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MAGNETOHYDRODYNAMIC VISCOPLASTIC FLOW OVER A VERTICAL PLATE WITH CONVECTIVE HEATING: NUMERICAL ANALYSIS

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ABSTRACT

The steady boundary layer Magnetohydrodynamicflow of a Casson fluid and heat transfer over vertical plate with conductive boundary situations, outcomes of MHD, convective heating is analyzed. The governing flow problem is primarily based on momentum equation and energy equation and these are further simplified with the help of non-similarity differences. The decreased, resulting incredibly nonlinear coupled ordinary differential equations are solved using the finite difference technique. The consequences of certain parameters on the dimensionless velocity and temperature are supplied in graphical notation and skin friction and heat transfer rate discussed via tabular notation. A contrast with published facts has been finished, and appropriate agreements are observed, The Velocity and Temperature profiles against η for different values of Casson Paramater (β), Convective heating (γ), Magnetic Parameter (M), Prandtl Number (Pr) and Two dimensional non similarity parameter (ξ) are discussed in detail.

Keywords: *Thermal convection; Convective boundary condition; Keller-box numerical method; Casson Viscoplastic model.*

NOMENCLATURE

B_0	externally imposed radial magnetic field
C_f	skin friction coefficient
f	non-dimensional stream function
g	acceleration due to gravity
Gr	Grashof (free convection) number
Nu	local Nusselt number
M	magnetic field parameter
Pr	Prandtl number
T	temperature
u, v	non-dimensional velocity components along the x- and y- directions, respectively
x	stream wise coordinate
y	transverse coordinate

GREEK SYMBOLS

α	thermal diffusivity
β	non-Newtonian Casson parameter
η	dimensionless transverse coordinate
ν	kinematic viscosity
γ	Convective Heating
θ	non-dimensional temperature
ξ	dimensionless steam wise coordinate
ψ	dimensionless stream function

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SUBSCRIPTS

w conditions on the wall
∞ free stream conditions

1. INTRODUCTION

Many present day engineering applications involve the study of non-Newtonian fluids. those consist of petroleum drilling muds [1], organic gels [2], polymer processing [3] and food processing [4]. Many of the fluids that are used in industries are shown (shows) non-Newtonian behavior, so the modern day investigators are showing (have) keen interest in those industrial non-Newtonian fluids, and their dynamics. A single constitutive equation is not enough to cover all properties of such non-Newtonian fluids, (all the properties of such non-Newtonian fluids are not covered by a single constitutive equation) and hence (So, for that) many non-Newtonian fluid models [5–8] have been proposed to clarify all physical behaviors. {Among them Casson fluid is one of the types of such non-Newtonian fluids}, which (that) behaves like an elastic solid, and for this a yield shear stress exists in the constitutive equation. Casson model is claimed to fit rheological data better than general viscoplastic models for many materials and is the (it also an) preferred rheological model for blood and chocolate [9]. Hiemenz [10] first studied two-dimensional stagnation flow using similarity transformations to reduce the Navier–Stokes equations to nonlinear ordinary differential equations. The Casson fluid flow in a pipe with a homogeneous porous medium was investigated by Dash *et al.* [11]. The Casson model fits the flow data better than the more general Herschel-Bulkley model by Joye [12] and Kirsanov and Remizo [13], which is a power-law formulation with yield stress as Bird *et al.* [14]. Bhattacharyya [15] examine the steady boundary layer stagnation point flow of Casson fluid and heat transfer towards a shrinking/stretching sheet. Hayat [16] considered the mixed convection stagnation point flow of an incompressible non-Newtonian fluid over a stretching sheet under convective boundary conditions. Ramesh [17] investigated the stagnation-point flow of an incompressible non-Newtonian fluid over a non-isothermal stretching sheet. The MHD and slip effect in the boundary layer viscous flow over a surface with heat transfer was examined by Bhattacharyya *et al.* [18].

In the current work, a mathematical model is advanced for steady, natural convection boundary layer flow in a Cassonviscoplastic polymeric fluid external to a vertical plate with Magneto-hydrodynamic, conductive heating effect and maintained at non-uniform surface temperature. A finite difference numerical solution is obtained for the transformed nonlinear two-point boundary value problem concern to physically appropriate boundary conditions at the cone surface and within the free stream. The impact of the emerging thermos-physical parameters i.e. Casson non-Newtonian parameter, MHD, conductive heating and Prandtl number on velocity, temperature, wall shear stress function and Nusselt number, inside the presence of wall suction, are provided graphically and in Tables. Validation with previous Newtonian study is included. exact evaluation of the physics is included. The existing problem has to the authors' expertise now not regarded so far in the experimental scientific literature and is applicable to thermal remedy of polymeric enrobing structures [19].

2. MATHEMATICAL FORMULATION

A study two-dimensional laminar boundary layer MHD flow of viscoplastic in compressible electrically conducting Casson fluid over a vertical plate with conductive boundary conditions within the presence of Magnetohydrodynamic impact. The coordinate system chosen in one of these way that the axis is along the sheet and y-axis is normal to the plate (see Fig.1). The magnetic Reynolds number is thought very small, and consequently the included magnetic field is negligible. It's also assumed the rheological equations of the state for an isotropic and incompressible go with the flow of Casson fluid can be written as (Mustafa *et al.* [20])

$$\tau_{ij} = \begin{cases} 2\left(\mu_\beta + \rho_y \big/ \sqrt{2\pi}\right) e_{ij},\ \pi > \pi_c \\ 2\left(\mu_\beta + \rho_y \big/ \sqrt{2\pi_c}\right) e_{ij},\ \pi < \pi_c \end{cases} \tag{1}$$

Where τ_{ij} is the $(i, j)^{th}$ component of the stress tensor, $\pi = e_{ij}e_{ij}$, e_{ij} is the $(i, j)^{th}$ component of deformation rate, π denotes the product of the component of deformation rate with itself, and ρ_y is the yield stress of a fluid, π_c is a critical value of this product based on the non-Newtonian model, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid.

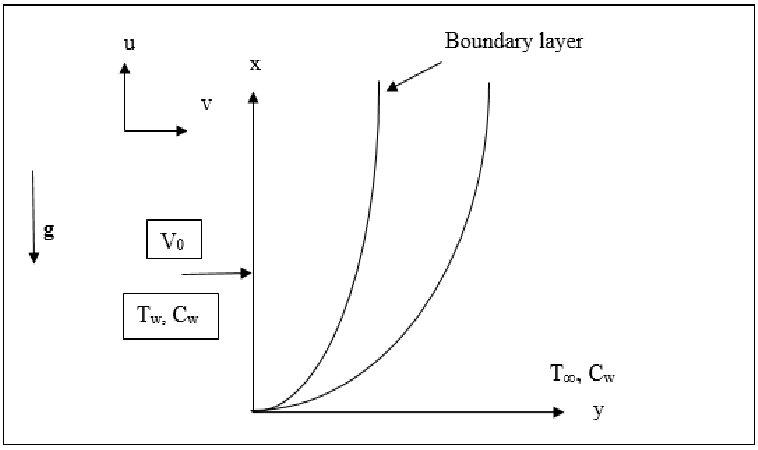

Figure-1: Non-Newtonian Heat transfer model

The equations for mass continuity, momentum and energy, can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}+g\Lambda(T-T_\infty)-\frac{\sigma B_0{}^2}{\rho}u \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The boundary conditions for the considered flow with conductive heating:

$$\text{At } y=0, u=0,\ v=0,\ -k\frac{\partial T}{\partial y}=h_w(T_w-T)$$

$$\text{As } y\to\infty,\ u\to 0,\ v\to 0,\ T\to T_\infty \tag{5}$$

The stream function ψ is defined by $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, and therefore, the continuity equation is automatically satisfied. In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced:

$$\xi=\frac{V_0 x}{\nu},\eta=\frac{y}{x}\left(Gr_x\right)^{1/4},\ \psi=4\nu\left(Gr_x\right)^{1/4}\left(f(\xi,\eta)+\frac{1}{4}\xi\right)$$

$$\theta(\xi,\eta)=\frac{T-T_\infty}{T_w-T_\infty},Gr_x=\frac{g\beta_T\left(T_w-T_\infty\right)x^3}{4\nu^2} \tag{6}$$

The emerging momentum and heat (energy) conservation equations in dimensionless from assume the following form:

$$\left(1+\frac{1}{\beta}\right)f'''+\left(3f+\xi\right)f''-2f'^2-Mf'+\theta=\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right) \tag{7}$$

$$\frac{\theta''}{\Pr}+\left(3f+\xi\right)\theta'=\xi\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right) \tag{8}$$

The transformed dimensionless boundary conditions are reduced to:

$$\text{At } \eta=0,\ f=0,\ f'=0,\theta=1+\frac{\theta'}{\gamma}$$

$$\text{As } \eta\to\infty,\ f'\to 0,\theta\to 0 \tag{9}$$

The skin-friction coefficient (cone surface shear stress function) and Nusselt number (heat transfer rate) can be defined using the transformations described above with the following expressions:

$$\frac{1}{4}Gr_x^{-3/4}C_f=\left(1+\frac{1}{\beta}\right)f''(\xi,0) \tag{10}$$

$$Gr_x^{-1/4}Nu=-\theta'(\xi,0) \tag{11}$$

3. NUMERICAL SOLUTIONS WITH KELLER BOX SCHEME

The coupled boundary layer equations in a (ξ,η) coordinate system remain strongly nonlinear. A numerical method, the Keller-Box implicit difference method, is therefore deployedto solve the boundary value problem defined by Ens. (7)-(8) with boundary conditions (9). This technique, despite recent developments in other numerical methods, remains a powerful and very accurate approach for parabolic boundary layer flows. It is unconditionally stable and achieves exceptional accuracy [21]. It has been used recently in Non-Newtonian fluid flow dynamics by Subba Rao *et al.* [22-24] and Amanulla *et al.* [25]. The key stages involved are as follows:

a. Reduction of the *Nth* order partial differential equation system to *N* first order equations
b. Finite difference discretization
c. Quasilinearization of non-linear Keller algebraic equations
d. Block-tridiagonal elimination of linear Keller algebraic equations

4. NUMERICAL RESULTS AND INTERPRETATION

As a way to confirm the accuracy of the Keller box solutions, computations arebenchmarked with in advance effects reported in [26-28] results as shown in **Table 1**. With $\beta \to \infty$, the present model reduces to the Newtonian model considered in[26-28]. Very close correlationis achieved between the Keller box computational results and confidence in the Keller box numerical code is consequently justifiably excessive.

Table-1. Skin friction coefficient for the boundary layer flow past a non-porous plate in the absence of magnetic field.

	[26]	[27]	[28]	Present
C_f	0.33206	0.332058	0.332058	0.332056

Table-2. Numerical values of $\left(1+\frac{1}{\beta}\right) f''(0)$ and $-\theta'(0)$ for various values of M, γ and β.

M	γ	β	$\left(1+\frac{1}{\beta}\right) f''(0)$	$-\theta'(0)$
0.0	0.5	1	0.6294	0.6535
0.5			0.5629	0.6384
1.0			0.5148	0.6276
2.0			0.4484	0.6131
1.0	0.25		0.3584	0.4043
	0.45		0.4979	0.6022
	0.75		0.5648	0.7043
	1.0		0.5894	0.7431
	0.5	1	0.5148	0.6276
		3	0.4777	0.6408
		5	0.4687	0.6442

Table 2 presents the influence of M,γ *and* β on the wall heat transfer rate i.e. Nusselt number, $-\theta'(0)$. An increase in Newtonian Heating (γ) and Casson parameter (β) induces a substantial decrease in the magnitude of $-\theta'(0)$. The contrary response is computed with an increase in Magnetic parameter (*M*). With increasing β values, less heat is transferred from the boundary layer regime to the plate surface (the fluid is heated and the plate surface is cooled). This manifests in a decrease in Nusselt numbers with greater viscoplastic effect (*larger* β *values).* Similar observations have been reported by for example Mustafa *et al*. [16] and Amanulla et al. [25]

Figures. 2a and 2b, depict the effect of Casson fluid parameter, β on velocity and temperatureprofiles. It is shown that the effect of β enhances velocity near the plate surface but depletesitis shown that the effect of β increases , the velocity profile increases near the plate but the trend gets reversed when $\eta \geq 2$ but temperature profiles are slightly decreases as β increases. further away. Increasing Casson parameter however consistently *weakly* decreases temperaturesthroughout the boundary layer. The influence on velocity field is significantly greater howeversince the viscoplastic effect is simulated solely in the momentum equation (7) via the shear term $\left(1+\frac{1}{\beta}\right) f'''$.

Figure 3a illustrates that the velocity is reduced with an increase in M . This indicates that the transverse magnetic field opposes the transport phenomena since an increase in M leads to an increase in the Lorentz force, which opposes the transport process. This stronger Lorentz force produces more resistance to the transport. The higher the value of M , the more prominent is the reduction in hydrodynamic boundary layer thickness. But from Fig.3b, the opposite phenomenon is observed with an increase in Magnetic field parameter M on temperature field.

Figures 4a and 4b illustrate the influence of convective heating parameter (γ) on the velocity and temperature. Both velocity and temperature are consistently suppressed with an increase in γ. Temperatures are strongly increases in particular at the plate surface. Greater convetive heating parameter therefore accelerates the flow and cools the boundary layer. Momentum boundary layer thickness is enhanced whereas thermal boundary layer thickness is increased with increasing convetive heating parameter. it is shown that the effect of γ increases , the velocity profile increases near the plate but the trend gets reversed when $\eta \geq 4$ but temperature profiles are slightly decreses as γ increases.

Figures 5a–5b, present the effect of Prandtl number (*Pr*) on the velocity and temperature profiles along the transverse coordinate i.e. normal to the plate surface.Fig.5a shows the effects of Prandtl number *Pr* on the heat transfer process. It is observed that the temperature of the fluid is depressed by larger Prandtl number. From a physical point of view, Prandtl number is a dimensionless number approximating the ratio of the momentum diffusivity to the thermal diffusivity. In short, a larger *Pr* feature lower thermal diffusion compared to viscous diffusion and hence it offers less penetration depth for temperature. Further, a decrease in the thermal boundary layer with strong Prandtl number is compensated with steeper temperature profiles in fig 5b.

Figures 6a – 6b depict the velocity and temperature distributions with radial coordinate, for various transverse (stream wise) coordinate values, ξ along with the variation in the Casson parameter (β). Clearly, from these figures it can be seen that as suction parameter ξ increases, the maximum fluid velocity decreases. This is due to the fact that the effect of the suction is to take away the warm fluid on the vertical plate and thereby decrease the maximum velocity with a decrease in the intensity of the natural convection rate. Fig. 6(b) shows the effect of the local suction parameter on the temperature profiles. It is noticed that the temperature profiles decrease with an increase in the suction parameter and as the suction is increased, more warm fluid is taken away and this the thermal boundary layer thickness decreases. It is also seen that an increase in β , the impedance offered by the fibres of the porous medium will increase and this will effectively decelerate the flow in the regime, as testified to by the evident decrease in velocities shown in fig. 6(a).

5. CONCLUSION

In this paper, a numerical study has been performed for Magnetohydrodynamicfree convective flow over a vertical plate in the presence of conductive conditions. The non-dimensional form of momentum and energy equations are solved numerically using Keller box implicit method. The following results are obtained.

1. The momentum boundary layer thickness decreases with increasing Casson parameter β , but the Skin friction coefficient increases with β
2. The velocity decreases with an increase of Magnetic Parameter (*M*), Prandtl number (*Pr*), Two dimensional non similarity parameter (ξ) and it increases with an increase of Casson Parameter β, Convective heating (γ)
3. The temperature decreases with an increase of Casson Parameter β, Prandtl number (*Pr*), Two dimensional non similarity parameter (ξ) and it increases with an increase of Magnetic Parameter (*M*), Convective heating (γ).

REFERENCES

1. S. Livescu, Mathematical modeling of thixotropic drilling mud and crude oil flow in wells and pipelinesA review, *J. Petroleum Science and Engineering*, 98/99, 174–184 (2012).
2. J. Hron, J. Málek, P. Pustějovská and K. R. Rajagopal, On the modeling of the synovial fluid, *Advances in Tribology,* Volume 2010 (2010), Article ID 104957, 12 pages. http://dx.doi.org/10.1155/2010/104957.
3. F. Loix, L. Orgéas, C. Geindreau, P. Badel, P. Boisse and J.-F. Bloch, Flow of non-Newtonian liquid polymers through deformed composites reinforcements, *Composites Science and Technology,* 69, 612–619 (2009).
4. Viviane Kechichian, Gabriel P. Crivellari, Jorge A.W. Gut, Carmen C. Tadini, Modeling of continuous thermal processing of a non-Newtonian liquid food under diffusive laminar flow in atubular system, *Int. J. Heat and Mass Transfer*, 55, 5783–5792 (2012).
5. W. L. Wilkinson. The drainage of a Maxwell liquid down a vertical plate. The Chemical Engineering Journal. Vol. 1, pp. 255–257(1970).
6. K. R. Rajagopal. Viscometric flows of third grade fluids. Mechanics Research Communications. Vol. 7, no. 1, pp. 21–25 (1980).

7. K. R. Rajagopal, T. Y. Na and A. S. Gupta. Flow of a viscoelastic fluid over a stretching sheet, RheologicaActa. Vol. 23, pp. 213–215 (1984).
8. C. Dorier and J. Tichy. Behavior of a Bingham-like viscous fluid in lubrication flows. Journal of Non-Newtonian Fluid Mechanics. Vol. 45, pp. 291–310 (1992).
9. N. Casson. Rheology of dispersed system. Oxford Pergamon Press. 84 (1959).
10. K. Hiemenz Die Grenzchicht an einem in den Gleichformingen Flussigkeitsstromeingetauchtengeraden Kreiszylinder. Dinglers Polytechnic J. Vol.326, pp. 321–410 (1911).
11. R. K, Dash, K. N. Mehta and G. Jayaraman. Casson fluid flow in a pipe filled with a homogeneous porous medium. Int. J. Eng. Sci. Vol. 34, pp.1145-1156 (1996).
12. D. D. Joye. Hazardous and Industrial Wastes. Proc. 30th Mid-Atlantic Industrial Waste Conf., Technomic Publ. Corp., Lancaster, PA/Basel,pp. 567–575(1998).
13. E. A. Kirsanov and S. V. Remizov. Application of the Casson model to thixotropic waxy crude oil. Rheol. Acta 38, 172–176(1999).
14. R. B. Bird, W. E. Stewart and E. N. Lightfoot. Transport Phenomena, Wiley, New York pp.94–95, 104–105 (1960).
15. Krishnendu Bhattacharyya. Boundary layer stagnation point flow of Casson fluid and heat transfer towards a shrinking/stretching sheet. Frontiers in Heat and Mass Transfer. Vol. 4, pp.1-9 (2013).
16. T. Hayat, S. A. Shehzad, A. Alsaediand M. S. Alhothuali. Mixed Convection Stagnation Point Flow of Casson Fluid with Convective Boundary Conditions. Chin. Phys. Lett. Vol.29, pp.1-4(2012).
17. G. K. Ramesh, B. C. Prasannakumara, B. J.Gireesha and M. M. Rashidi .Casson Fluid Flow near the Stagnation Point over a Stretching Sheet with Variable Thickness and Radiation. Journal of Applied Fluid Mechanics. Vol. 9, pp. 1115-1122 (2016).
18. K. Bhattacharyya, S. Mukhopadhyay, and G. C. Layek. MHD Boundary Layer Slip Flow and Heat Transfer over a Flat Plate.Chin. Phys. Lett. Vol. 28, pp. 1-4(2011).
19. J. Dealy and K. Wissbrun, Melt Rheology and its Role in Plastics Processing: Theory andApplications, Van Nostrand Reinhold, New York (1990).
20. M. Mustafa, T. Hayat, I. Pop and A. Aziz, Unsteady boundary layer flow of a Casson fluiddue to an impulsively started moving flat plate, Heat Transfer-Asian Research, Vol. 40, pp. 563–576(2011).
21. H.B. Keller, Numerical Solution of Two-Point Boundary Value Problems, SIAM Press, Philadelphia, USA (1976).
22. A. Subba Rao, C.H. Amanulla, N. Nagendra,O.A. Bég, A.Kadir. Hydromagnetic Flow and Heat Transfer in a Williamson Non-Newtonian Fluid from a Horizontal Circular Cylinder with Newtonian Heating. Int. J. Appl. Comput. Math. 1-21 (2017) http://dx.doi.org/10.1007/s40819-017-0304-x.
23. A. Subba Rao, VR. Prasad, N. Nagendra, N.B. Reddy, O.A. Bég. Non-Similar Computational Solution for Boundary Layer Flows of Non-Newtonian Fluid from an Inclined Plate with Thermal Slip. J. Applied Fluid Mechanics. 9(2), 795-807 (2016)
24. A. Subba Rao, N. Nagendra, and V.R. Prasad, Heat transfer in a non-Newtonian Jeffrey'sfluid over a non-isothermal wedge, Procedia Eng., Vol. 127, pp. 775-782(2015)., http://dx.doi.org/10.1016/j.proeng.2015.11.412
25. CH. Amanulla, N. Nagendra and M. Suryanarayana Reddy, Heat transfer in MHD Viscoplastic fluid flow from a Vertical Permeable Cone with Convective Heating,i-manager's Journal on Mathematics, Vol. 6, No.1l, 2017
26. L. Howarth. On the solution of the laminar boundary layer equations, Proc. Roy. Soc. London A, Vol. 164, pp. 547-579 (1938).
27. K. Bhattacharyya and G. C.Layek. Similarity solution of MHD boundary layer flow with diffusion and chemical reaction over a porous flat plate with suction/blowing, Meccanica, Vol. 47, pp. 1043-1048 (2012).
28. S. Mukhopadhyay and I.C. Mandal. Magnetohydrodynamic (MHD) mixed convection slip flow and heat transfer over a vertical porous plate, Engineering Science and Technology, an International Journal,18, 98-105 (2015).

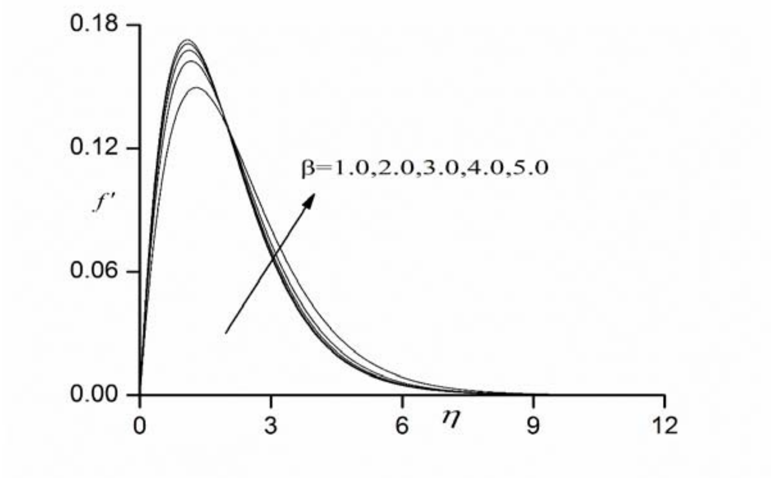

Figure-2a: Velocity profiles for different values of β

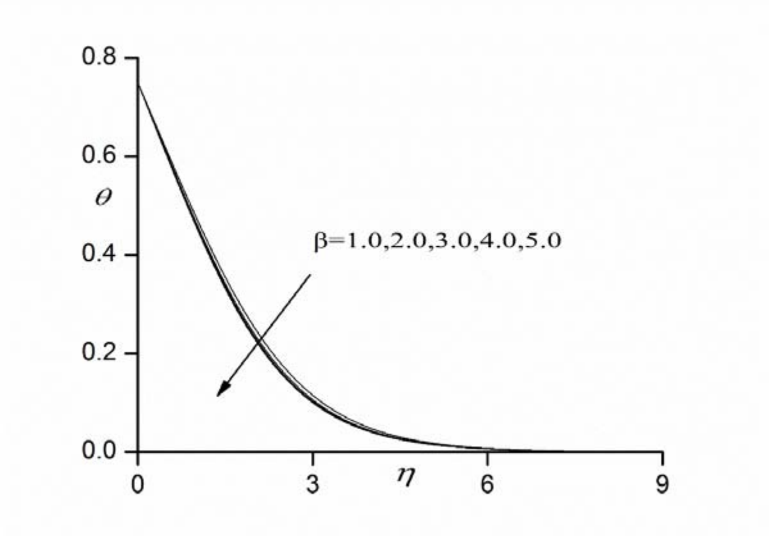

Figure-2b: Temperature profiles for different values of β

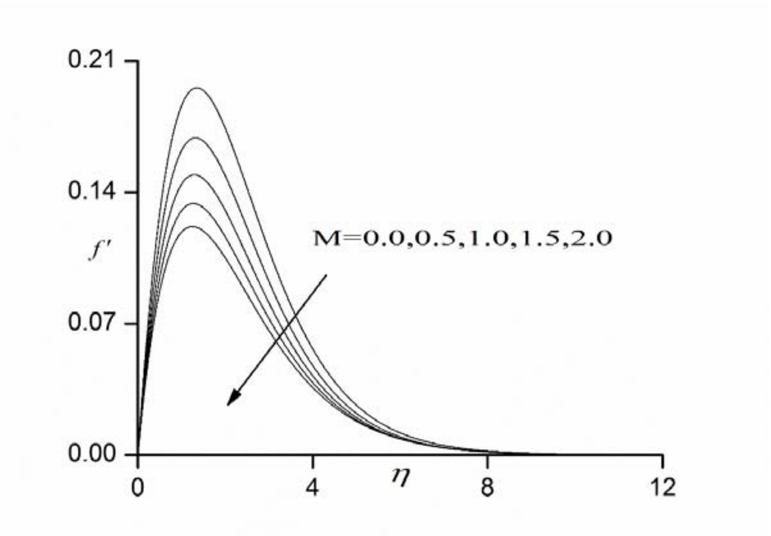

Figure-3a: Velocity profiles for different values of M

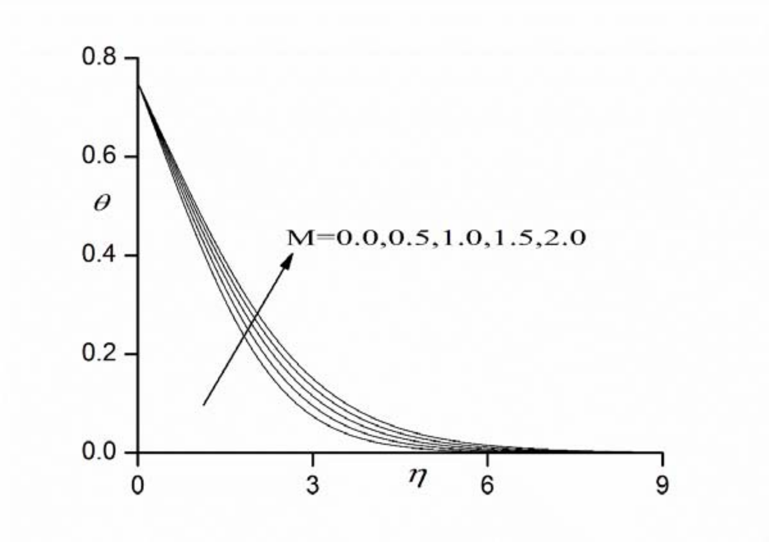

Figure-3b: Temperature profiles for different values of M

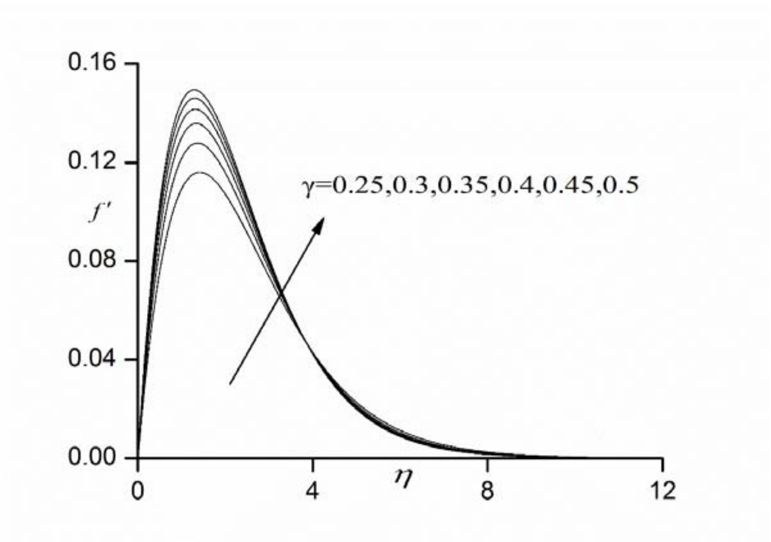

Figure-4a: Velocity profiles for different values of γ

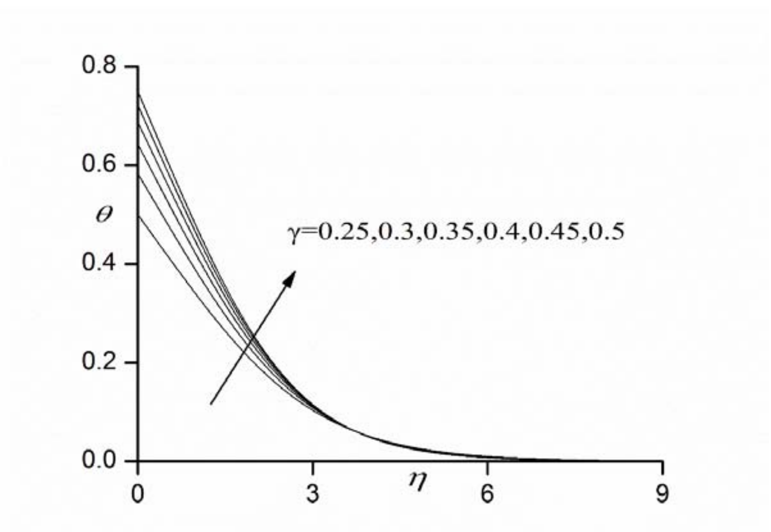

Figure-4b: Temperature profiles for different values of γ

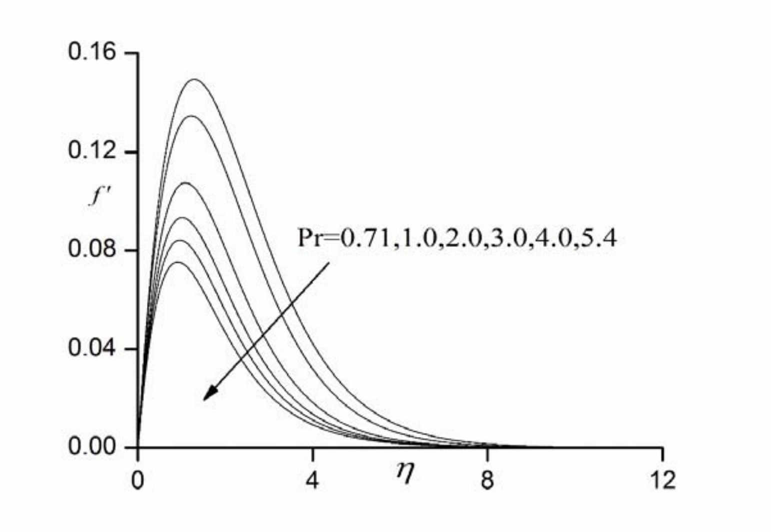

Figure-5a: Velocity profiles for different values of *Pr*

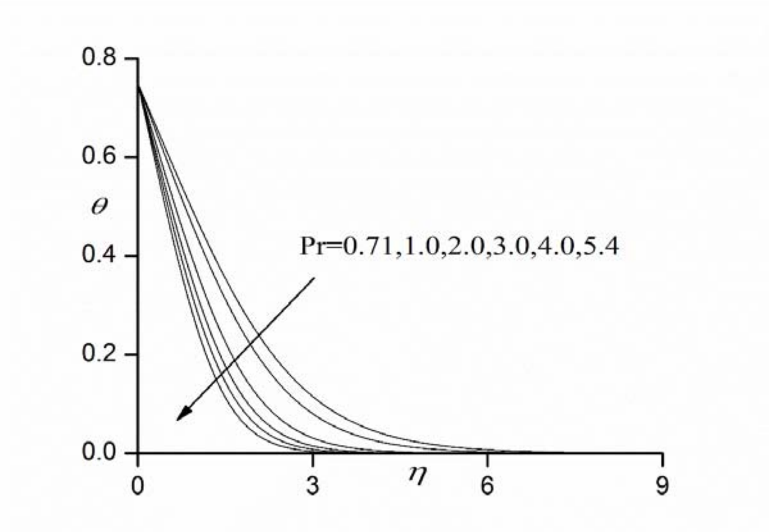

Figure-5b: Temperature profiles for different values of *Pr*

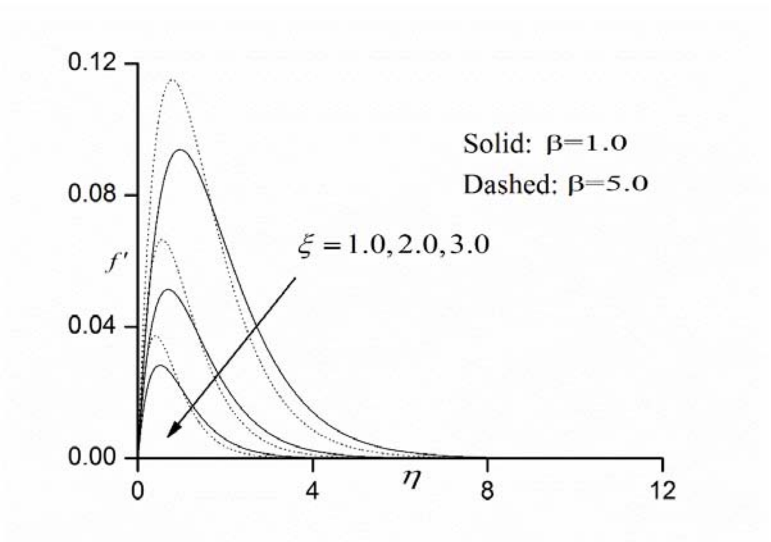

Figure-6a: Velocity profiles for different values of ξ

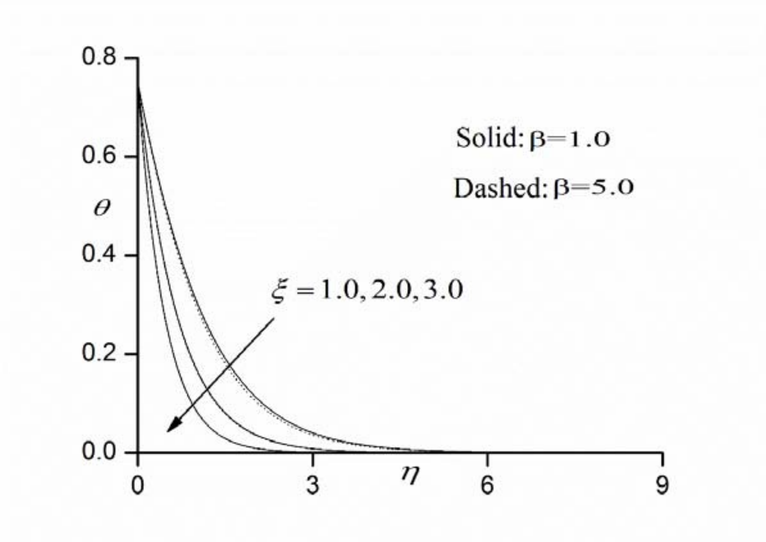

Figure-6b: Temperature profiles for different values of ξ

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