

NANO sg-INTERIOR AND NANO sg-CLOSURE IN NANO TOPOLOGICAL SPACES

S. CHANDRASEKAR*¹, M. SURESH² AND T. RAJESH KANNAN³

^{1,3}Department of Mathematics,
Arignar Anna Government Arts college, Namakkal (Dt.), Tamil Nadu, India.

²Department of Mathematics,
RMD Engineering College, Kavaraipettai, Gummidipoondi, Tamil Nadu, India.

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ABSTRACT

In this paper, we introduce Nsg-interior, Nsg-closure and some of its basic properties.

Keywords: Nsg-open; Nsg-closed; Nsg-int(A); Nsg-cl(A).

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1. INTRODUCTION

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in general topology concerns the variously modified forms of continuity, separation axioms etc., by utilizing generalized open sets. Levine [12] introduced the concept of generalized closed sets in topological space. Bhattacharyya and Lahiri [1] introduced semi-generalized closed sets in topology and investigated some of their properties. The concept of nano topology was introduced by Lellis Thivagar [13,14] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established the weak forms of nano open sets namely nano -open sets, nano semi open sets and nano pre open sets in a nano topological space. nano g closed and nano sg closed was introduced by K. Bhuvaneshwari [2,3] *et al.* Since the advent of these notions several research papers with interesting results in different respects came to existence.

In this paper, the notion of Nsg-interior is defined and some of its basic properties are investigated. Also we introduce the idea of Nsg-closure in Nano topological spaces using the notions of Nsg-closed sets and obtain some related results.

2. PRELIMINARIES

Definition 2.1[14]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

- (i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by X .
 - (ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$
 - (iii) The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not- X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.
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**Corresponding Author: S. Chandrasekar*¹, ¹Department of Mathematics,
Arignar Anna Government Arts college, Namakkal (DT), Tamil Nadu, India.**

Definition 2.2[14]: If (U, R) is an approximation space and $X, Y \subseteq U$, then

- (i) $L_R(X) \subseteq X \subseteq U_R(X)$
- (ii) $L_R(\phi) = U_R(\phi) = \phi$
- (iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- (iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
- (v) $L_R(X \cup Y) \subseteq L_R(X) \cup L_R(Y)$
- (vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
- (vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
- (viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
- (ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$
- (x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition 2.3[14]: Let U be an universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms

- (i) $U, \phi \in \tau_R(X)$
- (ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
- (iii) The intersection of the elements of any finite sub- collection of $\tau_R(X)$ is in $\tau_R(X)$. That is, $\tau_R(X)$ forms a topology on U called the nano topology on U with respect to X . We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano open sets.

Definition 2.4[14]: Let $(U, \tau_R(X))$ be a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then A is said to be

- (i) nano semi-open [14] if $A \subseteq \text{Ncl}(\text{Nint}(A))$.
- (ii) nano α -open [14] if $A \subseteq \text{Nint}(\text{Ncl}(\text{Nint}(A)))$,

The complements of the above mentioned nanooopen sets are called their respective nanoclosed sets.

Definition 2.5: A subset A of a space $(U, \tau_R(X))$ is called:

- (i) nano generalized closed (briefly, nano g-closed) if $\text{Ncl}(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open in U .
- (ii) a nano semi-generalized closed [2,3] (briefly Nsg closed) set if $\text{Nscl}(A) \subseteq G$ whenever $A \subseteq G$ and G is semi-open in U . The complement of nanosg-closed set is called nanosg-open.

Definition 2.6: Let $(U, \tau_R(X))$ be a Nano topological space and let $x \in U$.

A subset N of U is said to be Nsg-neighbourhood of x if there exists a Nsg-open set G such that $x \in G \subseteq N$.

3. NANO sg INTERIOR IN NANO TOPOLOGICAL SPACE

Definition 3.1: Let $(U, \tau_R(X))$ be a Nano topological space and let $x \in U$. A subset N of U is said to be Nsg-neighbourhood of x if there exists a Nsg-open set G such that $x \in G \subseteq N$.

Definition 3.2: Let A be a subset of $(U, \tau_R(X))$. A point $x \in A$ is said to be Nsg-interior point of A if A is a Nsg-neighbourhood of x . The set of all Nsg-interior points of A is called the Nsg-interior of A and is denoted by $\text{Nsg-int}(A)$.

Theorem 3.3: If A be a subset of $(U, \tau_R(X))$. Then $\text{Nsg-int}(A) = \bigcup \{ G : G \text{ is a Nsg-open, } G \subseteq A \}$.

Proof: Let A be a subset of $(U, \tau_R(X))$. $x \in \text{Nsg-int}(A) \Leftrightarrow x$ is a Nsg-interior point of A .
 $\Leftrightarrow A$ is a Nsg-nbhd of point x .

\Leftrightarrow there exists Nsg-open set G such that $x \in G \subset A$
 $\Leftrightarrow x \in \cup \{G : G \text{ is a Nsg-open, } G \subset A \}$

Hence $\text{Nsg-int}(A) = \cup \{G : G \text{ is a Nsg-open, } G \subset A \}$.

Theorem 3.4: Let A and B be subsets of $(U, \tau_R(X))$. Then

- (i) $\text{Nsg-int}(U) = U$ and $\text{Nsg-int}(\phi) = \phi$
- (ii) $\text{Nsg-int}(A) \subset A$.
- (iii) If B is any Nsg-open set contained in A , then $B \subset \text{Nsg-int}(A)$.
- (iv) If $A \subset B$, then $\text{Nsg-int}(A) \subset \text{Nsg-int}(B)$.
- (v) $\text{Nsg-int}(\text{Nsg-int}(A)) = \text{Nsg-int}(A)$.

Proof:

- (i) Since U and ϕ are Nsg open sets, by Theorem 3.3
 $\text{Nsg-int}(U) = \cup \{G : G \text{ is a Nsg-open, } G \subset U\} = U \cup \text{all Nsg open sets} = U$.
 (ie) $\text{int}(U) = U$. Since ϕ is the only Nsg- open set contained in ϕ , $\text{Nsg-int}(\phi) = \phi$
- (ii) Let $x \in \text{Nsg-int}(A) \Rightarrow x$ is a Nsg interior point of A .
 $\Rightarrow A$ is a nbhd of x .
 $\Rightarrow x \in A$.
 Thus, $x \in \text{Nsg-int}(A) \Rightarrow x \in A$.
 Hence $\text{Nsg-int}(A) \subset A$.
- (iii) Let B be any Nsg-open sets such that $B \subset A$. Let $x \in B$.
 Since B is a Nsg-open set contained in A , x is a Nsg-interior point of A .
 (ie) $x \in \text{Nsg-int}(A)$. Hence $B \subset \text{Nsg-int}(A)$.
- (iv) Let A and B be subsets of $(U, \tau_R(X))$ such that $A \subset B$. Let $x \in \text{Nsg-int}(A)$. Then x is a Nsg-interior point of A and so A is a Nsg-nbhd of x . Since $B \supset A$, B is also Nsg-nbhd of x . $\Rightarrow x \in \text{Nsg-int}(B)$. Thus we have shown that $x \in \text{Nsg-int}(A) \Rightarrow x \in \text{Nsg-int}(B)$.

Theorem 3.4: If a subset A of space $(U, \tau_R(X))$ is Nsg-open, then $\text{Nsg-int}(A) = A$.

Proof: Let A be Nsg-open subset of $(U, \tau_R(X))$. We know that $\text{Nsg-int}(A) \subset A$. Also, A is Nsg-open set contained in A . From Theorem (iii) $A \subset \text{Nsg-int}(A)$. Hence $\text{Nsg-int}(A) = A$. The converse of the above theorem need not be true, as seen from the following example.

Example 3.5: Let $U = \{1, 2, 3, 4, 5\}$ with $U/R = \{\{1\}, \{2,4\}, \{3,5\}\}$. Let $X = \{1, 2\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$.

Nano sg-open sets are $\text{NSGO}(U) = \{U, \phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{4, 5\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 2, 5\}, \{1, 3, 4\}, \{1, 3, 5\}, \{1, 4, 5\}, \{2, 4, 5\}, \{3, 4, 5\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}, \{1, 2, 4, 5\}, \{1, 3, 4, 5\}, \{2, 3, 4, 5\}\}$
 $\text{Nsg-int}(\{2, 3, 4\}) = \{2\} \cup \{3\} \cup \{2, 3\} \cup \{2, 4\} \cup \{\phi\} = \{2, 3, 4\}$.

But $\{2, 3, 4\}$ is not Nsg-open set in U .

Theorem 3.6: If A and B are subsets of $(U, \tau_R(X))$, then $\text{Nsg-int}(A) \cup \text{Nsg-int}(B) \subset \text{Nsg-int}(A \cup B)$.

Proof: We know that $A \subset A \cup B$ and $B \subset A \cup B$.

We have Theorem 2.2 (iv) $\text{Nsg-int}(A) \subset \text{Nsg-int}(A \cup B)$, $\text{Nsg-int}(B) \subset \text{Nsg-int}(A \cup B)$.

This implies that $\text{Nsg-int}(A) \cup \text{Nsg-int}(B) \subset \text{Nsg-int}(A \cup B)$.

Theorem 3.7: If A and B are subsets of $(U, \tau_R(X))$, then $\text{Nsg-int}(A \cap B) = \text{Nsg-int}(A) \cap \text{Nsg-int}(B)$.

Proof: We know that $A \cap B \subset A$ and $A \cap B \subset B$. We have $\text{Nsg-int}(A \cap B) \subset \text{Nsg-int}(A)$ and $\text{Nsg-int}(A \cap B) \subset \text{Nsg-int}(B)$. This implies that $\text{Nsg-int}(A \cap B) \subset \text{Nsg-int}(A) \cap \text{Nsg-int}(B)$ (1)

Again let $x \in \text{Nsg-int}(A) \cap \text{Nsg-int}(B)$. Then $x \in \text{Nsg-int}(A)$ and $x \in \text{Nsg-int}(B)$. Hence x is a Nsg-int point of each of sets A and B . It follows that A and B is Nsg-nbhds of x , so that their intersection $A \cap B$ is also a Nsg-nbhds of x . Hence $x \in \text{Nsg-int}(A \cap B)$. Thus $x \in \text{Nsg-int}(A) \cap \text{Nsg-int}(B)$ implies that $x \in \text{Nsg-int}(A \cap B)$.

Therefore $\text{Nsg-int}(A) \cap \text{Nsg-int}(B) \subset \text{Nsg-int}(A \cap B)$ (2)

From (1) and (2), We get $Nsg-int(A \cap B) = Nsg-int(A) \cap Nsg-int(B)$.

Theorem 3.7: If A is a subset of U , then $Nint(A) \subset Nsg-int(A)$.

Proof: Let A be a subset of U .

Let $x \in Nint(A) \Rightarrow x \in \{G : G \text{ is nano open, } G \subset A\}$.
 \Rightarrow there exists an nanoopen set G such that $x \in G \subset A$.
 \Rightarrow there exist a Nsg -open set G such that $x \in G \subset A$, as every open set is a Nsg -open set in U .
 $\Rightarrow x \in \{G : G \text{ is } Nsg\text{-open, } G \subset A\}$.
 $\Rightarrow x \in Nsg-int(A)$.

Thus $x \in Nint(A) \Rightarrow x \in Nsg-int(A)$. Hence $Nint(A) \subset Nsg-int(A)$.

Remark 3.8: Containment relation in the above theorem may be proper as seen from the following example.

Example 3.9: Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{1\}, \{2, 3\}, \{4\}\}$.

Let $X = \{1, 3\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$.

Nano sg-open sets are $NSGO = \{U, \phi, \{1\}, \{2\}, \{3\}, \{2, 3\}, \{1, 4\}, \{1, 2\}, \{1, 3\}, \{2, 3, 4\}, \{1, 3, 4\}, \{1, 2, 4\}, \{1, 2, 3\}\}$.

Let $A = \{1, 3\}$. Now $Nsg-int(A) = \{1, 3\}$ and $Nint(A) = \{1\}$.

It follows that $Nint(A) \subset Nsg-int(A)$ and $Nint(A) \neq Nsg-int(A)$.

Theorem 3.10: If A is a subset of U , then $Ng-int(A) \subset Nsg-int(A)$, where $Ng-int(A)$ is given by $Ng-int(A) = \{G : G \text{ is } Ng\text{-open, } G \subset A\}$.

Proof: Let A be a subset of $(U, \tau_R(X))$.

Let $x \in Ng-int(A) \Rightarrow x \in \{G : G \text{ is } Ng\text{-open, } G \subset A\}$.
 \Rightarrow there exists a Ng -open set G such that $x \in G \subset A$
 \Rightarrow there exists a Nsg -open set G such that $x \in G \subset A$, as every Ng -open set is a Nsg -open set in U
 $\Rightarrow x \in \{G : G \text{ is } Nsg\text{-open, } G \subset A\}$. $x \in Nsg-int(A)$.

Hence $Ng-int(A) \subset Nsg-int(A)$.

Remark 3.11: Containment relation in the above theorem may be proper as seen from the following example.

Example 3.12: Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{1\}, \{3\}, \{2, 4\}\}$.

Let $X = \{1, 2\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$.

Nano sg-open sets are $NSGO = \{U, \phi, \{1\}, \{2\}, \{4\}, \{2, 4\}, \{1, 4\}, \{1, 2\}, \{1, 3\}, \{1, 3, 4\}, \{1, 2, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$.

Nano g-open sets are $NGO = \{U, \phi, \{1\}, \{2\}, \{4\}, \{2, 4\}, \{1, 4\}, \{1, 2\}, \{1, 3, 4\}\}$.

Let $A = \{1, 3\}$, $Nsg-int(A) = \{1, 3\}$ & $Ng-int(A) = \{1\}$. It follows $Ng-int(A) \subset Nsg-int(A)$ and $Ng-int(A) \neq Nsg-int(A)$

4. NANO sg CLOSURE IN NANO TOPOLOGICAL SPACE

Definition 4.1: Let A be a subset of a space U . We define the Nsg -closure of A to be the intersection of all Nsg -closed sets containing A .

In symbols, $Nsg-cl(A) = \bigcap \{F : A \subset F \in NSGC(U)\}$

Theorem 4.2: If A and B are subsets of a space U . Then

- (i) $Nsg-cl(U) = U$ and $Nsg-cl(\phi) = \phi$
- (ii) $A \subset Nsg-cl(A)$.
- (iii) If B is any Nsg -closed set containing A , then $Nsg-cl(A) \subset B$.
- (iv) If $A \subset B$ then $Nsg-cl(A) \subset Nsg-cl(B)$.

Proof:

- (i) By the definition of Nsg-closure, U is the only Nsg-closed set containing U. Therefore $Nsg-cl(U) = \text{Intersection of all the Nsg-closed sets containing } U = \cap \{U\} = U$. That is $Nsg-cl(U) = U$. By the definition of Nsg-closure, $Nsg-cl(\phi) = \text{Intersection of all the Nsg-closed sets containing } \phi = \phi \cap \{ \phi \} = \phi$. That is $Nsg-cl(\phi) = \phi$.
- (ii) By the definition of Nsg-closure of A, it is obvious that $A \subset Nsg-cl(A)$.
- (iii) Let B be any Nsg-closed set containing A. Since $Nsg-cl(A)$ is the intersection of all Nsg-closed sets containing A, $Nsg-cl(A)$ is contained in every Nsg-closed set containing A. Hence in particular $Nsg-cl(A) \subset B$.
- (iv) Let A and B be subsets of U such that $A \subset B$. By the definition $Nsg-cl(B) = \cap \{ F : B \subset F \in NSGC(U) \}$. If $B \subset F \in NSGC(U)$, then $Nsg-cl(B) \subset F$. Since $A \subset B$, $A \subset B \subset F \in NSGC(U)$, we have $Nsg-cl(A) \subset F$. Therefore $Nsg-cl(A) \subset \cap \{ F : B \subset F \in NSGC(U) \} = Nsg-cl(B)$. (i.e) $Nsg-cl(A) \subset Nsg-cl(B)$.

Theorem 4.3: If $A \subset (U, \tau_R(X))$ is Nsg-closed, then $Nsg-cl(A) = A$.

Proof: Let A be Nsg-closed subset of U. We know that $A \subset Nsg-cl(A)$. Also $A \subset A$ and A is Nsg-closed. By theorem (iii) $Nsg-cl(A) \subset A$.

Hence $Nsg-cl(A) = A$.

Remarks 4.4: The converse of the above theorem need not be true as seen from the following example.

Example 4.5: Let $U = \{1, 2, 3, 4, 5\}$ with $U/R = \{\{1\}, \{2,4\}, \{3,5\}\}$.

Let $X = \{1, 2\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$.

Nano sg-closed sets are $NSGC(U) = \{U, \phi, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1,2\}, \{1,3\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\}, \{3,5\}, \{4,5\}, \{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{1,3,4\}, \{1,3,5\}, \{1,4,5\}, \{2,3,4\}, \{2,3,5\}, \{2,4,5\}, \{3,4,5\}, \{1,2,3,4\}, \{1,2,3,5\}, \{1,2,4,5\}, \{1,3,4,5\}, \{2,3,4,5\}\}$

$Nsg-cl(\{1,5\}) = \{1,3,5\} \cap \{1,2,5\} \cap \{1,2,3,5\} \cap \{1,2,4,5\} = \{1,5\}$.

But $\{1, 5\}$ is not sg-closed set in U.

Theorem 4.6: If A and B are subsets of a space $(U, \tau_R(X))$, then $Nsg-cl(A \cap B) \subset Nsg-cl(A) \cap Nsg-cl(B)$.

Proof: Let A and B be subsets of U. Clearly $A \cap B \subset A$ and $A \cap B \subset B$. By theorem $Nsg-cl(A \cap B) \subset Nsg-cl(A)$ and $Nsg-cl(A \cap B) \subset Nsg-cl(B)$. Hence $Nsg-cl(A \cap B) \subset Nsg-cl(A) \cap Nsg-cl(B)$.

Theorem 4.7: If A and B are subsets of a space $(U, \tau_R(X))$ then $Nsg-cl(A \cup B) = Nsg-cl(A) \cup Nsg-cl(B)$.

Proof: Let A and B be subsets of U. Clearly $A \subset A \cup B$ and $B \subset A \cup B$. We have

$$Nsg-cl(A) \cup Nsg-cl(B) \subset Nsg-cl(A \cup B) \tag{1}$$

Now to prove $Nsg-cl(A \cup B) \subset Nsg-cl(A) \cup Nsg-cl(B)$. Let $x \in Nsg-cl(A \cup B)$ and suppose $x \notin Nsg-cl(A) \cup Nsg-cl(B)$. Then there exists Nsg-closed sets A_1 and B_1 with $A \subset A_1$, $B \subset B_1$ and $x \notin A_1 \cup B_1$. We have $A \cup B \subset A_1 \cup B_1$ and $A_1 \cup B_1$ is Nsg-closed set by theorem such that $x \notin A_1 \cup B_1$. Thus $x \notin Nsg-cl(A \cup B)$ which is a contradiction to $x \in Nsg-cl(A \cup B)$.

$$\text{Hence } Nsg-cl(A \cup B) = Nsg-cl(A) \cup Nsg-cl(B) \tag{2}$$

From (1) and (2), we have $Nsg-cl(A \cup B) = Nsg-cl(A) \cup Nsg-cl(B)$.

Theorem 4.8: For an $x \in U$, $x \in Nsg-cl(A)$ if and only if $V \cap A \neq \phi$ for every Nsg-closed sets V containing x.

Proof: Let $x \in U$ and $x \in Nsg-cl(A)$. To prove $V \cap A \neq \phi$ for every Nsg-open set V containing x. Prove the result by contradiction. Suppose there exists a Nsg-open set V containing x such that $V \cap A = \phi$. Then $A \subset U - V$ and $U - V$ is Nsg-closed. We have $Nsg-cl(A) \subset U - V$. This shows that $x \notin Nsg-cl(A)$, which is a contradiction. Hence $V \cap A \neq \phi$ for every Nsg-open set V containing x. Conversely; let $V \cap A \neq \phi$ for every Nsg-open set V containing x. To prove $x \in Nsg-cl(A)$. We prove the result by contradiction. Suppose $x \notin Nsg-cl(A)$. Then $x \in U - F$ and $U - F$ is Nsg-open. Also $(U - F) \cap A = \phi$, which is a contradiction. Hence $x \in Nsg-cl(A)$.

Theorem 4.9: If A is a subset of a space $(U, \tau_R(X))$, then $Nsg-cl(A) \subset Ncl(A)$.

Proof: Let A be a subset of a space $(U, \tau_R(X))$. By the definition of nano closure, $Ncl(A) = \cap \{F: U \subset F \in C(U)\}$.

If $A \subset F \in NC(U)$, Then $A \subset F \in NSGC(U)$, because every closed set is Nsg-closed. That is $Nsg-cl(A) \subset F$. Therefore $Nsg-cl(A) \subset \cap \{F \subset X : F \in NC(U)\} = Ncl(A)$. Hence $Nsg-cl(A) \subset Ncl(A)$.

Remark 4.10: Containment relation in the above theorem may be proper as seen from the following example.

Example 4.11: Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{1\}, \{3\}, \{2, 4\}\}$.

Let $X = \{1, 2\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$.

Nano sg-closed sets are $NSGC(U) = \{U, \phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 3\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 3, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$.

Nano g-closed sets are $NGC(U) = \{U, \phi, \{3\}, \{2,3\}, \{3,4\}, \{1, 3\}, \{2,3,4\}, \{1,2,3\}, \{1, 3, 4\}\}$.

Let $A = \{2,4\}$, $Ng-cl(A) = \{2,3,4\}$ and $Nsg-cl(A) = \{2,4\}$.

It follows $Nsg-cl(A) \subset Ng-cl(A)$ and $Nsg-cl(A) \neq Ng-cl(A)$.

Theorem 4.12: If A is a subset of $(U, \tau_R(X))$, then $Nsg-cl(A) \subset Ng-cl(A)$, where $Ng-cl(A)$ is given by $Ng-cl(A) = \cap \{F \subset U : A \subset F \text{ and } f \text{ is a Ng-closed set in } U\}$.

Proof: Let A be a subset of U . By definition of $Ng-cl(A) = \cap \{F \subset U : A \subset F \text{ and } f \text{ is a Ng-closed set in } U\}$. If $A \subset F$ and F is Ng-closed subset of x , then $A \subset F \in NSGC(U)$, because every Ng closed is Nsg-closed subset in X . That is $Nsg-cl(A) \subset F$. Therefore $Nsg-cl(A) \subset \cap \{F \subset U : A \subset F \text{ and } f \text{ is a Ng-closed set in } U\} = Ng-cl(A)$. Hence $Nsg-cl(A) \subset Ng-cl(A)$.

Corollary 4.13: Let A be any subset of $(U, \tau_R(X))$. Then

- (i) $Nsg-int(A)^C = Nsg-cl(A^C)$
- (ii) $Nsg-int(A) = (Nsg-cl(A^C))$
- (iii) $Nsg-cl(A) = (Nsg-cl(A^C))$

Proof: Let $x \in Nsg-int(A)^C$. Then $x \notin Nsg-int(A)$. That is every Nsg-open set V containing x is such that $V \not\subset A$. That is every Nsg-open set V containing x is such that $V \not\subset A^C$. By theorem $x \in Nsg-int(A)^C$ and there fore $Nsg-int(A)^C \subset Nsg-cl(A^C)$. Conversely, let $x \in Nsg-cl(A^C)$. Then by theorem, every Nsg-open set V containing x is such that $V \cap A^C \neq \phi$. That is every Nsg-open set V containing x is such that $V \not\subset A$. This implies by definition of Nsg-interior of A , $x \notin Nsg-int(A)$. That is $x \in Nsg-int(A)^C$ and $Nsg-cl(A^C) \subset (Nsg-int(A)^C)$. Thus $Nsg-int(A)^C = Nsg-cl(A^C)$ (ii) Follows by taking complements in (i). (ii) Follows by replacing A by A^C in (i).

CONCLUSIONS

Many different forms of open functions have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, this paper we studied the concept of Nano sg-Interior and Nano sg-Closure in Nano topological spaces. This shall be extended in the future Research with some applications

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