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A NEW BANHATTI GEOMETRIC-ARITHMETIC INDEX

V. R. KULLI*

Department of Mathematics, Gulbarga University, Gulbarga 585106, India.

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ABSTRACT

In this paper, we introduce the Banhatti geometric-arithmetic index of a graph. A topological index is a numeric quantity from structural graph of a molecule. We determine Banhatti geometric-arithmetic index of some standard classes of graphs. Also we compute Banhatti geometric-arithmetic index of V-Phenylenic nanotubes and V-Phenylenic nanotorus.

Keywords: *molecular graph, Banhatti geometric-arithmetic index.*

Mathematics subject Classification: *05C05, 05C12.*

1. INTRODUCTION

Let $G = (V, E)$ be a finite, simple connected graph. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The degree of an edge $e = uv$ in G is defined by $d_G(e) = d_G(u) + d_G(v) – 2$. We refer to [1] for undefined term and notation.

A molecular graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms. A topological index is a numeric quantity from structural graph of a molecule. These indices are useful for establishing correlation between the sturucture of a molecular compound and its physico-chemical properties.

One of the well known and widely used topological index is the geometric-arithmetic index introduced by Vukičević and Furtula in [2].

Motivated by the definition of the geometric-arithmetic index and its applications, we introduce the Banhatti geometric-arithmetic index of a graph G as follows:

The Banhatti geometric-arithmetic index of a graph G is defined as

$$BGA(G) = \sum_{ue} \frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)}$$

where ue means that the vertex u and edge e are incident in G.

Many other Banhatti topological indices were studied, for example, in [3, 4, 5, 6, 7, 8, 9, 10]. Also geometric-arithmetic indices were studied, for example, in [11, 12]

In this paper, we determine Banhatti geometric-arithmetic index of some standard classes of graphs and also compute geometric-arithmetic index of V-Phenylenic nanotubes and V-Phenylenic nantorus.

*Corresponding Author: V. R. Kulli**
Department of Mathematics, Gulbarga University, Gulbarga 585106, India.

2. SOME STANDARD CLASSES OF GRAPHS

Proposition 1: Let C_n be a cycle with $n \geq 3$ vertices. Then $BGA(C_n) = 2n$.

Proof: Let $G = C_n$ be a cycle with $n \geq 3$ vertices. Every vertex of a cycle C_n is incident with exactly two edges and the number of edges in C_n is n.

$$BGA(G) = \sum_{ue} \frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)} = \sum_{uv\in E(G)} \left[\frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)} + \frac{2\sqrt{d_G(v)d_G(e)}}{d_G(v)+d_G(e)}\right]$$

$$= n\left[\frac{2\sqrt{2\times 2}}{2+2} + \frac{2\sqrt{2\times 2}}{2+2}\right] = 2n.$$

Proposition 2: Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$BGA(K_n) = \frac{2\sqrt{2}}{3n-5}\, n(n-1)\sqrt{(n-1)(n-2)}.$$

Proof: Let $G = K_n$ be a complete graph with $n \geq 3$ vertices. Every vertex of K_n is incident with $n - 1$ edges.

$$BGA(G) = \sum_{ue} \frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)} = \sum_{uv\in E(G)} \left[\frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)} + \frac{2\sqrt{d_G(v)d_G(e)}}{d_G(v)+d_G(e)}\right]$$

$$= \frac{n(n-1)}{2}\left[\frac{2\sqrt{(n-1)(2n-4)}}{(n-1)+(2n-4)} + \frac{2\sqrt{(n-1)(2n-4)}}{(n-1)+(2n-4)}\right]$$

$$= \frac{2\sqrt{2}}{3n-5} n(n-1)\sqrt{(n-1)(n-2)}.$$

Proposition 3: Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$, and $s \geq 2$ vertices. Then

$$BGA(K_{r,s}) = 2rs\sqrt{(r+s-2)}\left(\frac{\sqrt{s}}{(r+2s-2)} + \frac{\sqrt{r}}{(2r+s-2)}\right).$$

Proof: Let $G = K_{r,s}$ be a complete bipartite graph with $r+s$ vertices and rs edges such that $|V_1|= r$, $|V_2|= s$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \leq r \leq s$; $s \geq 2$. Every vertex of V_1 is incident with s edges and every vertex of V_2 is incident with r edges.

$$BGA(G) = \sum_{ue} \frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)} = \sum_{uv\in E(G)} \left[\frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)} + \frac{2\sqrt{d_G(v)d_G(e)}}{d_G(v)+d_G(e)}\right]$$

$$= rs\left[\frac{2\sqrt{s(r+s-2)}}{s+(r+s-2)} + \frac{2\sqrt{r(r+s-2)}}{r+(r+s-2)}\right]$$

$$= 2rs\sqrt{(r+s-2)}\left(\frac{\sqrt{s}}{r+2s-2} + \frac{\sqrt{r}}{2r+s-2}\right).$$

Corollary 4: Let $K_{r,r}$ be a complete bipartite gragh with $r \geq 2$. Then

$$BGA(K_{r,r}) = \frac{4\sqrt{2}}{3r-2} r^2\sqrt{r(r-1)}.$$

Corollary 5: Let $K_{1,s}$ be a star with $s \geq 2$. Then

$$BGA(K_{1,s}) = 2s\sqrt{s-1}\left(\frac{\sqrt{s}}{2s-1} + \frac{1}{s}\right).$$

Proposition 6: If G is an r-regular graph with n vertices and $r \geq 2$, then

$$BGA(G) = \frac{2nr\sqrt{2r(r-1)}}{3r-2}.$$

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Proof: Let G be an r-regular graph with n vertices, $r \geq 2$ and $\frac{nr}{2}$ edges. Every edge of G is incident with r edges.

$$BGA(G) = \sum_{ue} \frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)} = \sum_{uv \in E(G)} \left[\frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)} + \frac{2\sqrt{d_G(v)d_G(e)}}{d_G(v)+d_G(e)} \right]$$

$$= \frac{nr}{2}\left[\frac{2\sqrt{r(2r-2)}}{r+2r-2} + \frac{2\sqrt{r(2r-2)}}{r+2r-2} \right] = \frac{2nr\sqrt{2r(r-1)}}{3r-2}.$$

3. RESULTS FOR *V*-PHENYLENIC NANOTUBES AND NANOTORUS

Molecular graphs *V*-Phenylenic nanotubes *VPHX* [*m, n*] and *V*-Phenylenic nanotorus *VPHY*[*m, n*] belong to two different families of nanostructures whose structures are made up of cycles with length four, six and eight. Molecular graphs of *V*-Phenylenic nanotubes *VPHX* [*m, n*] and *V*-Phenylenic nanotorus *VPHY*[*m,n*] are shown in Figure 1 and 2 respectively.

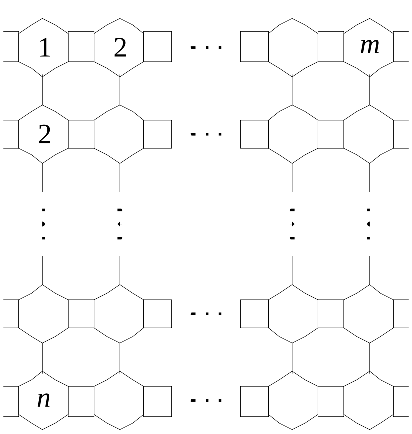

Figure-1

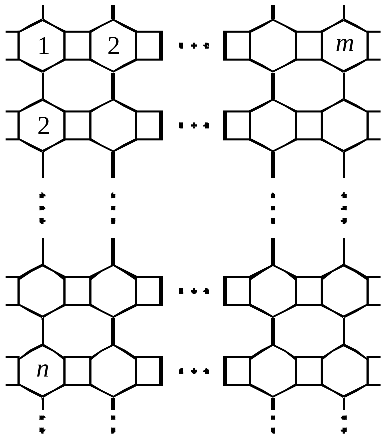

Figure-2

We determine the Banhatti geometric-arithmetic index of *V*-Phenylenic *VPHX*[*m,n*] nanotubes.

Theorem 7: Let G be *V*-Phenylenic nanotubes *VPHX*[*m, n*] for any $m, n \in N - \{1\}$. Then

$$BGA(G) = \frac{72\sqrt{3}}{7}mn + \left(1 + \frac{2\sqrt{6}}{5} - \frac{10\sqrt{3}}{7}\right)4m.$$

Proof: Let G be *V*-Phenylenic nanotubes *VPHX*[*m,n*], where m and n are the number of hexagons in the first row and first column in G, see Figure 1. By algebraic method, we get $|V(G)| = 6mn$ and $|E(G)| = 9mn - m$. Further, the edge degree partition of G is given in Table 1.

$d_G(u), d_G(v) / uv \in E(G)$	E_{23}= (2, 3)	E_{33}= (3, 3)
$d_G(e)$	3	4
Number of edges	$4m$	$9mn - 5m$

Table-1: Edge degree partition of *VPHX*[*m,n*]

$$BGA(G) = \sum_{ue} \frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)}$$

$$= \sum_{uv \in E_{23}} \left(\frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)} + \frac{2\sqrt{d_G(v)d_G(e)}}{d_G(v)+d_G(e)} \right) + \sum_{uv \in E_{33}} \frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)} + \left(\frac{2\sqrt{d_G(v)d_G(e)}}{d_G(v)+d_G(e)} \right)$$

$$= 4m\left(\frac{2\sqrt{2\times 3}}{2+3} + \frac{2\sqrt{3\times 3}}{3+3} \right) + (9mn - 5m)\left(\frac{2\sqrt{3\times 4}}{3+4} + \frac{2\sqrt{3\times 4}}{3+4} \right)$$

$$= \frac{72\sqrt{3}}{7}mn + \left(1 + \frac{2\sqrt{6}}{5} - \frac{10\sqrt{3}}{7}\right)4m.$$

We determine the Banhatti geometric-arithmetic index of *V*-Phenylenic *VPHY*[*m, n*] nanotorus.

Theorem 8: Let G be *V*-Phenylenic nanotorus *VPHY*[*m,n*] for any $m, n \in N - \{1\}$. Then

$$BGA(G) = \frac{72\sqrt{3}}{7}mn.$$

Proof: Let G be *V*-Phenylenic nanotorus *VPHY*[*m,n*], where m and n are the number of hexagons in the first row and first column in G, see Figure 2. By algebraic method, we get $|V(G)| = 6mn$ and $|E(G)|=9mn$. Further, the edge degree partition of G is given in Table 2.

$d_G(u),d_G(v)\backslash e = uv \in E(G)$	$E_{33}= (3, 3)$
$d_G(e)$	4
Number of edges	$9mn$

Table-2: Edge degree partition of *VPHY*[*m, n*]

$$BGA(G) = \sum_{ue} \frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)} = \sum_{ue}\left(\frac{2\sqrt{d_G(u)d_G(e)}}{d_G(u)+d_G(e)} + \sum_{ue}\frac{2\sqrt{d_G(v)d_G(e)}}{d_G(v)+d_G(e)}\right)$$

$$= 9mn\left(\frac{2\sqrt{3\times 4}}{3+4} + \frac{2\sqrt{3\times 4}}{3+4}\right) = \frac{72\sqrt{3}}{7}mn.$$

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