

PULSATILE FLOW OF BLOOD THROUGH AN INCLINED BELL SHAPE STENOSED TUBE UNDER PERIODIC BODY ACCELERATION WITH MAGNETIC EFFECT AND POROUS MEDIA

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ABSTRACT

This is about the mathematical model for blood flow through an inclined bell shape stenosed tube under periodic body acceleration with magnetic field and porous media. Using appropriate boundary conditions, analytical expressions for the velocity, the volumetric flow rate, the fluid acceleration have been derived. These expressions are computed numerically and the computational results are presented graphically.

Key Words: Pulsatile flow, stenosis, Bell shaped Stenosis, Periodic body acceleration, magnetic field, Inclined tubes, Porous media.

INTRODUCTION

Srivastava [1] studied the two-phase model of blood flow through stenosed tubes in the presence of a peripheral layer. Srivastava and Rastogi [2] studied the blood flow through stenosed catheterized artery: effect of haematocrit and stenosis shape. Srivastava, *et al.*, [3] studied the macroscopic two-phase blood flow through a bell shaped stenosis in an artery with permeable wall. Srivastav, *et al.*, [4] studied the blood flow through an overlapping stenosis in catheterized artery with permeable wall. Srivastav, *et al.*, [5] studied the two-phase model of blood flow through a composite stenosis in the presence of a peripheral layer. Srivastav [6] studied the mathematical model of blood flow through a composite stenosis in catheterized artery with permeable wall. Srivastava, *et al.*, [7] studied the response of composite stenosis to non-Newtonian blood in arteries. Srivastav and Srivastava [8] studied On two-fluid blood flow through stenosed artery with permeable wall. Tzirtzilakis [9] studied the biomagnetic fluid flow in a channel with stenosis. Young and Tsai [10] studied the flow characteristics in model of arterial stenosis-steady flow. Young [11] studied the effects of a time-dependent stenosis of flow through a tube. Srivastava [12] studied the flow of a couple stress fluid representing blood through stenotic vessels with a peripheral layer. Srivastava [13] studied the particulate suspension blood flow through stenotic arteries: Effect of hematocrit and stenosis shape. Gupta and Gupta [14] studied the unsteady blood flow in an artery through a nonsymmetrical stenosis. Paolo and Christopher [15] studied the boundary layer model for wall shear stress in arterial stenosis. Farzan and Xiaoyan [16] studied the turbulence detection in a stenosed artery bifurcation by numerical simulation of pulsatile blood flow using the low-Reynolds number turbulence model. Musad, *et al.*, [17] studied the effect of paired stenosis on blood flow through small artery. Kumar, *et al.*, [18] studied the effect of porous parameter for the blood flow in a time dependent stenotic artery. Verma [19] studied the mathematical model of unsteady blood flow through a narrow tapered vessel with stenosis. Wootton, *et al.*, [20] studied the mechanistic model of acute platelet accumulation in thrombogenic stenosis.

MATHEMATICAL FORMULATION:

Let us consider a one-dimensional pulsatile flow of blood through a uniform straight and stenosed inclined cylindrical tube by applying magnetic field and porous media with magnetic field and porous medium by considering blood as a couple stress fluid. The flow is considered as axially symmetric pulsatile and fully developed. The pressure gradient and body acceleration G are given by the expressions

$$-\frac{\partial P}{\partial z} = A_0 + A_1 \cos(\omega_p t) \tag{1}$$

$$G = a_0 \cos(\omega_b t + \phi) \tag{2}$$

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where A_0 and A_1 are pressure gradient of steady flow and amplitude of oscillatory part respectively, a_0 is the amplitude of body acceleration, $\omega_p = 2\pi f_p$, $\omega_b = 2\pi f_b$ with f_p is the pulse frequency and f_b is body acceleration frequency, ϕ is the phase angle of body acceleration G with respect to pressure gradient and time, t .

The pulsatile couple stress equation (Stokes), in cylindrical polar coordinates in an inclined tube under the periodic body acceleration with magnetic field and porous medium can be written in the form:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \rho G + \mu \nabla^2 u - \eta \nabla^2 (\nabla^2 u) + \rho g \sin \theta - \sigma B_0^2 u - \frac{\mu}{k} u \quad (3)$$

where
$$\nabla^2 = \frac{1}{\xi} \left(\frac{\partial}{\partial \xi} \left(\xi \frac{\partial}{\partial \xi} \right) \right)$$

Where $u(\xi, t)$ is axial velocity, ρ and μ are the density and viscosity of blood, η is the couple stress parameter and ξ is the radial coordinate.

Let us introduce the following dimensionless quantities:

$$u^* = \frac{u}{\omega R}, t^* = t\omega, A_0^* = \frac{R}{\mu\omega} A_0, A_1^* = \frac{R}{\mu\omega} A_1, a_0^* = \frac{\rho R}{\mu\omega} a_0, z^* = \frac{z}{R}, g^* = \frac{\rho R}{\mu\omega} g, k^* = \frac{k}{R^2} \quad (4)$$

Consider the axi symmetric flow of blood through a bell shaped stenosis, specified at the location as shown in Figure 1, in an artery. The geometry of the stenosis, assumed to be manifested in the arterial wall segment, is described (Srivastava *et al.* 2012) as

$$\frac{R(z)}{R_0} = \begin{cases} 1 - \frac{\delta}{R_0} \exp\left(\frac{-\omega^2 \varepsilon^2 z^2}{R_0^2}\right); & |z| \leq L_0 \\ 1; & \text{otherwise} \end{cases} \quad (5)$$

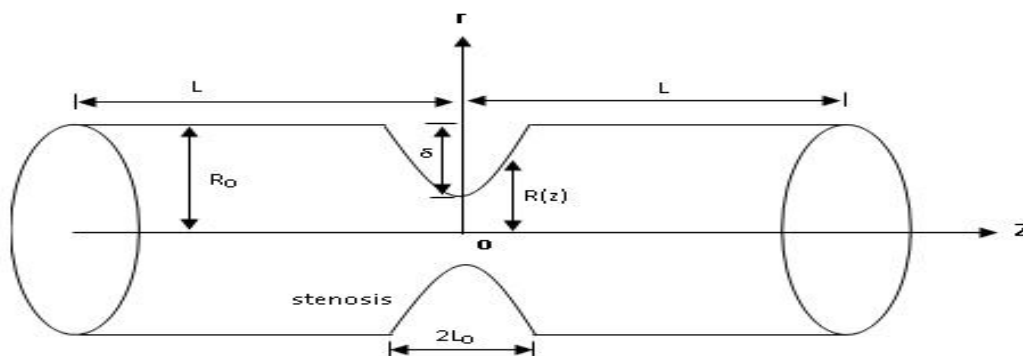


Figure-1: The geometry of a bell shaped stenosis in an artery

where R_0 is the radius of the arterial segment in the non-stenotic region, $R(z)$ is the radius of the stenosed portion located at the axial distance z from the left end of the segment, δ is the depth of stenosis at the throat and, ω is a parametric constant, ε is the relative length of the constriction, defined as the ratio of the radius to half length of the stenosis, i.e., $\varepsilon = R_0 / L_0$

Let us introduce a radial coordinate transformation given by:

$$\xi = \frac{r}{R(z)}$$

In terms of these variables, equation (3) [after dropping stars] becomes

$$\bar{\alpha}^2 \alpha^2 \frac{\partial u}{\partial t} = \bar{\alpha}^2 A_0 + \bar{\alpha}^2 A_1 \cos(bt) + \bar{\alpha}^2 a_0 \cos(ct + \phi) + \bar{\alpha}^2 \left(\frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial u}{\partial \xi} \right) \right) - \left(\frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial}{\partial \xi} \right) \right) \left(\frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial u}{\partial \xi} \right) \right) + \bar{\alpha}^2 g \sin \theta - \bar{\alpha}^2 H^2 u - \bar{\alpha}^2 h^2 u \quad (6)$$

Where $\bar{\alpha}^2 = R^2 \left(\frac{\mu}{\eta} \right)$ -- Couple stress parameter, $\alpha^2 = R^2 \left(\frac{\omega \rho}{\mu} \right)$ -- Womersley parameter

$b = \frac{\omega_1}{\omega}$, $c = \frac{\omega_2}{\omega}$, $H^2 = B_0^2 R^2 \left(\frac{\sigma}{\mu} \right)$ -- Hartman number,

$h^2 = \frac{1}{k}$ -- permeability of the porous media and R -- radius of the tube.

The initial and boundary conditions for this problem are:

$$u(\xi, 0) = 2 \sum_{n=1}^{\infty} \frac{J_0(\xi \lambda_n) \bar{\alpha}^2}{\lambda_n J_1(\lambda_n)} \cdot \frac{[A_0 + A_1 + a_0 \cos \Phi + g \sin \theta]}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \quad (7a)$$

$$u \text{ and } \nabla^2 u \text{ are all finite at } \xi = 0 \quad (7b)$$

$$u = 0, \nabla^2 u = 0 \quad \text{at } \xi = 1 \quad (7c)$$

INTEGRAL TRANSFORMS: If $f(\xi)$ satisfies Dirichlet conditions in closed interval (0,1) then it's finite Hankel transform, Sneddon is defined as

$$f^*(\lambda_n) = \int_0^1 \xi f(\xi) J_0(\xi \lambda_n) \quad (8)$$

where λ_n are the roots of $J_0(\lambda_n) = 0$. Then at each point of the interval at which $f(\xi)$ is continuous:

$$f(\xi) = 2 \sum_{n=1}^{\infty} f^*(\lambda_n) \frac{J_0(\xi \lambda_n)}{J_1^2(\lambda_n)}, \quad (9)$$

Where the sum is taken over all positive roots of $J_0(\xi) = 0$, J_0 and J_1 are Bessel functions of first kind.

The Laplace transform of any function is defined as:

$$\bar{f}(s) = \int_0^{\infty} e^{-st} f(t) dt, \quad s > 0 \quad (10)$$

SOLUTIONS: Employing the Laplace transforms (10) to equation (6) in the light (7b) we get:

$$\begin{aligned} \bar{\alpha}^2 \alpha^2 (s\bar{u} - u(\xi, 0)) &= \bar{\alpha}^2 \frac{A_0}{s} + \bar{\alpha}^2 \frac{A_1 s}{s^2 + b^2} + \bar{\alpha}^2 \left(\frac{a_0 (s \cos \varphi - c \sin \varphi)}{s^2 + c^2} \right) + \bar{\alpha}^2 \left(\frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial u}{\partial \xi} \right) \right) \\ &\quad - \left(\frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial}{\partial \xi} \right) \right) \cdot \left(\frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial \bar{u}}{\partial \xi} \right) \right) + \bar{\alpha}^2 g \sin \theta - \bar{\alpha}^2 H^2 u - \bar{\alpha}^2 h^2 u \end{aligned} \quad (11)$$

where

$$\bar{u}(\xi, s) = \int_0^{\infty} e^{-st} u(\xi, t) dt \quad (12)$$

Now applying the finite Hankel transform (8) to (11) and using (7c) we obtain:

$$\begin{aligned} \bar{u}(\lambda_n, s) &= \frac{J_1(\xi \lambda_n) \bar{\alpha}^2}{\lambda_n} \left[\frac{A_0}{s} + \frac{A_1 s}{s^2 + b^2} + \frac{a_0 (s \cos \varphi - c \sin \varphi)}{s^2 + c^2} + \frac{g \sin \theta}{s} + m \sum_{n=1}^{\infty} \frac{[A_0 + A_1 + a_0 \cos \varphi + g \sin \theta]}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \right] \\ &\quad \times \frac{1}{[sm + \lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \end{aligned}$$

Where $m = \bar{\alpha}^2 \cdot \alpha^2$

Now rearranging the terms for taking the inverse Laplace transform,

$$\begin{aligned} \bar{u}^*(\lambda_n, s) = & \frac{J_1(\xi\lambda_n)\bar{\alpha}^2}{\lambda_n} \left[\frac{A_0}{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]} \left(\frac{1}{s} - \frac{1}{s+h} \right) \right. \\ & + \frac{A_1[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]}{\{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]^2 + b^2m^2\}} \left\{ -\frac{1}{s+h} + \frac{s}{s^2 + b^2} + \frac{b^2m}{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)](s^2 + b^2)} \right\} \\ & + \frac{a_0[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)] \cos \phi}{\{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]^2 + c^2m^2\}} \left\{ -\frac{1}{s+h} + \frac{s}{s^2 + c^2} + \frac{c^2m}{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)](s^2 + b^2)} \right\} \\ & - \frac{a_0 c m \sin \phi}{\{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]^2 + c^2m^2\}} \left\{ \frac{1}{s+h} - \frac{s}{s^2 + c^2} + \frac{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]}{m(s^2 + c^2)} \right\} \\ & + \frac{g \sin \theta}{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]} \left\{ \frac{1}{s} - \frac{1}{s+h} \right\} + \sum_{n=1}^{\infty} \frac{[A_0 + A_1 + a_0 \cos \phi + g \sin \theta]}{(s+h)[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]} \end{aligned} \tag{13}$$

where
$$h_1 = \frac{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]}{m} \tag{14}$$

Now taking the inverse Laplace transform of (13) gives:

$$\begin{aligned} u^*(\lambda_n) = & \frac{J_1(\xi\lambda_n)\bar{\alpha}^2}{\lambda_n} \left\{ \frac{A_0}{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]} \right. \\ & + \frac{A_1[\{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)] \cos(bt) + m \sin(bt)\}]}{\{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]^2 + b^2m^2\}} \\ & + \frac{a_0[\{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)] \cos(ct + \phi) + cm \sin(ct + \phi)\}]}{\{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]^2 + c^2m^2\}} + \frac{g \sin \theta}{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]} \Big\} \\ & - e^{-ht} \left\{ \frac{A_0}{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]} + \frac{A_1[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]}{\{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]^2 + b^2m^2\}} \right. \\ & + \frac{a_0[\{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)] \cos \phi + cm \sin \phi\}]}{\{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]^2 + c^2m^2\}} + \frac{g \sin \theta}{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]} \\ & \left. - \sum_{n=1}^{\infty} \frac{(A_0 + A_1 + a_0 \cos \phi + g \sin \theta)}{[\lambda_n^2(\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2(H^2 + h^2)]} \right\} \end{aligned} \tag{15}$$

The finite Hankel inversion of (15) gives the final solution as:

$$u(\xi, t) = 2 \sum u^*(\lambda_n) \frac{J_0(\xi\lambda_n)}{J_1^2(\lambda_n)}$$

$$\begin{aligned}
 u(\xi, t) = & 2 \sum_{n=1}^{\infty} \frac{A_0 J_0(\xi \lambda_n) \bar{\alpha}^2}{\lambda_n J_1(\lambda_n)} \left[\left\{ \frac{1}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \right. \right. \\
 & + \frac{A_1/A_0 [\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\} \cos(bt) + m \sin(bt)]}{\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\}^2 + b^2 m^2} \\
 & + \frac{a_0/A_0 [\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\} \cos(ct + \varphi) + c m \sin(ct + \varphi)]}{[\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\}^2 + c^2 m^2]} \\
 & + \left. \frac{(1/A_0) g \sin \theta}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \right\} \\
 & - e^{-ht} \left\{ \left\{ \frac{1}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \right. \right. \\
 & + \frac{A_1/A_0 [\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]}{\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\}^2 + b^2 m^2} \\
 & + \frac{a_0/A_0 [\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\} \cos \varphi + c m \sin \varphi]}{\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\}^2 + c^2 m^2} \\
 & + \left. \frac{(1/A_0) g \sin \theta}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \right\} \\
 & - \left. \frac{[1 + (A_1/A_0) + (a_0/A_0) \cos \varphi + (1/A_0) g \sin \theta]}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \right\}] \quad (16)
 \end{aligned}$$

The expression for the flow rate Q can be written as:

$$Q = 2\pi \int_0^{\xi} \xi u \, d\xi \quad (17)$$

Then

$$\begin{aligned}
 Q(\xi, t) = & 4\pi \sum_{n=1}^{\infty} \frac{A_0 \bar{\alpha}^2}{\lambda_n^2} \left[\left\{ \frac{1}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \right. \right. \\
 & + \frac{A_1/A_0 [\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\} \cos(bt) + m \sin(bt)]}{\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\}^2 + b^2 m^2} \\
 & + \frac{a_0/A_0 [\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\} \cos(ct + \varphi) + c m \sin(ct + \varphi)]}{\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\}^2 + c^2 m^2} \\
 & + \left. \frac{(1/A_0) g \sin \theta}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \right\} \\
 & - e^{-ht} \left\{ \frac{1}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \right. \\
 & + \frac{A_1/A_0 [\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]}{\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\}^2 + b^2 m^2} \\
 & + \left. \frac{a_0/A_0 [\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\} \cos \varphi + c m \sin \varphi]}{\{\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)\}^2 + c^2 m^2} \right\}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{a_0/A_0 \{ [\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)] \cos \varphi + c m \sin \varphi \}}{\{ \lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2) \}^2 + c^2 m^2} \\
 & + \frac{(1/A_0) g \sin \theta}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \\
 & - \frac{[1 + (A_1/A_0) + (a_0/A_0) \cos \varphi + (1/A_0) g \sin \theta]}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \}] \quad (18)
 \end{aligned}$$

Similarly the expression for fluid acceleration F can be obtained from:

$$F(\xi, t) = \frac{\partial u}{\partial t} \quad (19)$$

Then we have

$$\begin{aligned}
 F(\xi, t) = & 2 \sum_{n=1}^{\infty} \frac{A_0 J_0(\xi \lambda_n) \bar{\alpha}^2}{\lambda_n J_1(\lambda_n)} \left[\left\{ \frac{A_1/A_0 [b m \cos (bt) - \{ \lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2) \} b \sin (bt)]}{\{ \lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2) \}^2 + b^2 m^2} \right. \right. \\
 & + \frac{a_0/A_0 c [c m \cos (ct + \varphi) - \{ \lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2) \} \sin (ct + \varphi)]}{\{ \lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2) \}^2 + c^2 m^2} \\
 & \left. + \frac{(1/A_0) g \sin \theta}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \right\} \\
 & + h e^{-ht} \left\{ \frac{1}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} + \frac{A_1/A_0 [\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]}{\{ \lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2) \}^2 + b^2 m^2} \right. \\
 & + \frac{a_0/A_0 [\{ \lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2) \} \cos \varphi + c m \sin \varphi]}{\{ \lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2) \}^2 + c^2 m^2} + \frac{(1/A_0) g \sin \theta}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \\
 & \left. - \frac{[1 + (A_1/A_0) + (a_0/A_0) \cos \varphi + (1/A_0) g \sin \theta]}{[\lambda_n^2 (\lambda_n^2 + \bar{\alpha}^2) + \bar{\alpha}^2 (H^2 + h^2)]} \right\} \quad (20)
 \end{aligned}$$

RESULTS

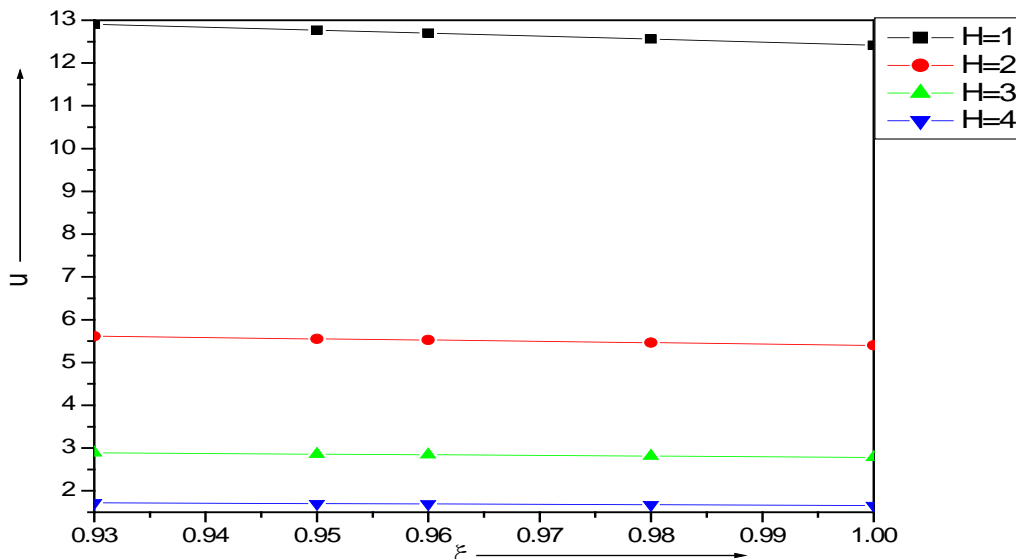


Fig.2. Variation of velocity profiles for different δ/R_0 against ξ with $\phi=30^\circ, \theta=40^\circ, t=1, g=9.8, k=0.1$

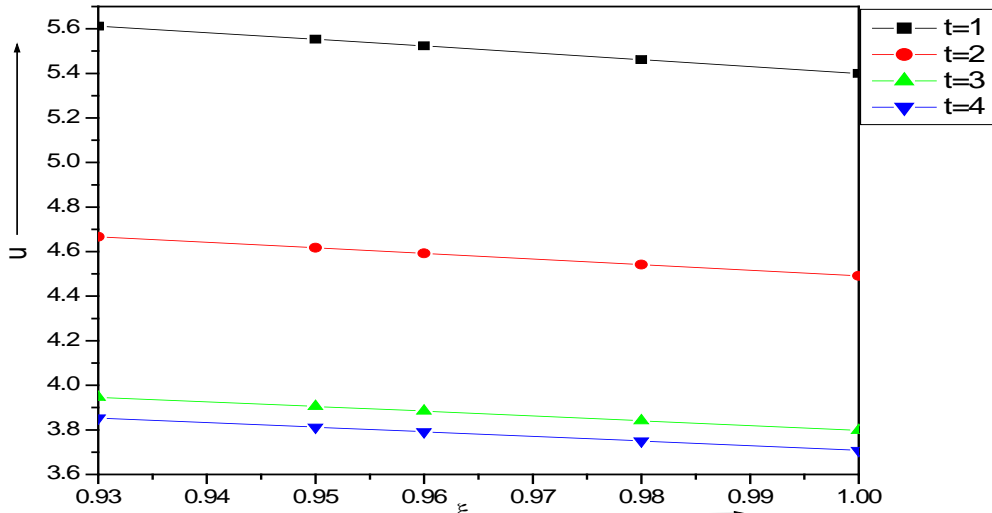


Fig.3.Variation of velocity profiles for different δ/R_0 against ξ with $\phi=30^\circ, \theta=40^\circ, H=2, g=9.8, k=0.1$

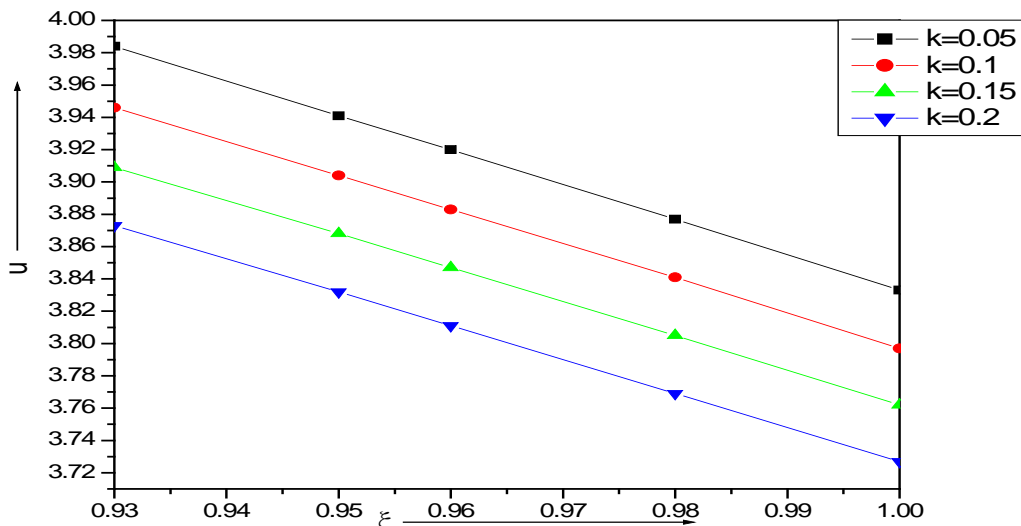


Fig.4.Variation of velocity profiles for different δ/R_0 against ξ with $\phi=30^\circ, \theta=40^\circ, H=2, g=9.8, t=3$

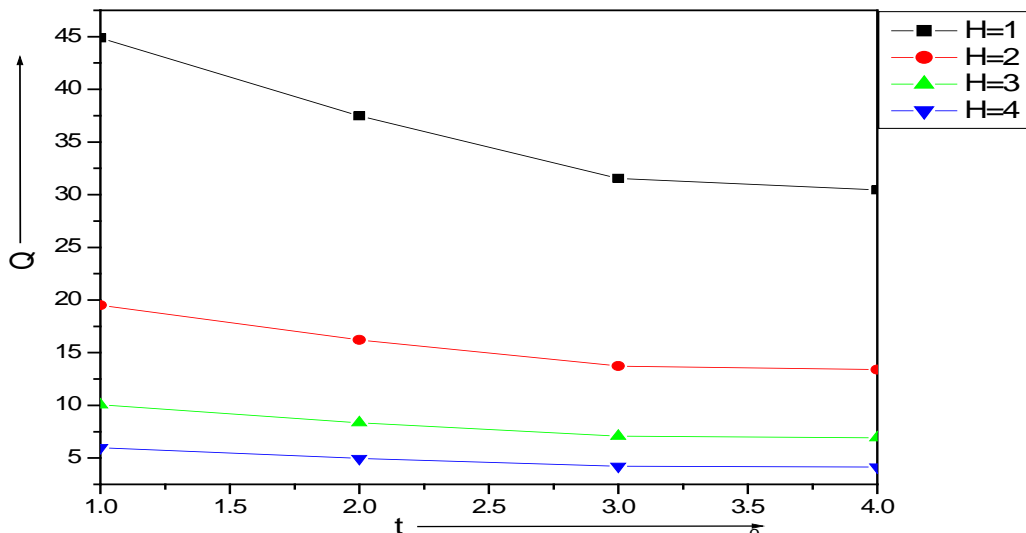


Fig.5.Variation of flow rate Q against t with $\phi=30^\circ, \theta=40^\circ, k=0.1, g=9.8$

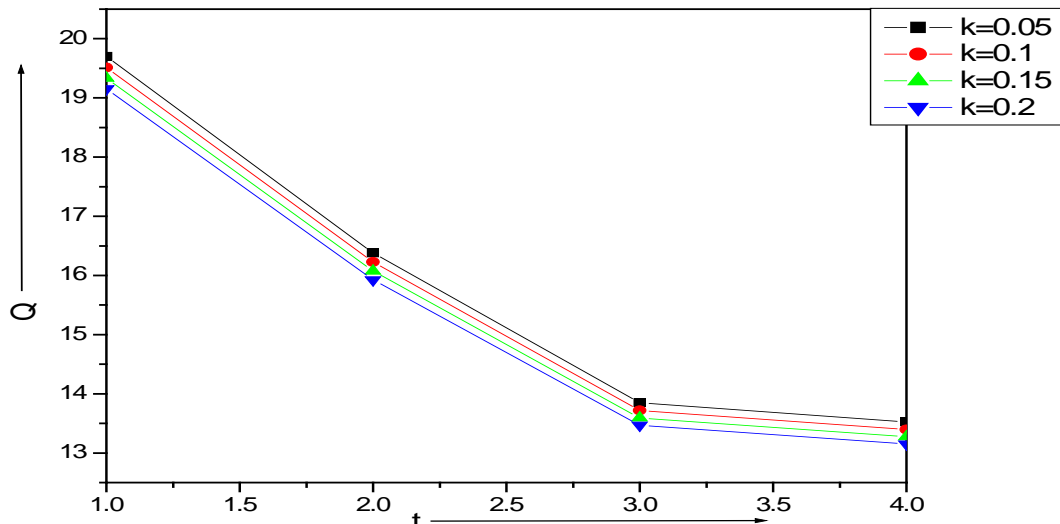


Fig.6.Variation of flow rate Q against t with $\phi=30^\circ$, $\theta=40^\circ$, $H=2$, $g=9.8$

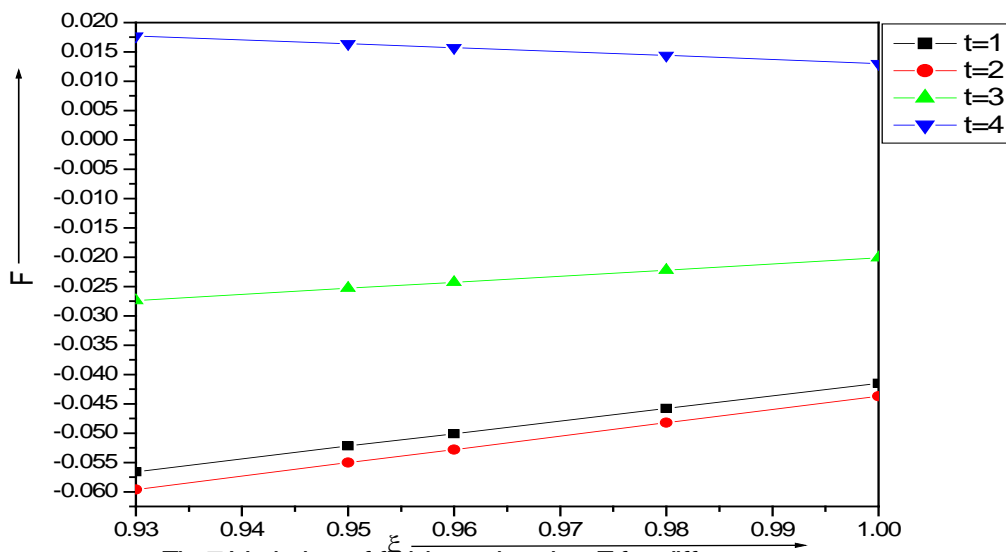


Fig.7.Variation of fluid acceleration F for different δ/R_0 against ξ with $\phi=30^\circ$, $\theta=40^\circ$, $H=2$, $g=9.8$, $k=0.1$

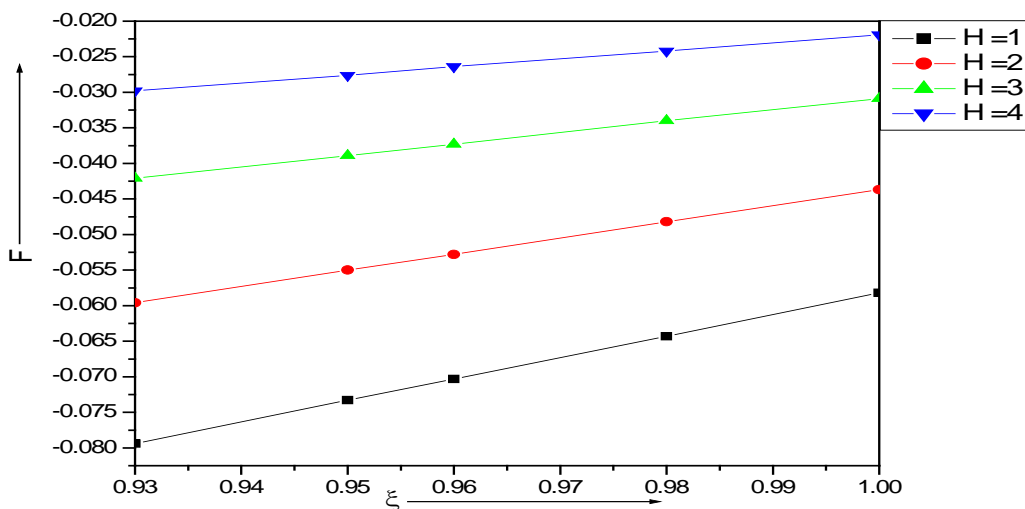


Fig.8.Variation of fluid acceleration F for different δ/R_0 against ξ with $\phi=30^\circ$, $\theta=40^\circ$, $t=2$, $g=9.8$

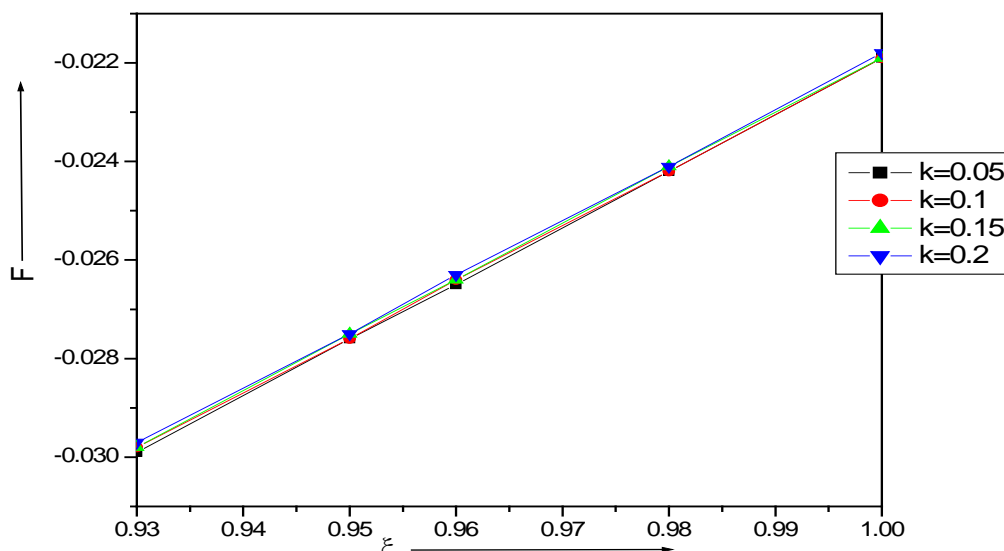


Fig.9.Variation of fluid acceleration F for different δ/R_0 against ξ with $\phi=30^\circ, \theta=40^\circ, t=2, H=4, g=9.8$

In Fig 2, velocity u versus ξ for different H is plotted. Velocity u decreases with increase in ξ and as H increases u decreases.

In Fig 3, velocity u versus ξ for different time t is plotted. It is observed that velocity u decreases with increase in ξ . As t increases, velocity u decreases.

In Fig 4, velocity u versus ξ for different k is plotted. Velocity u decreases with increase in ξ and as k increases u decreases.

In Fig 5, flow rate Q versus ξ for different H is plotted. As H increases, flow rate Q decreases and as t increases, Q decreases.

In Fig 6, flow rate Q versus ξ for different k is plotted. As t increases, flow rate Q decreases and as k increases, Q decreases.

In Fig 7, fluid acceleration F versus ξ for different time t is plotted. As ξ increases, F increases. Also, as t increases, F increases.

In Fig 8, fluid acceleration F versus ξ for different time H is plotted. As ξ increases, F increases and as H increases, F increases.

In Fig 9, fluid acceleration F versus ξ for different time k is plotted. As ξ increases, F increases and as k increases, F increases (slightly).

REFERENCES

- [1] Srivastava, V. P. (1996). "Two-phase model of blood flow through stenosed tubes in the presence of a peripheral layer", *J. Biomech.* 29, 1377-1382, 1996.
- [2] Srivastava, V. P. and Rastogi, R. (2010). "Blood flow through stenosed catheterized artery: effect of haematocrit and stenosis shape", *Comput. Math. Appl.* 59, 1377-1385.
- [3] Srivastava, V. P., Tandon, Mala, and Srivastav, Rupesh K., (2012). "A macroscopic two-phase blood flow through a bell shaped stenosis in an artery with permeable wall", *Appl. Appl. Math.* 7(1), pp.37-51.
- [4] Srivastav, R. K., Ahmad, Q. S. and Khan A. W. (2013a). "Blood flow through an overlapping stenosis in catheterized artery with permeable wall", *e-Journal of Science and Technology*, 8(2), 43-53.
- [5] Srivastav, R. K., Ahmad, Q. S. and Khan, A. W. (2013b). "Two-phase model of blood flow through a composite stenosis in the presence of a peripheral layer", *Journal of Multidisciplinary Scientific Research*, 1(5), 39-45.
- [6] Srivastav, R. K., (2014a). "Mathematical model of blood flow through a composite stenosis in catheterized artery with permeable wall", *Appl. Appl. Math.*, Vol. 9(1), 58-74.

- [7] Srivastava, V. P., Vishnoi, Rochana and Sinha, Poonam (2012a). Response of composite stenosis to non-Newtonian blood in arteries, e- jst, 7(2).
- [8] Srivastav, R. K., Srivastava, V. P. (2014b). "On two-fluid blood flow through stenosed artery with permeable wall", Appl. Bionics and Biomechanics, 11, 39-45.
- [9] Tzirtzilakis, E. E. (2008). "Biomagnetic fluid flow in a channel with stenosis", Physica D. 237, 66-81.
- [10] Young, D. F. and Tsai F. Y. (1973). "Flow characteristics in model of arterial stenosis-steady flow", Journal of Biomechanics, 6, pp. 395-410.
- [11] Young, D. F. (1968). "Effects of a time-dependent stenosis of flow through a tube", Journal of Eng. Ind., 90, pp. 248-254
- [12] Srivastava, V.P. Flow of a couple stress fluid representing blood through stenotic vessels with a peripheral layer. Indian J. pure appl. Math. 34 (12), p. 1727-1740, 2003.
- [13] Srivastava, V.P. Particulate suspension blood flow through stenotic arteries : Effect of hematocrit and stenosis shape. Indian J. pure appl. Math. 33(9), p. 1353-1360, 2002.
- [14] Gupta A.K. and Gupta, G.D. Unsteady blood flow in an artery through a nonsymmetrical stenosis. Acta Ciencia Indica, Vol. XXVII M, No. 2, p. 137-142, 2001.
- [15] Paolo P. Provenzano and Christopher J. Rutland, A boundary layer model for wall shear stress in arterial stenosis. Biorheology 39, p. 743-754, 2002.
- [16] Farzan Ghalichi and Xiaoyan Deng. Turbulence detection in a stenosed artery bifurcation by numerical simulation of pulsatile blood flow using the low-Reynolds number turbulence model. Biorheology 40, p. 637-654, 2003.
- [17] Musad Mohammed Musad Saleh and Mear Yaseen Ali Khan. Effect of paired stenosis on blood flow through small artery. Journal of Mathematics Research, 3(2), 224-229, 2011.
- [18] Kumar, Sanjeev and Dixit Archana. Effect of porous parameter for the blood flow in a time dependent stenotic artery. Indian Journal of Biomechanics: Special Issue (NCBM 7-8), 22-25, 2009.
- [19] Verma, S.R. Mathematical model of unsteady blood flow through a narrow tapered vessel with stneosis. Jour. PAS, Vol. 17, 96-108, 2011.
- [20] Wootton, D.M., Mar Kou, C.P., Hanson, S.R. and Ku, D.N. A mechanistic model of acute platelet accumulation in thrombogenic stenosis. Ann. Biom. Eng. 29, p. 321-329, 2001

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