$_5$, b.$_3$, c.$_1$} and τ_2 - Int(τ_1 - cl$_\mathfrak{C}$ λ) = {0}. Therefore λ ≥ τ_2 - Int (τ_1 - cl$_\mathfrak{C}$ (λ)). This implies that λ is fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi closed but λ is not fuzzy $\mathfrak{C}$ - τ_i - closed.

Remark 3.8: The concepts of fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi open (respectively fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi closed) and fuzzy $\mathfrak{C}$ - (τ_2, τ_1) - semi open (respectively fuzzy $\mathfrak{C}$ - (τ_2, τ_1) - semi closed) sets are independent.

Example 3.9: Let X = {a, b, c}, τ_1 = { 0, {a.$_2$, b.$_1$}, {a.$_1$, b.$_4$, c.$_8$}, {a.$_1$, b.$_1$}, {a.$_2$, b.$_4$, c.$_8$}, 1} and τ_2 = {0, {a.$_3$, c.$_6$}, {b.$_4$}, {a.$_3$, b.$_4$, c.$_6$}, 1}. Let $\mathfrak{C}$ (x) = $\frac{1-x}{1+3x}$, 0 ≤ x ≤ 1, be arbitrary complement function. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}$ (τ_1) = **{**1, {a.$_5$, b.$_7$, c$_1$}, {a.$_7$, b.$_3$, c.$_1$}, {a.$_7$, b.$_7$, c$_1$}, {a.$_5$, b.$_3$, c.$_1$}, 0} and $\mathfrak{C}$ (τ_2) = **{**1, {a.$_4$, b$_1$, c.$_{14}$}, {a$_1$, b.$_3$, c$_1$}, {a.$_4$, b.$_3$, c.$_{14}$}, 0}. Let λ = {a.$_3$, b.$_2$, c.$_1$}. Then it can be computed that τ_2 - cl$_\mathfrak{C}$ (τ_1 - Int(λ)) = τ_2 - cl$_\mathfrak{C}$ {a.$_2$, b.$_1$} = {a.$_4$, b.$_3$, c.$_1$}. Therefore λ ≤ τ_2 - cl$_\mathfrak{C}$ (τ_1 - Int λ). This implies that λ is fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi open. Now τ_1 - cl$_\mathfrak{C}$ (τ_2 - Int(λ)) = {0}. Therefore λ ≰ τ_1 - cl$_\mathfrak{C}$ (τ_2 - Int λ). This implies that λ is fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi open but λ is not fuzzy $\mathfrak{C}$ - (τ_2, τ_1) - semi open. Let λ = {a.$_1$, b.$_2$, c.$_1$}. Then it can be computed that τ_2 - Int (τ_1 - cl$_\mathfrak{C}$ (λ)) = τ_2 - Int{a.$_5$, b.$_3$, c.$_1$} = {0}. Therefore λ ≥ τ_2 - Int (τ_1 - cl$_\mathfrak{C}$ (λ)). This implies that λ is fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi closed. Also τ_1 - Int (τ_2 - cl$_\mathfrak{C}$ (λ)) = τ_1 - Int{a.$_4$, b.$_3$, c.$_{14}$} = {a.$_2$, b.$_1$, c$_0$}. This implies that λ ≱ τ_1 - Int (τ_2 - cl$_\mathfrak{C}$ (λ)). Therefore λ is not fuzzy $\mathfrak{C}$ - (τ_2 , τ_1) - semi closed.

Theorem 3.10: Let (X, τ_1, τ_2) be a fuzzy bitopological space and $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive properties. Then (i) Arbitrary union of fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open sets is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open. (ii) Arbitrary intersection of fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed sets is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed.

Proof

(i) Let {λ_α} be a collection of all fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open sets of a fuzzy bitopological space (X, τ_1, τ_2). Then for each α, there exists a $\mu_\alpha \in \tau_i$ such that $\mu_\alpha \le \lambda_\alpha \le \tau_j$ - cl$_\mathfrak{C}$ (μ_α). Thus $\vee\mu_\alpha \le \vee\lambda_\alpha \le \vee\ \tau_j$ - cl$_\mathfrak{C}$ (μ_α). Since $\mathfrak{C}$ satisfies the monotonic and involutive properties by applying Theorem 2.8, we have τ_j - cl$_\mathfrak{C}$ (μ_α) ≤ τ_j - cl$_\mathfrak{C}$ ($\vee\mu_\alpha$), that implies $\vee\ \tau_j$ - cl$_\mathfrak{C}$ (μ_α) ≤ τ_j - cl$_\mathfrak{C}$ ($\vee\mu_\alpha$). Hence $\vee\mu_\alpha \le \vee\lambda_\alpha \le \tau_j$ - cl$_\mathfrak{C}$ ($\vee\mu_\alpha$) and $\vee\mu_\alpha \in \tau_i$. By using Definition 3.1, we have $\vee\lambda_\alpha$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open.

(ii) Let {λ_α} be a collection of all fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed sets of a fuzzy bitopological space (X, τ_1, τ_2). Then for each α, there exists a fuzzy $\mathfrak{C}$ - τ_i - closed set μ_α such that $\mathfrak{C}\ \mu_\alpha \le \mathfrak{C}\ \lambda_\alpha \le \tau_j$ - cl$_\mathfrak{C}$ ($\mathfrak{C}\ \mu_\alpha$). Thus $\vee\ \mathfrak{C}\ \mu_\alpha \le \vee\ \mathfrak{C}\ \lambda_\alpha \le \vee\ \tau_j$ - cl$_\mathfrak{C}$ ($\mathfrak{C}\ \mu_\alpha$). Since $\mathfrak{C}$ satisfies the monotonic and involutive properties by applying Theorem 2.8, we have $\vee\tau_j$ - cl$_\mathfrak{C}$ ($\mathfrak{C}\ \mu_\alpha$) ≤ τ_j - cl$_\mathfrak{C}$ ($\vee\mu_\alpha$). This implies that $\vee\ \mathfrak{C}\ \mu_\alpha \le \vee\ \mathfrak{C}\ \lambda_\alpha \le \tau_j$ - cl$_\mathfrak{C}$ ($\vee\mathfrak{C}\ \mu_\alpha$). By using Lemma 2.6 $\mathfrak{C}$ ($\wedge\mu_\alpha$) ≤ $\mathfrak{C}$ ($\wedge\lambda_\alpha$) ≤ τ_j - cl$_\mathfrak{C}$ ($\mathfrak{C}$($\wedge\mu_\alpha$)). Since arbitrary intersection of fuzzy $\mathfrak{C}$ - τ_i - closed sets is fuzzy $\mathfrak{C}$ - τ_i - closed. That implies that $\wedge\mu_\alpha = \mu$ is fuzzy $\mathfrak{C}$ - τ_i - closed set. Therefore $\mathfrak{C}\ \mu \le \mathfrak{C}$ ($\wedge\lambda_\alpha$) ≤ τ_j - cl$_\mathfrak{C}$ ($\mathfrak{C}\ \mu$). By using Definition 3.1, we have $\wedge\lambda_\alpha$ is fuzzy $\mathfrak{C}$ - (τ_i , τ_j) - semi closed.

Remark 3.11: The following example shows that Intersection of two fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open sets need not be fuzzy $\mathfrak{C}$ - (τ_i , τ_j) - semi open.

Example 3.12: Let X = {a, b}, τ_1 = {0, {a.$_2$}, {b.$_6$ }, {a.$_2$, b.$_6$ }, 1} and τ_2 = {0, {a.$_3$}, {b.$_4$}, {a.$_3$, b.$_4$}, 1}. Let $\mathfrak{C}$ (x) =$\frac{1-x}{1+4x}$, 0 ≤ x ≤ 1, be a complement function. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}$ (τ_1) = **{**1, {a.$_4$, b$_1$}, {a$_1$, b.$_1$}, {a.$_4$, b.$_1$}, 0} and $\mathfrak{C}$ (τ_2) = **{** 1, {a.$_3$, b$_1$}, {a$_1$, b.$_2$}, {a.$_3$, b.$_2$}, 0}. Let λ = {a.$_3$, b.$_1$} and μ = {a.$_1$, b.$_7$}. Then it can be calculated that τ_2 - cl$_\mathfrak{C}$ (τ_1 - Int(λ)) = {a.$_3$, b.$_2$}. Therefore λ ≤ τ_2 - cl$_\mathfrak{C}$ (τ_1 - Int λ). Also τ_2 - cl$_\mathfrak{C}$ (τ_1 - Int(μ)) = {a.$_3$, b$_1$}. Therefore μ ≤ τ_2 - cl$_\mathfrak{C}$ (τ_1 - Int λ). This shows that λ and μ are fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi open. But λ∧μ = {a.$_1$, b.$_1$}. Now τ_2 - cl$_\mathfrak{C}$ (τ_1 - Int (λ∧μ)) = {0}. This shows that λ∧μ ≰ τ_2 - cl$_\mathfrak{C}$ (τ_1 - Int(λ∧μ)). This implies that λ∧μ need not be fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi open even if $\mathfrak{C}$ satisfies the monotonic and involutive conditions.

Remark 3.13: The following example shows that Union of two fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed sets need not be fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed set even though the complement function satisfies the monotonic and involutive condition.

Example 3.14: Let X = {a, b}, τ_1 = {0, {a.$_4$, b$_1$ }, {a $_1$, b.$_3$}, {a.$_4$, b.$_3$ }, 1} and τ_2 = { 0, {a.$_1$, b.$_1$}, {a.$_3$, b.$_1$}, {a.$_1$, b.$_5$}, {a.$_3$, b.$_5$ }, 1}. Let $\mathfrak{C}$ (x) = $\frac{1-x}{1+6x}$, 0 ≤ x ≤ 1, be a complement function. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}$ (τ_1) = **{**1, {a.$_2$, b$_0$}, {a$_0$, b.$_3$}, {a.$_2$, b.$_3$}, 0} and $\mathfrak{C}$ (τ_2) = **{**1, {a.$_6$, b.$_6$}, {a.$_3$, b.$_6$}, {a.$_6$, b.$_1$}, {a.$_3$, b.$_1$} 0}. Let λ = {a.$_{05}$, b$_0$} and μ = {a $_0$, b.$_{05}$}. Then it can be calculated that τ_2 - Int$_{\mathfrak{C}}$ (τ_1 - cl$_{\mathfrak{C}}$ (λ)) = {0} and τ_2 - Int$_{\mathfrak{C}}$ (τ_1 - cl$_{\mathfrak{C}}$ (μ)) = {0}. λ∨μ= {a.$_{05}$, b.$_{05}$}.Then it can be calculated that τ_2 - Int$_{\mathfrak{C}}$ (τ_1 - cl$_{\mathfrak{C}}$ (λ∨μ)) = {a.$_1$, b.$_1$} ≰ λ∨μ. This implies that λ∨μ is not fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi closed.

Remark 3.15: The following example shows that the intersection of fuzzy τ_i - open set, i=1,2 and fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open sets need not be fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open even if the complement function satisfies the monotonic and involutive properties.

Example 3.16: Let X = {a, b, c}, τ_1 = {0, {b.$_1$}, {a.$_2$}, {a.$_2$, b.$_1$}, {a.$_2$, b.$_8$, c.$_9$}, 1} and τ_2 = {0, {b.$_1$}, {a.$_2$, b.$_{05}$, c.$_3$}, {a.$_2$, b.$_1$, c.$_3$}, {b.$_{05}$}, 1}. Let $\mathfrak{C}$ (x) = $\frac{1-x}{1+x}$, be a complement function that satisfies the monotonic and involutive properties. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}$ (τ_1) = **{**0, {a$_1$, b.$_8$, c$_1$}, {a.$_7$, b$_1$, c$_1$}, {a.$_7$, b.$_8$, c$_1$}, {a.$_7$, b.$_1$, c.$_{05}$}, 1} and $\mathfrak{C}$ (τ_2) = **{**0, {a$_1$, b.$_8$, c$_1$}, {a.$_7$, b.$_9$, c.$_5$}, {a.$_7$, b.$_8$, c.$_5$}, {a$_1$, b.$_9$, c$_1$}, 1}. Let λ = {a.$_2$, b.$_{05}$, c.$_1$}. Then it can be calculated that τ_2 - cl$_{\mathfrak{C}}$ (τ_1 - Int(λ)) = {a.$_7$, b.$_8$, c.$_5$}. This shows that λ ≤ τ_2 - cl$_{\mathfrak{C}}$ (τ_1 - Int λ) and λ is fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi open. But μ = {b.$_1$} is fuzzy τ_i – open, i=1,2. Now μ∧λ = {b.$_{05}$}. It can be evaluated that τ_2 - cl$_{\mathfrak{C}}$ (τ_1 - Int (μ∧λ)) = τ_2 - cl$_{\mathfrak{C}}$ {0} = {0}. Therefore μ∧λ ≰ τ_2 - cl$_{\mathfrak{C}}$ (τ_1 - Int(μ∧λ)). This implies that μ∧λ is not fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi open.

Remark 3.17: The following example shows that the union of fuzzy $\mathfrak{C}$ - τ_i - closed set, i=1,2 and fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed sets need not be fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed even if the complement function satisfies the monotonic and involutive properties.

Example 3.18: Let X = {a, b, c}, τ_1 = {0, {a$_0$, b.$_2$, c$_0$}, {a.$_3$, b$_0$, c$_0$}, {a.$_3$, b.$_2$, c$_0$}, {a.$_3$, b.$_7$, c.$_8$}, 1} and τ_2 = {0, {b.$_2$}, {a.$_3$, b.$_{05}$, c.$_4$}, {a$_3$, b.$_2$, c.$_4$}, {a$_0$, b.$_{05}$, c.$_4$}, 1}. Let $\mathfrak{C}$ (x) =$\sqrt{1-x^2}$, be a complement function that satisfies the monotonic and involutive properties. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}$ (τ_1) = **{** 1, {a$_1$, b.$_{98}$, c$_1$}, {a.$_{95}$, b$_1$, c$_1$}, {a.$_{95}$, b.$_{98}$, c$_1$}, {a.$_{95}$, b.$_7$, c.$_6$}, 0} and $\mathfrak{C}$ (τ_2) = **{** 0, {a$_1$, b.$_{98}$, c$_1$}, {a.$_{95}$, b.$_{05}$, c.$_9$}, {a.$_{95}$, b.$_{98}$, c.$_9$}, {a$_1$, b.$_{05}$, c.$_9$}, 1}. Let λ = {a.$_{95}$, b.$_{99}$, c.$_{99}$}. Then it can be evaluated that τ_2 - Int (τ_1 - cl$_{\mathfrak{C}}$ (λ)) = {a.$_3$, b.$_2$, c.$_4$}. Therefore λ ≥ τ_2 - Int (τ_1 - cl$_{\mathfrak{C}}$ λ). That is λ is fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi closed. But μ = {a$_1$, b.$_{98}$, c$_1$} is fuzzy $\mathfrak{C}$ - τ_i - closed set, i=1, 2. Also μ∨λ = {a$_1$, b.$_{99}$, c$_1$}. Now τ_2 - Int (τ_1 - cl$_{\mathfrak{C}}$ (λ∨μ)) = τ_2 - Int{1} = {1}. This shows that μ∨λ ≱ τ_2 - Int (τ_1 - cl$_{\mathfrak{C}}$ (λ∨μ)). Therefore μ∨λ is not fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi closed.

Theorem 3.19: Let (X, τ_1, τ_2) be a fuzzy bitopological space and $\mathfrak{C}$ be a complement function satisfies the monotonic and involutive properties, then (i) If λ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open and λ ≤ λ_1 ≤ τ_j - cl$_{\mathfrak{C}}$ (λ), then λ_1 is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open set. (ii) If λ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed and τ_j - Int λ ≤ λ_1 ≤ λ, then λ_1 is fuzzy $\mathfrak{C}$ - (τ_i τ_j) - semi closed set.

Proof:

(i) Let λ be a fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open set. Then by Definition 3.1, there exists $\mu \in \tau_i$ such that $\mu \le \lambda \le \tau_j$ - cl$_{\mathfrak{C}}$ (μ). Since $\mathfrak{C}$ satisfies the monotonic and involutive properties by Theorem 2.8, we have τ_j - cl$_{\mathfrak{C}}$ (λ) ≤ τ_j - cl$_{\mathfrak{C}}$ (μ). Also $\lambda \le \lambda_1 \le \tau_j$ - cl$_{\mathfrak{C}}$ (λ) implies that $\mu \le \lambda \le \lambda_1 \le \tau_j$ - cl$_{\mathfrak{C}}$ (λ) ≤ τ_j - cl$_{\mathfrak{C}}$ (μ). Therefore $\mu \le \lambda_1 \le \tau_j$ - cl$_{\mathfrak{C}}$ (μ). Hence λ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open set.

(ii) Let λ be a fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed set. Then by Definition 3.1, there exists fuzzy $\mathfrak{C}$ - τ_i - closed set μ such that $\mathfrak{C}\,\mu \le \mathfrak{C}\,\lambda \le \tau_j$ - cl$_{\mathfrak{C}}$ ($\mathfrak{C}\,\mu$). Since $\mathfrak{C}$ satisfies the monotonic and involutive conditions by proposition 2.7, we have $\mu \ge \lambda \ge \tau_j$ - Int μ. Since τ_j -Int (λ) ≤ $\lambda_1 \le \lambda$, $\mathfrak{C}\,\lambda \le \mathfrak{C}\,\lambda_1 \le \tau_j$ - cl$_{\mathfrak{C}}$ ($\mathfrak{C}\,\lambda$). Therefore $\mathfrak{C}\,\mu \le \mathfrak{C}\,\lambda \le \mathfrak{C}\,\lambda_1$ ≤ τ_j - cl$_{\mathfrak{C}}$ ($\mathfrak{C}\,\lambda$) ≤ τ_j - cl$_{\mathfrak{C}}$ (τ_j - cl$_{\mathfrak{C}}$ ($\mathfrak{C}\,\mu$)) implies that $\mathfrak{C}\,\mu \le \mathfrak{C}\,\lambda_1 \le \tau_j$ - cl$_{\mathfrak{C}}$ ($\mathfrak{C}\,\mu$). Hence λ_1 is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed set.

4. Fuzzy $\mathfrak{C}$ - (τ_i , τ_j) - semi interior and Fuzzy $\mathfrak{C}$ - (τ_i , τ_j) - semi closure

In this section we define the notion of fuzzy $\mathfrak{C}$- (τ_i, τ_j) - semi interior and fuzzy $\mathfrak{C}$- (τ_i, τ_j) - semi closure operators and discussed some of their properties.

Definition 4.1: Let (X, τ_i, τ_j) be a fuzzy bitopological space and $\mathfrak{C}$ be a complement function. Then for a fuzzy subset λ of X, the fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi interior of λ (briefly (τ_i, τ_j) - SInt $_{\mathfrak{C}}$ λ), is the union of all fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open sets of X contained in λ.

That is (τ_i, τ_j) - SInt $_{\mathfrak{C}}$ (λ) = ∨ {μ : μ ≤ λ, μ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open}.

Definition 4.2: Let (X, τ_i, τ_j) be a fuzzy bitopological space and $\mathfrak{C}$ be a complement function. Then for a fuzzy subset λ of X, the fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closure of λ (briefly (τ_i, τ_j) - S $cl_{\mathfrak{C}}\,\lambda$), is the intersection of all fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed sets of X containing λ.

That is (τ_i, τ_j) - S $cl_{\mathfrak{C}}(\lambda) = \wedge\{\mu : \mu \geq \lambda, \mu$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed$\}$.

The concepts of fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closure and fuzzy (τ_i, τ_j) - semi closure are identical if $\mathfrak{C}$ is the standard complement function.

Theorem 4.3: Let (X, τ_i, τ_j) be a fuzzy bitopological space and $\mathfrak{C}$ be any complement function. Then for a fuzzy subset λ and μ of a fuzzy bitopological space X, we have

(i) (τ_i, τ_j)-$SInt_{\mathfrak{C}}(\lambda) \leq \lambda$.
(ii) λ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open $\Leftrightarrow (\tau_i, \tau_j)$ - $SInt_{\mathfrak{C}}(\lambda) = \lambda$.
(iii) (τ_i, τ_j) - $SInt_{\mathfrak{C}}((\tau_i, \tau_j)$ - $SInt_{\mathfrak{C}}(\lambda)) = (\tau_i, \tau_j)$ - $SInt_{\mathfrak{C}}(\lambda)$.
(iv) If $\lambda \leq \mu$ then (τ_i, τ_j) - $SInt_{\mathfrak{C}}(\lambda) \leq (\tau_i, \tau_j)$ - $SInt_{\mathfrak{C}}(\mu)$.

Proof:

(i) follows from Definition 4.1.
(ii) Let λ be a fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open.Since $\lambda \leq \lambda$, by Definition 4.1, $\lambda \leq (\tau_i, \tau_j)$ –$SInt_{\mathfrak{C}}(\lambda)$. By (i), (τ_i, τ_j) - S $Int_{\mathfrak{C}}(\lambda) = \lambda$. Conversely we assume that (τ_i, τ_j) - SInt $_{\mathfrak{C}}(\lambda) = \lambda$, By Definition 4.1 , λ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open.
(iii) By using (ii), we get (τ_i, τ_j) - SInt $_{\mathfrak{C}}((\tau_i, \tau_j)$ - SInt $_{\mathfrak{C}}(\lambda)) = (\tau_i, \tau_j)$ - SInt $_{\mathfrak{C}}(\lambda)$.
(iv) Since $\lambda \leq \mu$, by using (i), (τ_i, τ_j) - SInt $_{\mathfrak{C}}(\lambda) \leq \lambda \leq \mu$. This implies that (τ_i, τ_j) - SInt $_{\mathfrak{C}}((\tau_i, \tau_j)$ - S Int $_{\mathfrak{C}}(\lambda)) \leq (\tau_i, \tau_j)$ - S Int $_{\mathfrak{C}}(\mu)$. By using (iii), (τ_i, τ_j) - SInt $_{\mathfrak{C}}(\lambda) \leq (\tau_i, \tau_j)$ - SInt $_{\mathfrak{C}}(\mu)$.

Theorem 4.4: Let $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive properties. Then for a fuzzy subset λ of a fuzzy bitopological space X, we have

(i) $\mathfrak{C}((\tau_i, \tau_j)$ - SInt $_{\mathfrak{C}}(\lambda)) = (\tau_i, \tau_j)$ - S $cl_{\mathfrak{C}}(\mathfrak{C}\,\lambda)$.
(ii) $\mathfrak{C}((\tau_i, \tau_j)$ – S $cl_{\mathfrak{C}}(\lambda)) = (\tau_i, \tau_j)$ - SInt $_{\mathfrak{C}}(\mathfrak{C}\,\lambda)$.

Proof:

(i) By Definition 4.1, (τ_i, τ_j) - SInt $_{\mathfrak{C}}(\lambda) = \vee\{\mu : \mu \leq \lambda, \mu$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open$\}$. Taking complement on both sides, we get $\mathfrak{C}((\tau_i, \tau_j)$ - SInt $_{\mathfrak{C}}(\lambda)(x)) = \mathfrak{C}\{Sup\{\mu(x): \mu(x) \leq \lambda(x), \mu$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open$\}\}$. Since $\mathfrak{C}$ satisfies the monotonic and involutive properties, By Lemma 2.5, $\mathfrak{C}((\tau_i, \tau_j)$ - SInt $_{\mathfrak{C}}(\lambda)(x)) = Inf\{\mathfrak{C}\,\mu(x) : \mu(x) \leq \lambda(x), \mu$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open $\} = Inf\{\mathfrak{C}\,\mu(x) : \mathfrak{C}\,\mu(x) \geq \mathfrak{C}\,\lambda(x), \mu$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open$\} = Inf\{\eta(x) : \eta(x) \geq \mathfrak{C}\,\lambda(x), \eta$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed$\}$. By Definition 4.2, $\mathfrak{C}((\tau_i, \tau_j)$ - SInt $_{\mathfrak{C}}(\lambda)(x)) = (\tau_i, \tau_j)$ - S cl $_{\mathfrak{C}}(\mathfrak{C}\,\lambda)(x)$. Therefore $\mathfrak{C}((\tau_i, \tau_j)$ - SInt $_{\mathfrak{C}}(\lambda)) = (\tau_i, \tau_j)$ - S $cl_{\mathfrak{C}}(\mathfrak{C}\,\lambda)$.
(ii) By Definition 4.2, (τ_i, τ_j) - S $cl_{\mathfrak{C}}(\lambda) = \wedge\{\mu : \mu \geq \lambda, \mu$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed$\}$. Taking complement on both sides, we get $\mathfrak{C}((\tau_i, \tau_j)$ - S $cl_{\mathfrak{C}}(\lambda)(x)) = \mathfrak{C}\{Inf\{\mu(x) : \mu(x) \geq \lambda(x), \mu$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed$\}\}$. Since $\mathfrak{C}$ satisfies the monotonic and involutive properties, By using Lemma 2.6, $\mathfrak{C}((\tau_i, \tau_j)$ - S $cl_{\mathfrak{C}}(\lambda)(x)) = Sup\{\mathfrak{C}\,\mu(x): \mu(x) \geq \lambda(x), \mu$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed$\}$. By Definition $\mathfrak{C}((\tau_i, \tau_j)$ - S $cl_{\mathfrak{C}}(\lambda)(x) = Sup\{\mathfrak{C}\,\mu(x) : \mathfrak{C}\mu(x) \leq \mathfrak{C}\,\lambda(x), \mu$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed$\} = Sup\{\eta(x) : \eta(x) \leq \mathfrak{C}\,\lambda(x), \eta$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open$\}$. By using Definition 4.1, $\mathfrak{C}((\tau_i, \tau_j)$ - S $cl_{\mathfrak{C}}(\lambda)(x)) = (\tau_i, \tau_j)$ - SInt $_{\mathfrak{C}}(\mathfrak{C}\,\lambda)(x)$. Therefore $\mathfrak{C}((\tau_i, \tau_j)$ - S $cl_{\mathfrak{C}}(\lambda)) = (\tau_i, \tau_j)$ - SInt $_{\mathfrak{C}}(\mathfrak{C}\,\lambda)$.

Example 4.5: Let X = {a, b, c}, $\tau_1 = \{0, 1, \{c_{.3}\}, \{a_{.8}, b_{.7}\}, \{a_{.8}, b_{.7}, c_{.3}\}, 1\}$ and $\tau_2 = \{0, 1, \{c_{.9}\}, \{a_{.8}, b_{.9}\}, \{a_{.8}, b_{.9}\ c_{.9}\}, \{a_{.9}, b_{.9}, c_{.9}\}\}$. Let $\mathfrak{C}(x) = \frac{1}{1+3x}$, $0 \leq x \leq 1$, be a complement function does not satisfies involute property. Then the family of all fuzzy $\mathfrak{C}$- τ_i - closed sets are $\mathfrak{C}(\tau_1) = \{1, \{a_{.25}, b_{.25}, c_{.25}\}, \{a_1, b_1, c_{.526}\}, \{a_{.294}, b_{.3225}, c_1\}, \{a_{.294}, b_{.3225}, c_{.526}\}\}$ and $\mathfrak{C}(\tau_2) = \{1, \{a_{.25}, b_{.25}, c_{.25}\}, \{a_1, b_1, c_{.27}\}, \{a_{.29}, b_{.27}, c_1\}, \{a_{.29}, b_{.27}, c_{.27}\}, \{a_{.27}, b_{.27}, c_{.27}\}\}$. Let $\lambda = \{a_{.2}, b_{.2}, c_{.8}\}$. Then (τ_1, τ_2) - S $cl_{\mathfrak{C}}(\lambda) = \{a_{.8}, b_{.8}, c_{.5}\}$.Therefore $\mathfrak{C}((\tau_1, \tau_2)$ - S $cl_{\mathfrak{C}}(\lambda))=\{a_{.29}, b_{.29}, c_{.4}\}$. Now $= \mathfrak{C}\,\lambda = \{a_{.5}, b_{.4}, c_{.57}\}$, (τ_1, τ_2) - S $cl_{\mathfrak{C}}(\mathfrak{C}\,\lambda) = \{a_{.8}, b_{.7}, c_{.8}\}$. It can be calculated that (τ_1, τ_2) - $SInt_{\mathfrak{C}}(\lambda)=\{a_{.25}, b_{.25}, c_{.25}\}$ and $\mathfrak{C}((\tau_1, \tau_2)$ - $SInt_{\mathfrak{C}}(\lambda))=\{a_{.5}, b_{.5}, c_{.5}\}$.Also (τ_1, τ_2) - $SInt_{\mathfrak{C}}(\mathfrak{C}\,\lambda) = \{a_{.5}, b_{.4}, c_{.5}\}$.This implies that $\mathfrak{C}((\tau_1, \tau_2)$ - SInt $_{\mathfrak{C}}(\lambda))$ and (τ_1, τ_2) - S $cl_{\mathfrak{C}}(\mathfrak{C}\,\lambda)$ are not equal, $\mathfrak{C}((\tau_1, \tau_2)$ – S $cl_{\mathfrak{C}}(\lambda))$ and (τ_1, τ_2) - SInt $_{\mathfrak{C}}(\mathfrak{C}\,\lambda)$ are not equal .

Theorem 4.6: Let (X, τ_i, τ_j) be a fuzzy bitopological space and $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive properties. Then for a fuzzy subset λ and μ of a fuzzy bitopological space X, we have

(i) $\lambda \leq (\tau_i, \tau_j)$ - S $cl_{\mathfrak{C}}(\lambda)$.
(ii) λ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed $\Leftrightarrow (\tau_i, \tau_j)$ - S $cl_{\mathfrak{C}}(\lambda) = \lambda$.
(iii) (τ_i, τ_j) - S cl $_{\mathfrak{C}}((\tau_i, \tau_j)$ - S $cl_{\mathfrak{C}}(\lambda)) = (\tau_i, \tau_j)$ - S $cl_{\mathfrak{C}}(\lambda)$.
(iv) If $\lambda \leq \mu$ then (τ_i, τ_j) - S $cl_{\mathfrak{C}}(\lambda) \leq (\tau_i, \tau_j)$ - S $cl_{\mathfrak{C}}(\mu)$.

Proof:

(i) follows from Definition 4.2.

(ii) Let λ be a fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed. Since $\mathfrak{C}$ satisfies the monotonic and involutive conditions, By using Theorem 3.2, $\mathfrak{C}\ \lambda$ is fuzzy $\mathfrak{C}$- (τ_i, τ_j) - semi open. Therefore by Theorem 4.3, we get (τ_i, τ_j) - SInt $_{\mathfrak{C}}$ ($\mathfrak{C}\ \lambda$) = $\mathfrak{C}\ \lambda$. By Proposition 4.4, $\mathfrak{C}$ ((τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ)) = $\mathfrak{C}\ \lambda$. Taking complement on both sides, we get $\mathfrak{C}$ ($\mathfrak{C}$ ((τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ))) = $\mathfrak{C}$ ($\mathfrak{C}\ \lambda$). Therefore (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ) =λ. Conversely, we assume that (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ) =λ. Taking complement on both sides, we get $\mathfrak{C}$ ((τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ)) = $\mathfrak{C}\ \lambda$. By Theorem 4.4, (τ_i, τ_j) - SInt $_{\mathfrak{C}}$ ($\mathfrak{C}\ \lambda$) = $\mathfrak{C}\ \lambda$. By Theorem 4.3, $\mathfrak{C}\ \lambda$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open. Again By using Theorem 3.2, λ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed.

(iii) By Theorem 4.4, $\mathfrak{C}$ ((τ_i, τ_j) - S cl $_{\mathfrak{C}}$ (λ)) = (τ_i, τ_j) - SInt $_{\mathfrak{C}}$ ($\mathfrak{C}\ \lambda$). This implies that $\mathfrak{C}$ ((τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ)) is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open. By using Theorem 3.2, (τ_i, τ_j) - S cl $_{\mathfrak{C}}$ (λ)) is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed. Therefore By using (ii), we get (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ((τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ)) = (τ_i, τ_j) - S cl $_{\mathfrak{C}}$ (λ).

(iv) Suppose $\lambda \le \mu$ then $\mathfrak{C}\ \lambda \ge \mathfrak{C}\ \mu$. This implies that By Theorem 4.3, (τ_i, τ_j) - S Int $_{\mathfrak{C}}$ ($\mathfrak{C}\ \lambda$) $\ge$ (τ_i, τ_j) - S Int $_{\mathfrak{C}}$ ($\mathfrak{C}\ \mu$)). Since $\mathfrak{C}$ satisfies the monotonic and involutive and taking complement on both sides, we get (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ) $\le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (μ).

Proposition 4.7: Let (X, τ_1, τ_2) be a fuzzy bitopological space and $\mathfrak{C}$ be any complement function. Then for a fuzzy subsets λ and μ of a fuzzy bitopological space X, we have

(i) (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ ($\lambda \vee \mu$) $\ge$ (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ (λ) $\vee$ (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ (μ) and

(ii) (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ ($\lambda \wedge \mu$) $\le$ (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ (λ) $\wedge$ (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ (μ).

Proof:

(i) Since $\lambda \le \lambda \vee \mu$ and $\mu \le \lambda \vee \mu$. By Theorem 4.3, we have (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ (λ) $\le$ (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ ($\lambda \vee \mu$) and (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ (μ) $\le$ (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ ($\lambda \vee \mu$). This implies that (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ (λ) $\vee$ (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ (μ) $\le$ (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ ($\lambda \vee \mu$).

(ii) Since $\lambda \ge \lambda \wedge \mu$ and $\mu \ge \lambda \wedge \mu$. By Theorem 4.3, we have (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ ($\lambda \wedge \mu$) $\le$ (τ_i, τ_j) - S Int$_{\mathfrak{C}}$ (λ) and (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ ($\lambda \wedge \mu$) $\le$ (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ (μ). This implies that (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ ($\lambda \wedge \mu$) $\le$ (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ (λ) $\wedge$ (τ_i, τ_j) - SInt$_{\mathfrak{C}}$ (μ).

Proposition 4.8: Let (X, τ_1, τ_2) be a fuzzy bitopological space and $\mathfrak{C}$ be a complement function that satisfies these monotonic and involutive properties. Then for a fuzzy subsets λ and μ of a fuzzy bitopological space X, we have

(i) (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \vee \mu$) = (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ) $\vee$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (μ) and

(ii) (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \wedge \mu$) $\le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ) $\wedge$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (μ).

Proof:

(i) Since $\mathfrak{C}$ satisfies the involutive condition, (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \vee \mu$) = (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\mathfrak{C}$($\mathfrak{C}$($\lambda \vee \mu$))). Since $\mathfrak{C}$ satisfies the monotonic and involute properties, by Theorem 4.4, (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \vee \mu$) = $\mathfrak{C}$ ((τ_i, τ_j) – S In t$_{\mathfrak{C}}$ ($\mathfrak{C}$($\lambda \vee \mu$))). By Lemma 2.6, we have (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \vee \mu$) = $\mathfrak{C}$ ((τ_i, τ_j) – S In t$_{\mathfrak{C}}$ ($\mathfrak{C}\ \lambda \vee \mathfrak{C}\ \mu$)). ($\tau_i$, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \vee \mu$) $\le$ $\mathfrak{C}$ (((τ_i , τ_j)-Sint$_{\mathfrak{C}}$ ($\mathfrak{C}\ \lambda$)) $\wedge$ ((τ_i, τ_j)-SInt$_{\mathfrak{C}}$ ($\mathfrak{C}\ \mu$))). Again by Theorem 2.9, (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \vee \mu$) $\le$ $\mathfrak{C}$((τ_i, τ_j)-SInt $_{\mathfrak{C}}$ ($\mathfrak{C}\ \lambda$)) $\vee$ $\mathfrak{C}$((τ_i , τ_j)- S Int $_{\mathfrak{C}}$ ($\mathfrak{C}\ \mu$)). By Theorem 4.4, (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \vee \mu$) $\le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ) $\vee$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (μ). Also (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ) $\le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \vee \mu$) and (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (μ) $\le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \vee \mu$). This implies that (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ) $\vee$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (μ) $\le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \vee \mu$). Therefore (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \vee \mu$) = (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ) $\vee$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (μ).

(ii) Since (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \wedge \mu$) $\le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ) and (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \wedge \mu$) $\le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (μ) which implies that (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\lambda \wedge \mu$) $\le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ) $\wedge$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (μ).

Proposition 4.9: Let $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive properties. Then for any family {λ_α} of fuzzy subsets of a fuzzy bitopological space, we have

(i) $\vee$((τ_i τ_j) - S cl$_{\mathfrak{C}}$ (λ_α)) $\le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\vee \lambda_\alpha$)

(ii) (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\wedge \lambda_\alpha$) $\le$ $\wedge$((τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ_α))

Proof:

(i) For every β, $\lambda_\beta \le \vee \lambda_\alpha \le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\vee \lambda_\alpha$). By Theorem 4.6, (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ_β) $\le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\vee \lambda_\alpha$) for every β. Therefore $\vee$((τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ_β)) $\le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\vee \lambda_\alpha$). This proves (i).

(ii) For every β, $\wedge \lambda_\alpha \le \lambda_\beta$. Again by using Theorem 4.6, (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\wedge \lambda_\alpha$) $\le$ (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ_β). This implies that (τ_i, τ_j) - S cl$_{\mathfrak{C}}$ ($\wedge \lambda_\alpha$) $\le$ $\wedge$((τ_i, τ_j) - S cl$_{\mathfrak{C}}$ (λ_α)). This proves (ii).

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