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On rgwα- Open Sets in Topological Spaces

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ABSTRACT

In this paper, we introduced and studied rgwα-open sets in topological space and obtain some of their properties. Also we introduce rgwα-interior, rgwα-closure, rgwα- neighbourhood and rgwα-limit points in topological spaces.

Keywords: *rgwα-open sets, rgwα-interior, rgwα-closure, rgwα- neighbourhood, rgwα-limit points.*

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1] INTRODUCTION

Regular open sets have been introduced and investigated by stone [6]. P.Sundaram and and M.Sheik John [8] defined and studied w-closed sets in topological spaces. S.S Benchalli and R.S.Wali [12] introduced and studied rw-closed sets. N.Jasted [7] introduced and studied α-sets. S.S.Benchalli *et al.* [11] studied wα-closed sets in topological spaces. S.S.Benchalli *et al.* [10] introduced gwα-closed sets. and P.G.Patil *et al.* [9] introduced g*wα-closed set. A. Vadivel and Vairamanickam [2] introduced rgα-closed sets and rgα-open sets in topological spaces. In this paper we define rgwα-open sets, its properties and rgwα-interior, rgwα-closure, rgwα- neighbourhood and rgwα-limit points and obtain some of its basic properties.

2] PRELIMINARIES

Throughout the paper X and Y denote the topological space (X,) and (Y,) respectively. And on which no sepsration axioms are assumed unless otherwise explicitly stated. For a subset A of space X, cl(A), int(A), A^c , and rcl(A) denote the closure of A, Interior of A, complement of A and regular closure of A in X respectively.

Definition 2.1: A subset A of a space X is called
1) a regular open set [6] if A =int(cl(A)) and a regular closed set if A=cl(int(A)).
2) a α-open set [7] if A ⊆ int(cl(intA))) and α-closed set of cl (int(cl(A)) ⊆ A.
3) a weakely closed set (briefly, w-closed) [1] if cl(A) ⊆U whenever A ⊆U & U is semi open in X.
4) a weakely α-closed set (briefly, wα-closed) [11] if αcl(A)⊆U whenever A ⊆U & U is w- open in X.
5) a regular α-open set (2) if there is a regular open set U ∋ U⊆A⊆ αcl(U)

The intersection of all regular closed (resp. α-closed, wα- closed and regular α-closed) subsets of space X containing A is called regular closure (Resp. α-closure, wα- closure and regular α- closure) of A and denoted by rcl(A) (resp. αcl (A), wαcl(A) and rαcl(A)).

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Definition 2.2: A subset A of a space X is called

1) generalized α-closed set (briefly gα-closed) [4], if αcl(A) ⊆U whenever A⊆U and U is α-open in X.
2) generalized semi-pre closed set (briefly gsp-closed) [5] if spcl(A) ⊆U whenever A ⊆U and U is open in X.
3) generalized weak α-closed (briefly gwα-closed) set [10] if αcl(A) ⊆U whenever A⊆U & U is wα- open in X.
4) generalized star weakly α-closed set (briefly g*wα-closed) [9] if cl(A) ⊆U whenever A⊆U & U is wα- open in X.
5) regular generalized α-closed set (briefly rgα-closed) [2] if αcl(A) ⊆U whenever A ⊆U & U is regular α-open in X.

The complements of the above mentioned closed sets are respective open sets.

3. rgwα-closed sets in topological spaces.

Definition 3.1 [13]: A subset A of a space X is called regular generalized weakly α-closed set (briefly rgwα-closed) if rαcl (A) ⊆U whenever A⊆U & U is weak α-open set in X.

Results 3.2 from [13]:

1) Every closed set is rgwα-closed set in X.
2) Every regular closed set is rgwα-closed set in X.
3) Every weak- closed set is rgwα-closed set in X.
4) Every α-closed, gα-closed, rgα-closed, gwα-closed and g*wα-closed sets are rgwα-closed sets in X.
5) Every rw-closed, rα-closed, rs-closed and wα-closed sets are rgwα-closed sets in X.
6) Every rgwα-closed set is gβ-closed set in X.
7) The union of two rgwα-closed sets of X is rgwα-closed set in X.
8) The intersection of two rgwα-closed sets of X is need not be rgwα-closed set.

4. rgwα-open sets and their basic properties

In this section we introduce and study rgwα-open sets in topological spaces and obtain some of their properties.

Definition 4.1: A subset A of X is called regular generalized weakly-α open set (rgwα-open set) in X if A^c is rgwα-closed in X. We denote the family of all rgwα-open sets in X by RGWαO(X).

Theorem 4.2: If a subset A of a space X is w-open then it is rgwα-open set, but not conversely.

Proof: Let A be a w-open set in a space X. Then A^c is w-closed set. By result 3.2(3) A^c is rgwα-closed. Therefore A is rgwα-open set in X. The converse of this theorem need not be true as seen from the following example.

Example 4.3: Let X={a, b, c, d, e} with topology τ={X,ϕ,{a},{d},{e},{a,d},{a,e},{d,e}, {a,d,e}}. Then the set A={c} is rgwα-open set but not w-open set in X.

Corollary 4.4: Every open set is rgwα-open set but not conversely.

Proof: Follows from definition and theorem 4.2.

Corollary 4.5: Every regular open set is rgwα-open set but not conversely.

Proof: Follows from definition and theorem 4.2.

Theorem 4.6: If a subset A of a space X is rgwα-open, then it is gβ-open set in X.

Proof: Let A be rgwα-open set in X. Then A^c is rgwα-closed set in X. By result 3.2(6) A^c is gβ-closed set in X. Therefore A is gβ-open set in space X. The converse of this theorem need not be true as seen from the following example.

Example 4.7: In example 4.3 the subset {b, c} of X is gβ-open set but not rgwα-open set.

Theorem 4.8: If a subset A of X is gα-open set then it is rgwα-open set in X, but not conversely.

Proof: Let A be a gα-open set in a space X. Then A^c is gα-closed set. By result 3.2(4) A^c is rgwα-closed. Therefore A is rgwα-open set in X. The converse of this theorem need not be true as seen from the following example.

Example 4.9: In example 4.3 the subset A={a, b} of X is rgwα-open set but not gα-open set in X.

Theorem 4.10: If a subset A of X is gwα-open set then it is rgwα-open set in X, but not conversely.

Proof: Let A be a gwα-open set in a space X. Then A^c is gwα-closed set. By result 3.2 (4) A^c is rgwα-closed. Therefore A is rgwα-open set in X. The converse of this theorem need not be true as seen from the following example.

Example 4.11: In example 4.3 the sub set A= {b} of X is rgwα-open set but not gwα-open set in X.

Corollary 4.12: If a subset A of X is g*wα-open set then it is rgwα-open set in X, but not conversely.

Proof: it follows from the theorem 4.10 and the implication gwα ⇒ g*wα set.

Theorem 4.13: If A and B are rgwα-open sets in a space X. Then $A\cap B$ is also rgwα-open set in X.

Proof: If A and B are rgwα-open sets in a space X. Then A^c and B^c are rgwα-closed sets in a space X. By result 3.2(7). $A^c \cup B^c$ is also rgwα-closed set in X. That is $A^c \cup B^c = (A\cap B)^c$ is a rgwα-closed set in X. Therefore $A\cap B$ is rgwα-open set in X.

Remark 4.14: The union of two rgwα-open sets in X is generally not a rgwα-open in X.

Example 4.15: In example 4.3 the sets A={a,b} and B={c} are rgwα-open sets in X, But $A\cup B$={a,b,c} is not argwα-open set in X.

Theorem 4.16: If a set A is rgwα-open in a space X, then G=X, whenever G is wα-open and $int(A)\cup A^c \subset G$.

Proof: Suppose that A is rgwα-open in X. Let G be weak α-open and $int(A)\cup A^c \subset G$. This implies $G^c \subset (int(A)\cup A^c)^c = (int(A))^c \cap A$. That is $G^c \subset (int(A))^c - A^c$. Thus $G^c \subset cl(A)^c - A^c$, since $(int(A))^c = cl(A^c)$. Now G^c is also weak α-open and A^c is rgwα-closed then by theorem it follows that $G^c=\phi$. Hence G=X. The converse of this theorem need not be true as seen from the following example.

Example 4.17: In Example 4.3 RGWαO(X)={X, ϕ, {a}, {b}, {c}, {d}, {e}, {a, b}, {a, c}, {a, d}, {a, e}, {b, d}, {d, e}, {c, d},{c. e}, {a, b, d}, {a, b, e}, {a, c, d}, {a, c, e}, {a, d, e}, {b, c, d}, {b, d, e}, {c, d, e}, {a, b, c, d}, {a, b, d, e}, {a, b, c, e},{a, c, d, e},{b, c, d, e}}. And WαO(X) = {X, ϕ, {a}, {b},{d}, {e}, {b, d}, {a, d}, {a, b}, {a, c}, {a, e}, {b, e}, {d, e}, {a, b, c}, {a, b, d}, {a, b, e}, {b, d, e}, {a, d, e}, {a, b, d, e}, {a, c, d, e}}. Take A = {a, c, d}. then A is not rgwα-open. However $int(A)\cup A^c$ ={a, d}∪{b, e}= {a, b, d. e}. so for some weak α-open G, we have $int(A)\cup(A)^c$={a, b, d, e}⊂G gives G=X, but A is not rgwα-open.

Theorem 4.18: A subset A of (X, τ) is rgwα-open set if and only if U ⊆ rαint(A) whenever U is wα-closed and U⊆A.

Proof: Assume that A is rgwα-open in X and U is wα-closed set of (X,) such that U⊆A. Then X-A is rgwα-closed set in (X,). Also X-A ⊆ X-U and X-U is wα-open set of (X,). This implies that rαcl(X-A)⊆X-U. But rαcl(X-A) = X-rαint(A). Thus X-rαint(A)⊆X-U. So U ⊆ rαint(A).

Conversely, Suppose U ⊆ rαint(A) whenever U is wα-closed and U⊆A, To prove that A is rgwα-open in X. Let G be wα-open set of (X,) s.t. X-A ⊆ G. Then X-G ⊆ A. Now X-G is wα-closed set containing A. So X-G ⊆ rαint(A), X-rαint(A)⊆G, But rαcl(X-A)= X- rαint(A). Thus rαcl(X-A) ⊆ G. *i.e* X-A is rgwα-closed set. Hence A is rgwα-open set.

Theorem 4.19: If A is wα-open and rgwα-closed set then A is rα-closed.

Proof: Since A⊆A and A is wα-open and rgwα-closed we have rαcl(A) ⊆ A. Thus rαcl(A)=A. Hence A is rα-closed set of (X, τ).

Theorem 4.20: If rαint(A) ⊆ B ⊆A and A is rgwα-open set in X, then B is rgwα-open set in X.

Proof: If rαint(A) ⊆ B ⊆A, then X-A ⊆ X-B ⊆ X-rαint(A)=rαcl(X-A). Since (X-A) is rgwα-closed set, then by theorem 3.15 [13] X-B is also rgwα-closed set set in X. Therfore B is rgwα-open set in X.

Theorem 4.21: If A is rgwα-closed set in X, then rαcl(A)-A is rgwα-open set in X.

Proof: Let A be rgwα-closed set in X, Let F be an wα-open s.t. F ⊆ rαcl(A)-A. Since A is rgwα-closed, then by theorem 3.12[13] rαcl(A)-A does not contain any non empty wα-closed set in X. Thus F=ϕ. Then F ⊆ rαint(rαcl(A)-A). Therefore by theorem 4.18 rαcl(A)-A is rgwα-open set in X.

Theorem 4.22: If A and B be subsets of space (X, τ). If B rgwα-open and rαint(B) ⊆ A, then AαB is rgwα-open set in X.

Proof: Let B is rgwα-open in X. rαint(B) ⊆ A and rαint(B) ⊆ B is always true, then rαint(B) ⊆ AαB. also rαint(B) ⊆ AαB ⊆B and B is rgwα-open set then by theorem 4.20 AαB is also rgwα-open set in X.

5. rgwα-Closure and rgwα-Interior

In this section the notation of rgwα-Closure and rgwα-Interior is defined and some of its basic properties are studied.

Definition 5.1: For a subset A of X, rgwα-Closure of A is denoted by rgwαcl(A) and defined as rgwαcl(A)= ∩{G: A⊆G, G is rgwα-closed in X} or ∩{G: A⊆G, G∈ RGWαC(X)}.

Theorem 5.2: If A and B are subsets of a space X then

i) rgwαcl(X)=X, rgwαcl(φ)=φ.
ii) A⊆ rgwαcl(A).
iii) If B is any rgwα-closed set containing A, then rgwαcl(A)⊆B.
iv) If A ⊆B then rgwαcl(A)⊆rgwαcl(B).
v) rgwαcl(A)= rgwαcl(rgwαcl(A)).
vi) rgwαcl(A∪B)= rgwαcl(A) ∪ rgwαcl(B).

Proof: i) By definition of rgwα-Closure, X is Only rgwα-closed set containing X, therefore rgwαcl(X) = Intersection of all the rgwα-closed set containing X=∩{X}=X, therefore rgwαcl(X) = X. Again By the Definition of rgwα-Closure rgwαcl(ϕ) = Intersection of all rgwα-closed set containing ϕ = ϕ∩ any rgwα-closed set containing ϕ = ϕ. Therefore rgwαcl(ϕ) = ϕ.
ii) By definition of rgwα-Closure of A it is obious that A⊆ rgwαcl(A).

iii) Let B be any rgwα-closed set containing A, Since rgwαcl(A) is the intersection of all rgwα-closed set containing A, rgwαcl(A) is contained in every rgwα-closed set containing A. Hence in particular rgwαcl(A)⊆B.

iv) Let A and B be subsets of X, such that A⊆B by definition of rgwα-Closure, rgwαcl(B)= ∩{F: B⊆F∈ RGWαC(X)}. If B⊆F∈ RGWαC(X), then rgwαcl(B)⊆F. Since A⊆B, A⊆B⊆F∈ RGWαC(X), we have rgwαcl(A)⊆F, rgwαcl(A)⊆ ∩{F:B⊆F∈ RGWαC(X)} = rgwαcl(B). Therefore rgwαcl(A)⊆rgwαcl(B).

v) Let A be any subset of X by definition of rgwα-Closure, rgwαcl(A)= ∩{F: A⊆F∈ RGWαC(X)}. Therefore A⊆F∈ RGWαC(X) then rgwαcl(A)⊆F, Since F is rgwα-closed set containing rgwαcl(A) by (iii) rgwαcl(rgwαcl(A))= ∩{ F:A⊆F∈RGWαC(X)}= rgwαcl(A). therefore rgwαcl(rgwαcl(A))= rgwαcl(A)

vi) Let A and B be subsets of X, clearly A⊆A∪B, B⊆A∪B from (iv) rgwαcl(A)⊆rgwαcl(A∪B), rgwαcl(B)⊆rgwαcl(A∪B). Hence rgwαcl(A)∪ rgwαcl(B)⊆ rgwαcl(A∪B). Now we have to prove rgwαcl(A∪B)⊆ rgwαcl(A)∪ rgwαcl(B).

Suppose x∉ rgwαcl(A) U rgwαcl(B) then ∃ rgwα-closed set A_1 and B_1 with A⊆A_1, B⊆B_1 & x∉A_1UB_1. We have AUB⊆ A_1U B_1 and A_1U B_1 is the rgwα-closed set such that x∉ A_1U B_1, Thus x∉ rgwαcl(AUB) hence rgwαcl(AUB) ⊆ rgwαcl(A)U rgwαcl(B) (2). From (1) and (2) we have rgwαcl(AUB)= rgwαcl(A)U rgwαcl(B).

Theorem 5.3: If A⊆X is rgwα-closed set then rgwαcl(A)=A.

Proof: Let A be rgwα-closed subset of X. We know that A⊆rgwαcl(A) - (1) . Also A⊆A and A is rgwα-closed set by theorem 5.2 (iii) rgwαcl (A) ⊆A- (2). Hence rgwαcl(A)=A.

The converse of the above need not be true as seen from the following example.

Example 5.4: Let X={a, b, c, d, e} with topology τ= {x, ϕ, {a}, {d}, {e}, {a, d}, {a, e} {d, e}, {a, d, e} } here A={a, d} and rgwαcl (A)={a, d}=A but A is not rgwα-closed set.

Theorem 5.5: If A and B are subsets of Space X then rgwαcl (A∩B) ⊆ rgwαcl (A) ∩ rgwαcl (B).

Proof: Let A and B be subsets of X, clearly A∩B ⊆A, A∩B ⊆B, by theorem 5.2 (iv)
rgwαcl (A∩B)⊆ rgwαcl(A), rgwαcl(A∩B) ⊆ rgwαcl(B), hence rgwαcl(A∩B) ⊆ rgwαcl(A) ∩ rgwαcl(B).

Remark 5.6: In general rgwαcl(A) ∩ rgwαcl(B)⊈ rgwαcl(A∩B).

Theorem 5.7: For an x∈X, x∈ rgwαcl(X) if and only if A∩V ≠ ϕ for every rgwα-open set V containing x.

Proof: Let x∈ rgwαcl(A). To prove A∩V ≠ ϕ for every rgwα-open set V containing x by contradiction. Suppose ∃ rgwα-open set V containing x s.t. A∩V = ϕ then A⊆X-V, X-V is rgwα-closed set, rgwαcl(A)⊆X-V. This Shows that x∉ rgwαcl(A) which is contradiction. Hence A∩V ≠ ϕ for every rgwα-open set V containing x.

Conversely: Let A∩V ≠ ϕ for every rgwα-open set V containing x. To prove x∈rgwαcl(A). We prove the result by contradiction. Suppose x∉ rgwαcl(A) then there exist a rgwα-closed subset F containing A s.t. x∉F. Then x∈X-F is rgwα-open. Also, (X-F) ∩A= ϕ which is contradiction. Hence x∈rgwαcl(A).

Theorem 5.8: If A is subset of space X, then

i) rgwαcl(A)⊆ cl (A)
ii) rgwαcl(A)⊆ rαcl (A)

Proof: Let A be subset of space X by definition of closure cl (A)=$\cap$\{F: A$\subseteq$F$\in$C(X)\} If A$\subseteq$F$\in$C(X) then A$\subseteq$F$\in$RGWαC(X) because every closed set is rgwα-closed that is rgwαcl(A) $\subseteq$ F, therefore rgwαcl(A) $\subseteq$ $\cap$\{F:A$\subseteq$F$\in$C(x)\} Hence rgwαcl(A) $\subseteq$ cl (A).

ii) let A be subset of space X by definition of rα-closure rαcl(A)= $\subseteq\cap$\{F:A$\subseteq$F$\in$rαC(x)\}, If A$\subseteq$F$\in$rαC(x) then A$\subseteq$F$\in$rgwαC(x) because every rα-closed set is rgwα-closed that is rgwαcl(A) $\subseteq$F therefore rgwαcl(A) $\subseteq$ $\cap$\{F:A$\subseteq$F$\in$rαC(x)\}= rαcl(A). Hence rgwαcl(A)$\subseteq$ rαcl(A).

Remark 5.9: Containment relation in the above theorem 5.8 may be proper as seen from following example.

Example 5.10: Let X={a, b, c, d, e,} with topology τ= {X, ϕ, {a}, {d},{e},{a, d}, {a, e} ,{d, e}, {a, d, e}, A= {a, b, d, e,} cl(A)={X}, rgwαcl(A)={a, b, d, e} & rαcl(A)={X}. It follows that rgwαcl(A) $\subset$ Cl (A) and rgwαcl(A) $\subset$ rαcl(A).

Theorem 5.11: If A is subset of space X then gspcl(A) $\subseteq$ rgwαcl(A) where gspcl (A)= $\subseteq\cap$\{F: A$\subseteq$F$\in$ GSPC (X)\}.

Proof: Let A be subset of X by definition of rgwα-closure rgwαcl(A)= $\cap$\{F: A$\subseteq$F$\in$ RGWαC(X)\}. If A$\subseteq$F$\in$ RGWαC(X) then A$\subseteq$F$\in$ GSPC(X), because every rgwα-closed is gsp-closed i.e. gspcl (A) $\subseteq$F. therefore gspcl (A) $\subseteq\cap$\{F: A$\subseteq$F$\in$ RGWαC(X)\}= rgwαcl(A).
Hence gspcl(A) $\subseteq$ rgwαcl(A).

Theorem 5.12: rgwα-Closure is a kuratowski-Closure operator on a space X.

Proof: Let A and B be the subsets of space X. i) rgwαcl(x) =x, rgwαcl(ϕ) = ϕ ii) A$\subseteq$ rgwαcl(A) iii) rgwαcl(A) =rgwαcl(rgwαcl (A)) iv) rgwαcl (AUB) = rgwαcl (A) U rgwαcl (B) by theorem 5.2 Hence, rgwα-Closure is a Kuratowski-Closure operator on a space X.

Definition 5.13: For a subset A of X, rgwα-Interior of A is denoted by rgwαint (A) and defined as rgwαint (A)=U\{G:G$\subseteq$A and G is rgwα-open in X\} or U \{G: G$\subseteq$A and G $\in$RGWαO(X)\}.
i.e. rgwα-int(A) is the union of all rgwα-open set contained in A.

Theorem 5.14: Let A and B be subset of space x then

i) rgwαint (X)=X, rgwαint (ϕ)=ϕ
ii) rgwαint(A)$\subseteq$A
iii) If B is any rgwα-open set contained in A then B $\subseteq$ rgwαint(A)
iv) If A$\subseteq$B then rgwαint(A) $\subseteq$ rgwαint(B)
v) rgwαint(A)= rgwαint (rgwαint(A)).
vi) rgwαint(A$\cap$B) = rgwαint(A) $\cap$ rgwαint(B)

Proof: i) and ii) by definition of rgwα-Interior of A, it is obvious.
iii) Let B be any rgwα-open set such that B$\subseteq$A. Let x$\in$B, B is an rgwα-open set contained in A,
x is an element of rgwα-Interior of A i.e. x $\in$ rgwαint (A). Hence B$\subseteq$ rgwαint (A).
iv), v) vi) similar proof as theorem 5.2 and definition of rgwα-Interior.

Theorem 5.15: If a subset A of X is rgwα-open then rgwαint (A) = A.

Proof: Let A be rgwα-open subset of X. We know that rgwαint (A) $\subseteq$ A –(1) Also A is rgwα-open set contained in A from theorem 5.13 iii) A$\subseteq$ rgwαint (A) –(2) hence from (1) and (2) rgwαint (A)=A.

Theorem 5.16: If A and B are subsets of space X then rgwαint (A) U rgwαint (B) $\subseteq$ rgwαint (AUB)

Proof: We know that A$\subseteq$ (AUB) and B$\subseteq$ (AUB) we have theorem 5.13 iv) rgwαint (A) $\subseteq$ rgwαint (AUB) and rgwαint (B) $\subseteq$ rgwαint (AUB). This implies that rgwαint (A) U rgwαint (B) $\subseteq$ rgwαint (AUB).

Remarks 5.17: The converse of the above theorem need not be true.

Theorem 5.18: If A is a subset of X then i) int (A) $\subseteq$ rgwαint (A) ii) rαint (A) $\subseteq$ rgwαint (A).

Proof: Let A be a subset of a space X. Let x$\in$int (A)$\Rightarrow$ x$\in$U{G:G is open, G$\subseteq$ A}
$\Rightarrow$ $\exists$ an open set G s.t. x$\in$G $\subseteq$ A$\Rightarrow\exists$ an rgwα-open set G s.t. x$\in$G $\subseteq$ A as every open set is rgwα-open set in X $\Rightarrow$ x$\in$U{G:G is rgwα-open set in X} αx$\in$ rgwαint (A), thus x$\in$int (A) $\Rightarrow$x$\in$ rgwαint (A), Hence, int (A) $\subseteq$ rgwαint (A).
ii) Let A be a subset of space X. Let x$\in$ rαint (A), $\Rightarrow$ x$\in$U{G:G is r$\propto$-open G$\subseteq$ A}
$\Rightarrow$ $\exists$ an rα-open set G s.t. x$\in$ G$\subseteq$ A
$\Rightarrow$ $\exists$ an rgwα-open set G s.t. x$\in$ G$\subseteq$ A, as every rα-open set is an rgwα-open set in X$\Rightarrow$ x$\in$U{G:G is rgwα-open set in X} $\Rightarrow$ x$\in$ rgwαint (A).

Thus x$\in$ rαint (A) $\Rightarrow$ x$\in$ rgwαint (A).

Hence rαint (A) $\subseteq$ rgwαint (A).

Remark 5.19: Containment relation in the above theorem may be proper as seen from the following example.

Example 5.20: Let x={a, b, c, d, e} with topology τ={x, ϕ, {a}, {d}, {e}, {a, d}, {a, e}, {a, d, e}} A= {a, b} int(A)={a}, rαint(A)={a}, rgwαint (A)= {a, b} therefore int (A) $\subset$rgwαint (A) and rαint (A) $\subset$ rgwαint (A)

Theorem 5.21: If A is subset of X, then rgwαint (A) $\subseteq$ gspint (A), where gspint (A) is given by gspint (A)=U{G$\subseteq$ X:G is gsp-open, G$\subseteq$A}.

Proof: Let A be a subset of a space X. Let x $\in$ rgwαint (A) $\Rightarrow$x$\in$U{G:G is rgwα-open G$\subseteq$ A}
$\Rightarrow$ $\exists$ an rgwα-open set G s.t. x$\in$ G$\subseteq$ A, as every rgwα-open set is an gsp-open set in X $\Rightarrow$ x$\in$U{G:G is gsp-open G$\subseteq$ A} $\Rightarrow$ x$\in$ gspint (A).
Thus, x$\in$ rgwαint (A) } $\Rightarrow$ x$\in$ gspint (A) Hence, rgwαint (A) $\subseteq$ gspint(A).

Theorem 5.22: For any subset A of X
i) X- rgwαint (A)= rgwαcl (X-A)
ii) X- rgwαcl (A)= rgwαint (X-A)

Proof: x$\in$X- rgwαint (A), then x is not in rgwαint (A) i.e. every rgwα-open set G containing x such that G $\subseteq$ A. This implies every rgwα-open set G containing x intersects (X-A) i.e. G $\cap$(X-A)$\neq$ϕ. Then by theorem 5.7 x$\in$ rgwαcl (X-A) Therefore X- rgwαcl (A) $\subseteq$ rgwαcl (x-A)---(1)
and let x$\in$ rgwαcl (X-A) , then every x$\in$ rgwα-open set G containing x interests X-A i.e. G$\cap$(X-A)$\neq$ϕ. i.e. every rgwα-open set G containing x s.t. G$\subseteq$ A. Then by definition 5.12. x is not in rgwαcl (A), i.e. x$\in$X-rgwαint (A) and so rgwαcl (x-A) $\subseteq$ x-rgwαint (A)---(2)

Thus X-rgwαint (A)= rgwαcl(X-A). Similarly we can prove ii).

6. rgwα-Neighbourhood and rgwα-Limit points

In this section we define the notation of rgwα-Neighbourhood, rgwα-Limit points and rgwα-Derived set and some of their basic properties and analogous to those for open sets.

Definition 6.1: Let (X,τ) be a topological space and let $x\in X$, A subset N of X is said to be rgwα-Neighbourhood of x if there exists an rgwα-open set G s.t. $x\in G\subseteq N$.

Definition 6.2: i) Let (X, τ) be a topological space and A be a subset of X, A subset N of X is said to be rgwα Neighbourhood of A, if there exists an rgwα-open set G s.t. $A\subseteq G\subseteq N$
ii) The collection of all rgwα-Neighbourhood of $x\in X$ called rgwα-Neighbourhood system at x and shall be denoted by rgwαN (x)

Definition 6.3: i) Let (X, τ) be a topological space and A be a subset of X, then a point $x\in X$ is called a rgwα-Limit point of A if every rgwα-Neighbourhood of x contains a point of A distinct from x i.e. $(N-\{x\} \cap A \neq \phi$ for each rgwα-Neighbourhood N of x. Also equivalently iff, every rgwα-open set G containing x contains a point of A other than x.
ii) The set of all rgwα-Limit points of the set A is called Derived set of A and is denoted by rgwαd(A).

Theorem 6.4: Every neighbourhood N of $x\in X$ is called is a rgwα-Neighbourhood of $x\in X$.

Proof: Let N be neighbouhood of point $x\in X$. To prove that N is a rgwα-Neighbourhood of x by definition of neighbourhood, $\exists$ an open set G s.t. $x\in G\subset N \Rightarrow\exists$ an rgwα-open set G s.t. $x\in G\subset N$, as every open set is rgwα-open set. Hence N is rgwα-Neighbourhood of x,

Remark 6.5: In general, a is rgwα-nbhd N of $x\in X$. need not be a nbhd of x in X, as seen from the following example.

Example 6.6 : Let X={a, b, c, d, e} with topology τ = {x, ϕ, {a}, {d}, {e}, {a, d,}, {a, e}, {d, e}, {a, d, e}}. The set {a, b} is rgwα-Neighbourhood of the point b, since $\exists$ the rgwα-open set {b} s.t. $b\in\{b\}\subset\{a, b\}$, However the set {a, b} is not a nbhd of the point b. Since no open set G exists s.t. $b\in G \subseteq\{a, b\}$

Theorem 6.7: If a subset N of a space X is rgwα-open, then N is rgwα-nbhd of each of its points.

Proof: Suppose N is rgwα-open. Let $x\in N$. We claim that N is rgwα-nbhd of x. For N is a is rgwα-open set such that $x\in N\subset N$. Since x is an arbitrary point of N, it follows that N is a rgwα-nbhd of each of its points.

Remark 6.8: Let X= {a, b, c, d, e} with topology τ= {X, ϕ, {a}, {d}, {e},{a, d,}, {a, e}, {d, e}, {a, d, e}}. The set {b, c} is a is rgwα-nbhd of the point b, since the rgwα-open set {b} is s.t. $b\in\{b\}\subset\{b, c\}$, Also the set {b, c} is rgw∝-nbhd of the point c, Since the rgwα-open set {c} is s.t. $c\in\{c\} \subset \{b, c,\}$. That is {b, c,} is a rgwα-nbhd of each of its points. However the set {b, c} is not rgwα-open set in X.

Theorem 6.10: Let X be a topological space. If F is a rgwα-closed subset of X, and $x\in F^{c.}$. Prove that there exists argwα-nhbd N of x such that $N\cap F=\phi$. **Proof:** let F be rgwα-closed subset of X and $x\in F^c$. Then F^c is rgwα-open set of X. So by theorem 6.7 F^c contains a rgwα-nbhd of each of its points. Hence there exists a rgwα-nbhd of N of x such that $N\in F^c$. That is $N\cap F=\phi$.

Theorem 6.11: Let X be a topological space and for each $x\in X$, Let rgwα-N (x) be the collection of all rgwα-nbhd of x. Then we have following results.

i) $\forall x\in X$, rgwα-N (x) $\neq\phi$.

ii) $N \in$ rgwα-N (x) $\Rightarrow x \in N$.

iii) $N \in$ rgwα-N (x), $M \supset N \Rightarrow M \in$ rgwα-N (x)

iv) $N \in$ rgwα-N (x), $M \in$ rgwα-N (x) $\Rightarrow N \cap M \in$ rgwα-N (x)

v) $N \in$ rgwα-N (x)$\Rightarrow$ There exists $M \in$ rgwα-N (x) such that $M \in N$ & $M \in$ rgwα-N (y), for every $y \in M$.

Proof: i) Since X is a rgwα-open set, it is rgwα-nbhd of every $x \in X$. Hence there exists at least one rgwα-nbhd (namely_X) for each $x \in X$. Hence rgwα-N (x) $\neq \phi$ for every $x \in X$.
ii) If $N \in$ rgwα-N (x), then N is a rgwα-nbhd of x, so by definition of rgwα-nbhd, $x \in N$. Let $N \in$ rgwα-N (x) and $M \in N$. Then there is a rgwα-open set G such that $x \in G \subset N$. Since $N \subset M$, $x \in G \subset M$ and so M is rgwα-nbhd of x. Hence $M \in$ rgwα-N (x).
iv) Let $N \in$ rgwα-N (x) and $M \in$ rgwα-N (x). Then by definition of rgwα-nbhd there exists rgwα-open sets G_1 and G_2 such that $x \in G_1 \subset N$ and $x \in G_2 \subset M$.

Hence $x \in G_1 \cap G_2 \subset N \cap M$---(1). Since $G_1 \cap G_2$ is a rgwα-open set (being the intersection of two rgwα-open sets) it follows from (1) that $N \cap M$ is also rgwα-nbhd of x. Hence $N \cap M \in$ rgwα-N$\subset$ x).
v) If $N \in$ rgwα-N(x), then there exists a rgwα-open set M such that $x \in M \subset N$. Since M is rgwα-open set, it is rgwα-nbhd of each of its points. Therefore $M \in$ rgwα-N(y) for every $y \in M$.

Theorem 6.12: Let X be a non empty set, and for each $x \in X$, let rgwα-N(x) be a nonempty collection of subsets of X satisfying following conditions.

i) $N \in$ rgwα-N(x) $\Rightarrow x \in N$

ii) $N \in$ rgwα-N(x), $M \in$ rgwα-N(x$\Rightarrow N \cap M \in$ rgwαN(x)

Let τ consists of the empty set and all those non-empty subsets of G of X having the property that $x \in G$ implies that there exists an $N \in$ rgwα-N(x) such that $x \in N \subset G$. Then τ is a topology for X.

Proof:

(i) $\phi \in T$ by definition. We now show that $X \in \tau$. Let x be any arbitrary element of X. Since rgwα-N(x) is nonempty, there is $N \in$ rgwα-N(x) and so $x \in N$ by (i). Since N a subset of X, we have $x \in N \subset X$. Hence $X \in \tau$.

(ii) Let $G_1 \in \tau$ and $G_2 \in \tau$. if $x \in G_1 \cap G_2$ Then $x \in G_1$, $x \in G_2$. Since $G_1 \in \tau$, $G_2 \in \tau$, there exist $N \in$ rgwα-N(x) and $M \in$ rgwα-N(x) , such that $x \in N \subset G_1$ and $x \in M \subset G_2$. Then $x \in N \cap M \subset G_1 \cap G_2$. But $N \cap M \in$ rgwα-N(x) by theorem 6.11 (iv) Hence $G_1 \cap G_2 \subset \tau$.

(iii) Let $G_\lambda \in \tau$ for every $\lambda \in \Lambda$. If $x \in U\{ G_\lambda : \lambda \in \Lambda\}$, then $x \in G_{\lambda x}$ for some $\lambda_x \in \Lambda$. Since $G_{\lambda x} \in \tau$, there exists an $N \in$ rgwα-N(X) such that $x \in N \subset G_{\lambda x}$ and consequently $x \in N \subset U\{G_{\lambda} : \lambda \in \Lambda\}$. Hence $U \{G_{\lambda} : \lambda \in \Lambda\} \in \tau$. It follows that τ is a topology for X.

Theorem 6.13: Let X be a topological space then

i) rgwαd(A)=ϕ

ii) If $A \subset B \Rightarrow$ rgwαd(A) $\subset$ rgwαd(B)

iii) rgwαd(AUB) = rgwαd(A) U rgwαd(B)

Proof: i) Suppose that rgwαd(A) $\neq \phi$ then rgwαd(A) contains at least one element. Therefore let $x \in$ rgwαd(ϕ) then x is a rgwα-Limit point of ϕ therefore for every rgwα-open set G containing 'x', (G-{x}) $\cap \phi \neq \phi$, But this is not true. Since intersection of ϕ with any set is again a ϕ. Therefore rgwαd(A)= ϕ.
ii) Given $A \subset B$ to prove rgwαd(A) $\subset$ rgwαd(B). let $x \in$ rgwαd(A). $\Rightarrow$ x is a rgw∝-limit point of A. Therefore by definition, $\exists$ an rgwα-open set G containing x such that (G-{x})$\cap A \neq \phi$---(1). But $A \subset B \Rightarrow$A-{x} $\subset$B-{x} $\Rightarrow$(G-{x})$\cap B \neq \phi$. $\Rightarrow$x is a rgwα-limit point of B $\Rightarrow x \in$ rgwαd(B). Thus $x \in$ rgw∝d(A) $\Rightarrow x \in$ rgw∝d(B). Therefore rgwαd(A) $\subset$ rgwαd(B)
iii)We have $A \subset AUB$ and $B \subset AUB$. Therefore rgwαd(A)$\subset$ rgwαd(AUB) and rgwαd(B) $\subset$ rgwαd(AUB). Therefore rgwαd(A)U rgwαd(B) $\subset$rgwαd(AUB). ---(1). To prove rgwαd(AUB)$\subset$ rgwαd(A)U rgwαd(B). Let $x \in$ rgwαd(AUB) $\Rightarrow$ x is rgwα-limit point of (AUB).
$\Rightarrow$(G-{x})$\cap$(AUB) $\neq \phi$ for every rgwα-open set G containing x. $\Rightarrow$[(G-{x})$\cap$A] U [(G-{x})$\cap$B] $\neq \phi$ $\Rightarrow$(G-{x})$\cap A \neq \phi$ or (G-{x})$\cap B \neq \phi \Rightarrow$ x is a rgwα-limit point of A or x is a rgwα-limit point of B. i.e. $x \in$rgwαd(A) or $x \in$rgwαd(B) therefore $x \in$rgwαd(A) U rgwαd(B).
For $x \in$rgwαd(AUB) $\Rightarrow x \in$rgwαd(A) U rgwαd(B). $\Rightarrow$rgwαd(A$\cup$B)$\subset$ rgwαd(A) U rgwαd(B)---(2)
$\Rightarrow$From (1) and (2) rgwαd(AUB)= rgwαd(A) U rgwαd(B)

Theorem 6.14: Let X be a topological space and A⊂ X. Then AUrgwαd(A) is rgwα-closed set in X.

Proof: To prove AUrgwαd(A) is a rgwα-closed set in X. that is to prove X- AUrgwαd(A) is an rgwα-open set in X. Let x∈ X- AUrgwαd(A) ⇒x∈X & x∉ AUrgwαd(A) ⇒x∈X & (x∉A & x∉rgwαd(A)) ⇒x∈X & (x∉A & x is not a limit point of A). ⇒x∈X ,x∉A , there exist an rgwα-open set G containing x s.t. G∩(A-{x})=ϕ i.e. G∩A=ϕ. Further, G∩rgwαd(A)=ϕ. Let y∈G. then y∉A because G∩A=ϕ. Now G is an rgwα-open set containing y and G∩A=ϕand y∈A. therefore G is an rgwα-open set containing y s.t. G∩(A-{y}) =ϕ. Therefore there exist an rgwα-open set G containing y s.t. G∩(A-{y}) =ϕ. Therefore y is not a limit point of A. i.e. y∉rgwαd(A). y∈G, y∉rgwαd(A). therefore G∩rgwαd(A)=ϕ. Thus we have G∩A=ϕ and G∩rgwαd(A)=ϕ. ⇒(G∩A)∪ (G∩rgwαd(A))=ϕ. □G ∩AUrgwαd(A)=ϕ ⇒G⊂{X AUrgwαd(A)}. Thus for all x∈{ X- (AUrgwαd(A))} there exist an open set G s.t. x∈G ⊂ {X (AUrgwαd(A))} ⇒X (AUrgwαd(A)) is an rgwα-open set. Therefore AUrgwαd(A) must be argwα- closed set in X.

Theorem 6.15: Let X be a topological space and A⊂ X, then A is rgwα-closed iff A ⊃ rgwαd(A) i.e. A is rgwα-closed if and only if A contains all its rgwα-limit points. i.e. A is rgwα-closed if and only if rgwαd(A) ⊂ A.

Proof: Suppose A is rgwα-closed set, To prove A ⊃ rgwαd(A) i.e rgwαd(A) ⊂ A. Let x∉A, we prove x∉ rgwαd(A). Since x∉A, we have x∈X-A.

Now X-A is an rgwα-open set containing x and (X-A) ∩ A=ϕ. i.e (X-A) ∩ (A-{x})=ϕ. There exist an rgwα-open set (X-A) containing x s.t. (X-A) ∩ (A-{x})=ϕ. Therefore x is not a limit point of A. x∉ rgwαd(A). Thus x∉A ⇒x∉ rgwαd(A). therefore A ⊃ rgwαd(A) i.e rgwαd(A) ⊂ A .

Conversely, on the other hand suppose A ⊃ rgwαd(A) i.e rgwαd(A) ⊂ A. we prove A is rgwα-closed set i.e we prove X-A is rgwα-open set .

Let x∈X-A ⇒ x∉A ⇒x∉ rgwαd(A). ⇒x is not a limit point of A.⇒there exist an rgwα-open set G containing x s.t. G∩(A-{x})=ϕ ⇒there exist an rgwα-open set G containing x s.t. G∩A=ϕ ⇒there exist an rgwα-open set G containing x s.t. G ⊂XA ⇒there exist an rgwα-open set G containing x s.t. x∈G ⊂XA. for all x∈X-A there exist an rgwα-open set G containing x s.t. x∈G ⊂ XA. therefore (X-A) must be an rgwα-open set. Therefore A must be a rgwα-closed set.

Theorem 6.16: Let X be topological space and A⊂ X then rgwαcl(A)=AU rgwαd(A).

Proof: w.k.t. AU rgwαd(A) is rgwα-closed set in X. Also we have A⊂ AU rgwαd(A). Therefore AU rgwαd(A) is a closed set containing A. But rgwαcl(A) is the smallest closed set containing A. Therefore rgwαcl(A) ⊂ AU rgw∝-d(A). __(1)

Further we have A⊂ rgwαd(A)__(i). To prove rgwαd(A) ⊂ rgwαcl(A). Let x∈rgwαd(A). ⇒x is a rgwα-limit point of A. We prove that x∈rgwαcl(A). If possible let x∉rgwαcl(A). Then x∈X- rgwαcl(A), Therefore X- rgwαcl(A) is an rgwα-open set containing x and [X- rgwαcl(A)]∩ [A-{x}]=ϕ. Therefore x is not a limit point of A. Which is wrong. Therefore x∈rgwαcl(A). If x∈rgwαd(A) then x∈rgwαcl(A) ⇒ rgwαd(A) ⊂ rgwαcl(A)---(ii)

From (i) and (ii) AU rgwαd(A) ⊂rgwαcl (A)---(2)

From (1) and (2) rgwαcl (A)=AU rgwαd(A).

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