# NANO sg-INTERIOR AND NANO sg-CLOSURE IN NANO TOPOLOGICAL SPACES

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### ABSTRACT

*$I$n this paper, we introduce Nsg-interior, Nsg-closure and some of its basic properties.*

*Keywords: Nsg-open; Nsg-closed; Nsg-int(A); Nsg-cl(A).*

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## 1. INTRODUCTION

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in general topology concerns the variously modified forms of continuity, separation axioms etc., by utilizing generalized open sets. Levine [12] introduced the concept of generalized closed sets in topological space. Bhattacharyya and Lahiri [1] introduced semi-generalized closed sets in topology and investigated some of their properties The concept of nano topology was introduced by Lellis Thivagar [13,14]which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established the weak forms of nano open sets namely nano -open sets, nano semi open sets and nano pre open sets in a nano topological space. nano g closed and nano sg closed was introduced by K. Bhuvaneswari [2,3] *et al.* Since the advent of these notions several research papers with interesting results in different respects came to existence.

In this paper, the notion of Nsg-interior is defined and some of its basic properties are investigated. Also we introduce the idea of Nsg-closure in Nano topological spaces using the notions of Nsg-closed sets and obtain some related results.

## 2. PRELIMINARIES

**Definition 2.1[14]:** Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by X.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by U_R(X).That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X = \phi\}$

(iii) The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

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**Definition 2.2[14]:** If (U, R) is an approximation space and X, Y $\subseteq$ U, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$

(ii) $L_R(\phi) = U_R(\phi) = \phi$

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X \cup Y) \subseteq L_R(X) \cup L_R(Y)$

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$

(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$

(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

**Definition 2.3[14]:** Let U be an universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq Y$. $\tau_R(X)$ satisfies the following axioms

(i) U, $\phi \in \tau_R(X)$

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$

(iii) The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$. That is, $\tau_R(X)$ forms a topology on U called the nano topology on U with respect to X. We call (U, $\tau_R(X)$) as the nano topological space. The elements of $\tau_R(X)$ are called nano open sets.

**Definition 2.4[14]:** Let (U,$\tau_R$(X)) be a Nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U, then A is said to be

(i) nano semi-open [14] if A $\subseteq$ Ncl(Nint(A)).
(ii) nano α-open [14] if A $\subseteq$ N int(Ncl(Nint(A))),

The complements of the above mentioned nanoopen sets are called their respective nanoclosed sets.

**Definition 2.5:** A subset A of a space (U,$\tau_R$(X)) is called:

(i) nano generalized closed (briey, nano g-closed) if Ncl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano open in U.
(ii) a nano semi-generalized closed [2,3] (briefly Nsg closed) set if Nscl(A) $\subseteq$ G whenever A $\subseteq$ G and G is semi-open in U. The complement of nanosg-closed set is called nanosg-open.

**Definition 2.6:** Let $(U, \tau_R(X))$ be a Nano topological space and let x $\in$ U.

A subset N of U is said to be Nsg-neighbourhood of x if there exists a Nsg-open set G such that x $\in$ G $\subset$ N.

## 3. NANO sg INTERIOR IN NANO TOPOLOGICAL SPACE

**Definition 3.1:** Let $(U, \tau_R(X))$ be a Nano topological space and let x $\in$ U. A subset N of U is said to be Nsg-neighbourhood of x if there exists a Nsg-open set G such that x $\in$ G $\subset$ N.

**Definition 3.2:** Let A be a subset of $(U, \tau_R(X))$. A point x $\in$ A is said to be Nsg-interior point of A is A is a Nsg-neighbourhood of x. The set of all Nsg-interior points of A is called the Nsg-interior of A and is denoted by Nsg-int(A).

**Theorem 3.3:** If A be a subset of $(U, \tau_R(X))$. Then Nsg-int(A) = $\cup$ { G : G is a Nsg-open, G $\subset$ A }.

**Proof:** Let A be a subset of $(U, \tau_R(X))$. x $\in$ Nsg-int(A) $\Leftrightarrow$ x is a Nsg-interior point of A.
$\Leftrightarrow$ A is a Nsg-nbhd of point x.

$\Leftrightarrow$ there exists Nsg-open set G such that x$\in$G$\subset$ A

$\Leftrightarrow$ x $\in \cup$ {G:G is a Nsg-open, G$\subset$ A }

Hence Nsg-int(A) = $\cup$ {G : G is a Nsg-open, G$\subset$ A }.

**Theorem 3.4:** Let A and B be subsets of $(U, \tau_R(X))$. Then
  (i)   Nsg-int(U) = U and Nsg-int($\phi$ ) = $\phi$
  (ii)  Nsg-int(A) $\subset$ A.
  (iii) If B is any Nsg-open set contained in A, then B $\subset$Nsg - int(A).
  (iv)  If A $\subset$B, then Nsg-int(A) $\subset$Nsg-int(B).
  (v)   Nsg-int(Nsg-int(A)) = Nsg-int(A).

**Proof:**
  (i)   Since U and $\phi$ are Nsg open sets, by Theorem 3.3
        Nsg-int(U) =$\cup$ {G : G is a Nsg-open, G$\subset$ U }= U $\cup$all Nsg open sets = U.
        (ie) int(U) = U. Since $\phi$ is the only Nsg- open set contained in $\phi$, Nsg-int($\phi$) = $\phi$
  (ii)  Let x$\in$Nsg-int(A) $\Longrightarrow$x is a Nsg interior point of A.
        $\Longrightarrow$A is a nbhd of x.
        $\Longrightarrow$x $\in$A.
        Thus, x$\in$Nsgint(A) $\Longrightarrow$x$\in$A.
         Hence Nsg-int(A) $\subset$A.
  (iii) Let B be any Nsg-open sets such that B $\subset$A.Let  Let x$\in$B.
        Since B is a Nsg-open set contained in A. x is a Nsg-interior point of A.
        (ie) x$\in$Nsg-int(A).Hence B $\subset$Nsg-int(A).
  (iv)  Let A and B be subsets of $(U, \tau_R(X))$ such that A $\subset$B. Let x$\in$Nsg-int(A). Then x is a Nsg-interior point of A
        and so A is a Nsg-nbhd of x. Since B$\supset$ A, B is also Nsg-nbhd of x. $\Longrightarrow$x$\in$Nsg-int(B). Thus we have shown that
        x$\in$Nsg-int(A) $\Longrightarrow$x$\in$Nsg-int(B).

**Theorem 3.4:** If a subset A of space $(U, \tau_R(X))$ is Nsg-open, then Nsg-int(A) =A.

**Proof:** Let A be Nsg-open subset of $(U, \tau_R(X))$. We know that Nsg-int(A) $\subset$ A. Also, A is Nsg-open set contained in A. From Theorem (iii) A$\subset$Nsg-int(A). Hence Nsg-int(A) = A. The converse of the above theorem need not be true, as seen from the following example.

**Example 3.5:** Let U= {1, 2, 3, 4,5}  with U/R={{1},{2,4},{3,5}}.Let  X={1, 2}$\subseteq$ U. Then $\tau_R(X)$  = {U, $\phi$ ,{1}, {1, 2, 4},{2, 4}}.

Nano sg-open sets are NSGO(U)={U, $\phi$ , {1}, {2},{3}, {1, 2},{1, 3},{1,4}{1,5},{2,3}, {2, 4}, {2,5}, {3,4}, {3,5}, {4,5} {1,2,3}, {1,2,4},{1,2,5},{1,3,4},{13,5}, {14,5},{2,4,5},{3,4,5},{1,2,3,4},{1,2,3,5{1,2,4,5},{1,3,4,5},{2,3,4,5}}
Nsg-int({2,3,4}) ={2}$\cup$ {3} $\cup${2,3} $\cup$ {2,4} $\cup${ $\phi$ } = {2,3,4}.

But {2, 3, 4} is not Nsg-open set in U.

**Theorem 3.6:** If A and B are subsets of $(U, \tau_R(X))$, then Nsg-int(A) $\cup$Nsg-int(B) $\subset$Nsg-int(A$\cup$ B).

**Proof:** We know that A $\subset$A$\cup$ B and B$\subset$ A$\cup$B.

We have Theorem 2.2 (iv) Nsg-int(A) $\subset$Nsg-int(A$\cup$ B), Nsg-int(B) $\subset$Nsg-int(A$\cup$ B).

This implies that Nsg-int(A) $\cup$Nsg-int(B) $\subset$Nsg-int(A$\cup$B).

**Theorem 3.7:** If A and B are subsets of $(U, \tau_R(X))$, then Nsg-int(A$\cap$ B) = Nsg-int(A)$\cap$Nsg-int(B).

**Proof:** We know that A$\cap$ B $\subset$A and A$\cap$ B$\subset$ B. We have Nsg-int(A$\cap$ B) $\subset$Nsg-int(A) and Nsg-int(A $\cap$B) $\subset$Nsg-int(B). This implies that Nsg-int(A$\cap$ B) $\subset$Nsg-int(A) $\cap$Nsg-int(B)                    (1)

Again let x$\in$Nsg-int(A)$\subset$Nsg-int(B). Then x$\in$Nsg-int(A) and x$\in$Nsg-int(B). Hence x is a Nsg-int point of each of sets A and B. It follows that A and B is Nsg-nbhds of x, so that their intersection A$\cap$ B is also a Nsg-nbhds of x. Hence x $\in$Nsg-int(A $\cap$B). Thus x $\in$Nsg-int(A) Nsg-int(A) implies that x $\in$Nsg-int(A$\cap$ B).

Therefore Nsg-int(A) $\cap$Nsg-int(B) $\subset$Nsg-int(A $\cap$B)                    (2)

From (1) and (2), We get Nsg-int(A∩ B)=Nsg-int(A) ∩Nsg-int(B).

**Theorem 3.7:** If A is a subset of U, then Nint(A) ⊂Nsg-int(A).

**Proof:** Let A be a subset of U.

Let x ∈ Nint(A) $\Longrightarrow$ x∪{G : G is nano open, G⊂ A}.
  $\Rightarrow$ there exists an nanoopen set G such that x∈ G ⊂A.
  $\Rightarrow$ there exist a Nsg-open set G such that x∈G ⊂A, as every open set is a Nsg-open set in U .
  $\Rightarrow$ x ∈∪{G : G is Nsg- open, G⊂ A}.
  $\Rightarrow$ x∈Nsg-int(A).

Thus x∈Nint(A) $\Longrightarrow$ x∈Nsg-int(A). Hence Nint(A) ⊂Nsg-int(A).

**Remark**.**3.8:** Containment relation in the above theorem may be proper as seen from the following example.

**Example 3.9:** Let U= {1, 2, 3, 4} with U/R= {{1},{2, 3},{4}}.

Let X ={1, 3}⊆ U. Then $\tau_R(X)$ = {U, φ ,{1}, {2, 3}, {1, 2, 3}}.

Nano sg-open sets are NSGO={U,φ, {1}, {2}, {3},{2, 3}, {1, 4}, {1, 2}, {1, 3}, {2, 3, 4},{1, 3, 4},{1, 2, 4},{1, 2, 3}}.

Let A = {1,3}. Now Nsg-int(A) = {1,3} and Nint(A) = {1}.

It follows that N int(A) ⊂Nsg-int(A) and Nint(A) ≠Nsg-int(A).

**Theorem 3.10:** If A is a subset of U, then Ng-int(A) ⊂Nsg-int(A), where Ng-int(A) is given by
Ng-int(A) = ∪{G: G is Ng-open, G ⊂A}.

**Proof:** Let A be a subset of $(U, \tau_R(X))$.

Let x ∈Nint(A) $\Longrightarrow$ x∈{G : G is Ng-open, G⊂ A}.
  $\Rightarrow$ there exists a Ng-open set G such that x∈ G⊂ A
  $\Rightarrow$ there exists a Nsg-open set G such that x∈ G⊂ A, as every Ng- open set is a Nsg-open set in U
  $\Rightarrow$ x ∈{G: G is Nsg-open, G ⊂A}. x ∈Nsg-int(A).

Hence Ng-int(A) ⊂Nsg-int(A).

**Remark 3.11:** Containment relation in the above theorem may be proper as seen from the following example.

**Example 3.12:** Let = {1, 2, 3, 4} with U/R = {{1}, {3}, {2, 4}}.

Let X={1, 2}⊆ U. Then$\tau_R(X)$  = {U, φ, {1},{1, 2, 4},{2, 4}}.

Nano sg-open sets are NSGO={U, φ , {1}, {2},{4},{2, 4},{1, 4}, {1, 2},{1, 3}, {1, 3, 4}, {1, 2, 4}, {1, 2, 3},{2, 3, 4}}.

Nano g-open sets are NGO={U, φ , {1}, {2},{4},{2, 4},{1, 4}, {1, 2},{1, 3, 4}}.

Let A = {1, 3}, Nsg-int(A) = {1, 3} & Ng-int(A) = {1}. It follows Ng-int(A) ⊂ Nsg-int(A) and Ng-int(A) ≠ Nsg-int(A)

## 4. NANO sg CLOSURE IN NANO TOPOLOGICAL SPACE

**Definition 4.1:** Let A be a subset of a space U. We define the Nsg-closure of A to be the intersection of all Nsg-closed sets containing A.

In symbols, Nsg-cl(A) = ∪{F:A⊂ F ∈NSGC(U)}

**Theorem 4.2:** If A and B are subsets of a space U. Then
  (i)   Nsg-cl(U) = U and Nsg-cl(φ) = φ
  (ii)  A⊂Nsg-cl(A).
  (iii) If B is any Nsg-closed set containing A, then Nsg-cl(A) ⊂ B.
  (iv) If A⊂ B then Nsg-cl(A) ⊂Nsg-cl(B).

**Proof:**
  (i)   By the definition of Nsg-closure, U is the only Nsg-closed set containing U. Therefore
        Nsg-cl(U) = Intersection of all the Nsg-closed sets containing U =∩ {U} = U.That is Nsg-cl(U) = U. By the
        definition of Nsg-closure, Nsg-cl(ϕ )=Intersection of all the Nsg-closed sets containing= ϕ ∩{ ϕ} = ϕ .
        That is Nsg-cl(ϕ ) = ϕ .
  (ii)  By the definition of Nsg-closure of A, it is obvious thatA ⊂Nsg-cl(A).
  (iii) Let B be any Nsg-closed set containing A. Since Nsg-cl(A) is the intersection of all Nsg-closed sets containing
        A, Nsg-cl(A) is contained in every Nsg-closed set containing A. Hence in particular Nsg-cl(A)⊂B
  (iv)  Let A and B be subsets of U such that A⊂ B. By the definition Nsg-cl(B) = ∩{ F: B ⊂F∈ NSGC (U)}. If
        B⊂ F NSGC (U), then Nsg-cl(B) F. Since A ⊂B, A⊂ B ⊂F ∈ NSGC (U), we have Nsg-cl(A) ⊂F. Therefore
        Nsg-cl(A)⊂∩{F: B⊂ F∈NSGC(U)} = Nsg-cl(B). (i.e) Nsg-cl(A) ⊂Nsg-cl(A).

**Theorem 4.3:** If A ⊂ $(U, \tau_R(X))$ is Nsg-closed, then Nsg-cl(A) = A.

**Proof:** Let A be Nsg-closed subset of U. We know that A ⊂Nsg-cl(A). Also A⊂ A and A is Nsg-closed. By theorem
(iii) Nsg-cl(A) ⊂A.

Hence Nsg-cl(A) = A.

**Remarks 4.4:** The converse of the above theorem need not be true as seen from the following example.

**Example 4.5:** Let U= {1, 2, 3, 4, 5} with U/R={{1},{2,4},{3,5}}.

Let X={1, 2}⊆ U. Then$\tau_R(X)$  = {U, ϕ, {1}, {1, 2, 4},{2, 4}}.

Nano sg-closed sets are NSGC(U) = {U, ϕ,{1},{2},{3},{4},{5},{1,2}{1,3},{2,3},{2,4},{2,5},{3,4} {3,5}, {4,5},
{1,2,3},{1,2,4},{1,2,5},{1,3,4},{1,3,5},{1,4,5},{2,3,4},  {2,3,5},  {2,4,5},  {3,4,5},  {1,2,3,4},  {1,2,3,5}  {1,2,4,5},
{1,3,4,5},{2,3,4,5}}

Nsg-cl({1,5}) ={1,3,5} ∩ {1,2,5}∩{1,2,3,5}∩{1,2,4,5}={1,5}.

But {1, 5} is not sg-closed set in U.

**Theorem 4.6:** If A and B are subsets of a space $(U, \tau_R(X))$, then Nsg-cl(A∩ B) ⊂Nsg-cl(A) ∩Nsg-cl(B).

**Proof:** Let A and B be subsets of U. Clearly A∩ B ⊂A and A∩ B⊂ B. By theorem Nsg-cl(A∩ B) ⊂Nsg-cl(A) and
Nsg-cl(A∩ B) ⊂Nsg-cl(B). Hence Nsg-cl(A∩ B) ⊂Nsg-cl(A) ∩Nsg-cl(B).

**Theorem 4.7:** If A and B are subsets of a space $(U, \tau_R(X))$ then Nsg-cl(A∪ B)= Nsg-cl(A)∪Nsg-cl(B).

**Proof:** Let A and B be subsets of U. Clearly A ⊂A∪ B and B⊂A∪ B. We have
        Nsg-cl(A) ∪Nsg-cl(B) ⊂Nsg-cl( A∪ B)                                                      (1)

Now to prove Nsg-cl( A∪ B) ⊂Nsg-cl(A)∪Nsg-cl(B). Let x∈Nsg-cl(A∪ B) and suppose x≠Nsg-cl(A) ∪Nsg- cl(B).
Then there exists Nsg-closed sets $A_1$ and $B_1$ with A⊂$A_1$, B ⊂$B_1$ and x ≠$A_1$∪ $B_1$. We have A∪ B⊂ $A_1$∪ $B_1$ and $A_1$∪ $B_1$
is Nsg-closed set by theorem such that x ≠ $A_1$ ∪ $B_1$. Thus x ≠ Nsg-cl(A ∪ B) which is a contradiction to
x ∈Nsg-cl(A∪ B).

Hence Nsg-cl(A∪ B) Nsg-cl(A) ∪Nsg-cl(B)                                                      (2)

From (1) and (2), we have Nsg-cl(A∪ B)= Nsg-cl(A) ∪Nsg-cl(B).

**Theorem 4.8:** For an x ∈U, x ∈Nsg-cl(A) if and only if V∩ A≠ ϕfor every Nsg-closed sets V containing x.

**Proof:** Let x ∈U and x∈Nsg-cl(A). To prove V∩A≠ ϕ for every Nsg-open set V containing x. Prove the result by
contradiction. Suppose there exists a Nsg-open set V containing x such that V∩ A =ϕ . Then A ⊂U-V and U-V is
Nsg-closed. We have Nsg-cl(A) ⊂U - V. This shows that x≠Nsg-cl(A), which is a contradiction. Hence V∩ A≠ ϕ for
every Nsg-open set V containing x. Conversely; let V∩ A ≠ ϕfor every Nsg-open set V containing x. To prove
x∈Nsg-cl(A). We prove the result by contradiction. Suppose x ≠Nsg-cl(A). Then x ∈U – F and S – F is Nsg-open. Also
(U–F) ∩A = ϕ, which is a contradiction. Hence x ∈Nsg-cl(A).

**Theorem 4.9:** If A is a subset of a space $(U, \tau_R(X))$, then Nsg-cl(A) $\subset$ Ncl(A).

**Proof:** Let A be a subset of a space $(U, \tau_R(X))$. By the definition of nano closure, Ncl(A) = $\cap$\{F: U$\subset$F $\in$C(U)\}.

If A$\subset$ F $\in$NC(U)\}, Then A$\subset$ F$\in$NSG(U), because every closed set is Nsg-closed. That is Nsg-cl(A) $\subset$ F. Therefore Nsg-cl(A) $\subset$$\cap$\{F$\subset$ X :F $\in$NC(U)\} = Ncl(A). Hence Nsg-cl(A) $\subset$Ncl(A).

**Remark 4.10:** Containment relation in the above theorem may be proper as seen from the following example.

**Example 4.11:** Let U = \{1, 2, 3, 4\} with U/R = \{\{1\},\{3\},\{2, 4\}\}.

Let X = \{1, 2\}$\subseteq$ U. Then $\tau_R(X)$ = \{U, $\phi$ , \{1\}, \{1, 2, 4\},\{2, 4\}\}.

Nano sg-closed sets are NSGC(U) = \{U, $\phi$ , \{1\}, \{2\},\{3\},\{4\},\{1, 3\}, \{2, 3\},\{2, 4\},\{3, 4\},\{1, 3, 4\}, \{1, 2, 3\},\{2, 3, 4\}\}.

Nano g-closed sets are NGC(U) = \{U, $\phi$, \{3\}, \{2,3\},\{3,4\},\{1, 3\},\{2,3,4\}, \{1,2,3\},\{1, 3, 4\}\}.

Let A = \{2,4\}, Ng-cl(A) = \{2,3,4\} and Nsg-cl(A) =\{2,4\}.

It follows Nsg-cl(A)$\subset$Ng-cl(A) and Nsg-cl(A) $\neq$Ng-cl(A) .

**Theorem 4.12:** If A is a subset of $(U, \tau_R(X))$, then Nsg-cl(A) $\subset$Ng-cl(A), where Ng-cl(A) is given by Ng-cl(A) = $\cap$\{F$\subset$U : A$\subset$ F and f is a Ng-closed set in U\}.

**Proof:** Let A be a subset of U. By definition of Ng-cl(A) = $\cap$\{F $\subset$U : A$\subset$ F and f is a Ng-closed set in U\}. If A$\subset$ F and F is Ng-closed subset of x, then A$\subset$ F$\in$NSGC(U), because every Ng closed is Nsg-closed subset in X. That is Nsg-cl(A) $\subset$F. Therefore Nsg-cl(A)$\subset$ $\cap$ \{F $\subset$U : A$\subset$ F and f is a Ng-closed set in U\} = Ng-cl(A). Hence Nsg-cl(A) $\subset$Ng-cl(A).

**Corollary 4.13:** Let A be any subset of $(U, \tau_R(X))$. Then
  (i)   Nsg-int(A))$^C$ = Nsg-cl(A$^C$)
  (ii)  Nsg-int(A) = (Nsg-cl(A$^C$))
  (iii) Nsg-cl(A) = (Nsg-cl(A$^C$))

**Proof:** Let x $\in$Nsg-int(A))$^C$. Then x$\notin$Nsg-int(A). That is every Nsg-open set V containing x is such that V$\not\subset$ A. That is every Nsg-open set V containing x is such that V$\not\subset$ A$^C$ . By theorem x$\in$Nsg-int(A))$^C$ and there fore Nsg-int(A))$^C$$\subset$Nsg-cl(A$^C$). Conversely, let x$\in$Nsg-cl(A$^C$). Then by theorem, every Nsg-open set V containing x is such that V$\cap$A$^C$$\neq$ $\phi$. That is every Nsg-open set V containing x is such that V$\not\subset$ A. This implies by definition of Nsg-interior of A, x$\notin$Nsg-int(A). That is x$\in$Nsg-int(A))$^C$ and Nsg-cl(A$^C$) $\subset$ (Nsg-int(A))$^C$. Thus Nsg-int(A))$^C$ = Nsg-cl(A$^C$) (ii) Follows by taking complements in (i). (ii) Follows by replacing A by A$^C$ in (i).

**CONCLUSIONS**

Many different forms of open functions have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, this paper we studied the concept of Nano sg-Interior and Nano sg-Closure in Nano topological spaces. This shall be extended in the future Research with some applications

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