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DECOMPOSITION OF GδS-HOMEOMORPHISM IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce the concept of gδsg-homeomorphisms and study some of their properties. Also the diagram of implications is given.

Keywords: *gδs-closed set, gδs-continuous map, gsc-homeomorphism, sgc-homeomorphism, gsg-homeomorphism, gδsc-homeomorphism, gδsg-homeomorphism.*

1. INTRODUCTION

Levine [8] has generalized the concept of closed sets to generalized closed sets. Devi, Balachandran and Maki [2] defined two classes of maps called semi generalized homeomorphisms and generalized semi homeomorphisms and also defined two classes of maps called sgc-homeomorphisms and gsc-homeomorphisms. In this paper, we introduce the class of maps called gδsg-homeomorphisms and study their properties.

2. PRELIMINARIES

Throughout this paper (X, τ) (or simply X) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), int(A) and Ac denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1 [5]: A subset A of X is called generalized δ-semiclosed (briefly gδs − closed) set if sCl(A)⊆U whenever A ⊆U and U is δ-open in X.

Definition 2.2 [4]: A function f: X → Y is called gδs-continuous if the inverse image of every closed set in Y is gδs-closed set in X.

Definition 2.3 [6]: A function f: X → Y is said to be gδs-closed (resp.gδs-open) if f(V) is gδs-closed (resp.gδs-open) in Y for every closed (resp-open) set V in X.

Definition 2.4 [6]: A function f: X → Y is gδs-irresolute if f^{-1}(V) is gδs-closed in X for every gδs-closed set V in Y

Definition 2.5 [5]: A bijective function f: X → Y is said to be gδs-homoeomorphism if f is both gδs-continuous and gδs-open, equivalently, if f and f^{-1} both are gδs-continuous.

Definition 2.6 [2]: A bijective map f: X → Y is called sgc-homeomorphism if the function f and the inverse function f^{-1} are both sg-irresolute.

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Definition 2.8 [3]: A bijective map f: X → Y is called gsg-homeomorphism if the function f and the inverse function f^{-1} are both gsg-irresolute.

Definition 2.9 [7]: A topological space X is called Tgδs-space if every gδs-closed set in it is closed.

Definition 2.10 [7]: A topological space X is called gδs$T_{1/2}$-space if every gδs-closed set in it is semiclosed.

3. GδSG-homeomorphism

Definition 3.1: A bijective map f: X → Y is called gδsc-homeomorphism if the function f and the inverse function f^{-1} are both gδs-irresolute.

Example 3.2: Let X = Y = {a, b, c}, τ = {X, φ, {a} and σ = {Y, φ, {b} be topologies on X and Y respectively. Define a function f : X → Y by f(a) = a, f(b) = b and f(c) = c. Then f is gδsc-homeomorphism.

Remark 3.3: Every semi-homeomorphism is gδsc-homeomorphism. But converse need not be as shown in the following example.

Example 3.4: Let X = Y = {a, b, c}, τ = {X, φ, {a}} and σ = {Y, φ, {a}, {b}, {a, b}} be topologies on X and Y respectively. Define a function f: X → Y as f(a) = a, f(b) = b and f(c) = c. Then f is gδsc-homeomorphism but not semi-homeomorphism, since for an open set {b} in Y , f^{-1}({b}) = {b} is not open in X.

Remark 3.5: Every homeomorphism is gδsc-homeomorphism. But converse need not be true as shown in the following example.

Example 3.6: Let X = Y = {a, b, c}, τ = {X, φ, {b}, {b, c}} and σ = {Y, φ, {a, b}} be topologies on X and Y respectively. Then the identity function f: X → Y is gδsc-homeomorphism but not homeomorphism, since for an open set {a, b} in Y , f^{-1}({a, b}) = {a, b} is not open in X.

Remark 3.7: Every gsc-homeomorphism is gδsc-homeomorphism. But converse need not be true as shown in the following example.

Example 3.8: Let X = Y = {a, b, c}, τ = {X, φ, {b}, {a, b}, {b, c}} and σ = {Y, φ, {a}, {b}} be topologies on X and Y respectively. Define a function f: X → Y as f(a) = a, f(b) = b and f(c) = c. Then f is gδsc-homeomorphism but not gsc-homeomorphism, since the set {a, b} is gs-closed in Y , but the set f^{-1}({a, b}) = {a, b} is gs-closed in X.

Remark 3.9: Every gδsc-homeomorphism is gδs-homeomorphism. But converse need not be true as shown in the following example.

Example 3.10: Let X = Y = {a, b, c}, τ = {X, φ, {a}, {c}, {a, c}} and σ = {Y, φ, {b}, {a, c}} be topologies on X and Y respectively. Define a function f: X → Y as f(a) = a, f(b) = b and f(c) = c. Then f is gδs-homeomorphism but not gδsc-homeomorphism, since for an open set {b} in Y, f^{-1}({b}) = {b} is not gδs-open in X.

Remark 3.11: Every sgc-homeomorphism is gδsc-homeomorphism. But converse need not be true as shown in the following example.

Example 3.12: Let X = Y = {a, b, c}, τ = {X, φ, {b}, {b, c}} and σ = {Y, φ, {a}, {b}, {a, b}} be topologies on X and Y respectively. Define a function f: X → Y as f(a) = a, f(b) = b and f(c) = c. Then f is gδsc-homeomorphism but not sgc-homeomorphism, since the set {b} is sg-closed in Y, f^{-1}({b}) = {b} is not sg-closed in X.

Theorem 3.13: If f: X → Y and g: Y → Z be two gδsc-homeomorphism functions, then (g ∘ f) is gδsc-homeomorphism.

Proof: Let f: (X, τ) → (Y, σ) be bijective map.

(i)To prove (g ∘ f) is gδs-irresolute map.
Let V be gδs-closed set in Z. Since g is gδs-irresolute map, g^{-1}(V) is gδs-closed set in Y . Since f is gδs-irresolute map, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is gδs-closed set in X. Therefore (g ∘ f) is gδs-irresolute.

(ii)To prove (g ∘ f)−1 is gδs-irresolute map.
Let V be gδs-closed set in X. Since f^{-1} is gδs-irresolute, $(f^{-1})^{-1}(V) = f(V)$ is gδs-closed set in Y . Since g^{-1} is gδs-irresolute $(g^{-1})^{-1}(f(V)) = gf(V) = g \circ f(V)$ is gδs-closed set in Z. Therefore $(g \circ f)^{-1}$ is gδs-irresolute map.
Hence from (i) and (ii) (g ∘ f) is gδsc-homeomorphism.

Definition 3.14: A bijection map f : X → Y is said to be gδsg-irresolute map, if $f^{-1}(A)$ is sg-closed in X for every gδs-closed set A of Y

Example 3.15: Let X = Y = {a, b, c}, τ = {X, φ, {b}, {a, c}} and σ = {Y, φ, {a}, {b, c}} be topologies on X and Y respectively. Define a function f: X → Y as f(a) = b, f(b) = a and f(c) = c. Then f is gδsg-irresolute map.

Definition 3.16: A bijection map f: X → Y is said to be gδsg-homeomorphism if the function f and the inverse function f^{-1} are both gδsg-irresolute maps. If there exists a gδsg-homeomorphism from X to Y, then the spaces (X, τ) and (Y, σ) are said to be gδsg-homeomorphic.

The family of all gδsg-homeomorphism of any topological spaces (X, τ) is denoted by gδsgh(X, τ)

Remark 3.17: The following two examples shows that the concepts of homeomorphism and gδsg-homeomorphism are independent of each other.

Example 3.18: Let X = {a, b, c}, τ = {X, φ, {a}, {a, c}} be topology on X. Define a function f_X : (X, τ) → (X, τ) as $f_X(a) = a$, $f_X(b) = b$ and $f_X(c) = c$. Then f_X is homeomorphism but not gδsg-homeomorphism, since the set {a, b} is gδs-closed but the set f_X^{-1} ({a, b}) = {a, b} is not sg-closed in X.

Example 3.19: Let X be any set which contains at least two elements, τ and σ be discrete and indiscrete topologies on X respectively. The identity map I_X: (X, τ) → (X, σ) is a gδsg-homeomorphism but is not a homeomorphism.

Remark 3.20: Every gδsg-homeomorphism is gsc-homeomorphism. But converse need not be true as shown in the following example.

Example 3.21: Let X = {a, b, c}, τ = {X, φ, {a}, {a, b}} be topologies on X. Define a function f_X : (X, τ) → (X, τ) as $f_X(a) = a$, $f_X(b) = b$ and $f_X(c) = c$. Then f is gsc-homeomorphism but not gδsg-homeomorphism, since the set {a, b} is gδs-closed set but the set f_X^{-1} ({a, b}) = {a, b} is not sg-closed in X.

Theorem 3.22: Every gδsg-homeomorphism is gδsc-homeomorphism.

Proof: Let f: (X, τ) → (Y, σ) be bijective map.

(i)To prove f is gδs-irresolute map.
Let V be a gδs-closed set in Y. Since f is gδs-irresolute map, $f^{-1}(V)$ is sg-closed set in X. We know that, every sg-closed set is gδs-closed set. Therefore, $f^{-1}(V)$ is gδs-closed set in X. Hence f is gδs-irresolute map.

(ii)To prove f^{-1} is gδs-irresolute map.

Let V be a gδs-closed set in X. Since f^{-1} is gδsg-irresolute map, $(f^{-1})^{-1}(V) = f(V)$ is sg-closed set in Y. We know that, every sg-closed set is gδs-closed set.

Therefore, f(V) is gδs-closed set in Y. Hence f^{-1} is gδs-irresolute map.

Hence from (i) and (ii) f is gδsc-homeomorphism.

Remark 3.23: Converse of the above theorem need not be true as shown in the following example.

Example 3.24: Let X = {a, b, c}, τ = {X, φ, {b}, {a, b}} be topology on X. Define a function f_X : (X, τ) → (X, τ) as $f_X(a) = a$, $f_X(b) = b$ and $f_X(c) = c$. Then f_X is gδsc-homeomorphism but not gδsg-homeomorphism, since the set {a, b} is gδs-closed but the set f_X^{-1} ({a, b}) = {a, b} is not sg-closed in X.

Theorem 3.25: Every gδsc-homeomorphism from $g\delta sT_{1/2}$ space onto itself is gδsg-homeomorphism.

Proof: Let f: (X, τ) → (Y, σ) be bijective map.

(i)To prove f is gδsg-irresolute map.
Let V be a gδs-closed set in Y. Since f is gδs-irresolute map, $f^{-1}(V)$ is gδsclosed set in X. Since X is $g\delta sT_{1/2}$ space, $f^{-1}(V)$ is semiclosed in X. We know that every semiclosed set is sg-closed set. Therefore, $f^{-1}(V)$ is sg-closed set in X.
(ii)To prove f^{-1} is gδsg-irresolute map.
Let V be a gδs-closed set in X. Since f^{-1} is gδs-irresolute map, $(f^{-1})^{-1}(V) = f(V)$ is gδs-closed set in Y. Since Y is $g\delta sT_{1/2}$ space, f(V) is semiclosed. We know that, every semiclosed set is sg-closed set. Therefore, f(V) is sg-closed set in Y . Hence from (i) and (ii) f is gδsg − homeomorphism.

Theorem 3.26: Every gδsg-homeomorphism is gδs-homeomorphism.

Proof: Let f : (X, τ) → (Y, σ) be bijective map.

(i)To prove f is gδs-continuous map.
Let V be a closed set in Y. Since f is gδsg-irresolute map, f^{-1}(V) is sg-closed set in X. We know that, every sg-closed set is gδs-closed set. Therefore, f^{-1}(V) is gδs-closed set in X.

(ii)To prove f−1 is gδs-continuous map.
Let V be a closed set in X. We know that every closed set is gδs-closed set. Since f^{-1} is gδsg-irresolute map, $(f^{-1})^{-1}$(V) = f(V) is sg-closed set in Y, but we know that, every sg-closed set is gδs-closed set. Therefore, f(V) is gδs-closed set in Y.

Hence from (i) and (ii) f is gδs-homeomorphism.

Remark 3.27: Converse of the above theorem need not be true as shown in the following example.

Example 3.28: Let X = Y = {a, b, c}, τ = {X, φ, {b}, {c}, {b, c}} and σ = {Y, φ, {a}, {b, c}} be topology on X. Define a function f: (X, τ) → (Y, σ) as f(a) = a, f(b) = b and f(c) = c. Then f is gδs-homeomorphism but not gδsg-homeomorphism, since the set {b, c} is gδs-closed set in Y, but the set f^{-1}({b, c}) = {b, c} is not sg-closed in X.

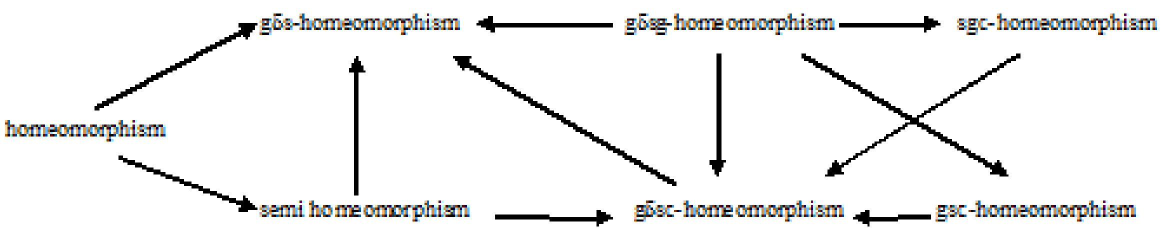
Theorem 3.29: Every gδs-homeomorphism from Tgδs space onto itself is gδsg-homeomorphism.

Proof: Let f : (X, τ) → (Y, σ) be bijective map.

(i)To prove f is gδsg-irresolute map.
Let V be a gδs-closed set in Y. Since f is gδs-continuous map, f^{-1}(V) is gδs-closed set in X. Since X is Tgδs space, f^{-1}(V) is closed in X. We know that every closed set is sg-closed set. Therefore, f^{-1}(V) is sg-closed set in X.

(ii)To prove f^{-1} is gδsg-irresolute map.
Let V be a gδs-closed set in X. Since f^{-1} is gδs-continuous map, $(f^{-1})^{-1}$(V) = f(V) is gδs-closed set in Y. Since Y is gδsT space, f(V) is closed. We know that, every closed set is sg-closed set. Therefore, f(V) is sg-closed set in Y .
Hence from (i) and (ii) f is gδsg-homeomorphism.

Remark 3.30: From the all above statement, we have the following diagram.



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