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A RELAXED ABSOLUTE DIVISOR CORDIAL GRAPHS

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ABSTRACT

A relaxed absolute divisor cordial labeling of a graph G with vertex set V is a bijection from V to $\{-1, 0, 1\}$ *such that each edge uv is assigned the label 1 if* $|f(u) - f(v)|$ *is even, otherwise 0 with the condition that* $|e_f(0) - e_f(1)| \le 1$*. The graph that admits a relaxed absolute divisor cordial labeling is called a relaxed absolute divisor cordial graph. In this paper, we prove some standard graphs such as path, cycle, wheel, star, and bistar are relaxed absolute divisor cordial graphs.*

Keywords: *Relaxed cordial labeling, Relaxed cordial graph, Divisor cordial graph.*

AMS Subject classification: *05C78.*

1. INTRODUCTION

All graphs considered here are finite, simple and undirected. Gallian [1] has given a dynamic survey of graph labeling. For graph theoretic terminologies and notations we follow Harary [2]. The origin of graph labeling can be attributed to Rosa [3]. A path related relaxed cordial graphs were introduced by Dr.A.Nellai Murugan and R.Megala [4]. This definition motivates us to define a Relaxed absolute divisor cordial labeling of a graph and we prove some standard graphs such as path, cycle, wheel, star and bistar are relaxed absolute divisor cordial graphs.

2. PRELIMINARIES

Definition 2.1: Let $G = (V(G),E(G))$ be a simple graph and $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ be a bijection. For each edge uv, assign the label 1if either $f(u) \mid f(v)$ or $f(v) \mid f(u)$ and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \le 1$. A graph which admits a divisor cordial labeling is called a divisor cordial graph.

Definition 2.2: Let $G = (V, E)$ be a graph with p vertices and q edges. A Relaxed Cordial labeling of a graph G with vertex set V is bijection from V to $\{-1, 0, 1\}$ such that each edge uv is assigned the label 1 if $|f(u)+f(v)| = 1$or 0 if $|f(u) + f(v)| = 0$ with the condition that $|e_f(0) - e_f(1)| \le 1$.

Definition 2.3: A Relaxed absolute divisor cordial labeling of a graph G with vertex set V is a bijection from V to $\{-1, 0, 1\}$ such that each edge uv is assigned the label 1 if $|f(u) - f(v)|$ is even, otherwise 0 with the condition that $|e_f(0) - e_f(1)| \le 1$. The graph that admits a relaxed absolute divisor cordial labeling is called a relaxed absolute divisor cordial graph.

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3. MAIN RESULTS

Theorem 3.1: Path P_n is a relaxed absolute divisor cordial graph.

Proof: Let $V(P_n) = \{u_i : 1 \le i \le n\}$ and $E(P_n) = \{u_iu_{i+1} : 1 \le i \le n-1\}$. Then $|V(P_n)| = n$ and $|E(P_n)| = n - 1$.

Define $f: V(P_n) \to \{-1, 0, 1\}$ by

$$f(u_i) = \begin{cases} 0 & i \equiv 1 \pmod 4 \\ -1 & i \equiv 0 \pmod 2 \\ 1 & i \equiv 3 \pmod 4 \end{cases} \qquad 1 \le i \le n$$

The induced edge labeling are

$$f^*(u_iu_{i+1}) = \begin{cases} 0 & i \equiv 0,1 \pmod 4 \\ 1 & i \equiv 2,3 \pmod 2 \end{cases} \qquad 1 \le i \le n\text{-}1$$

Here, $e_f(1) = e_f(0)$ for $n \equiv 1, 3 \pmod 4$
$e_f(1) = e_f(0) + 1$ for $n \equiv 0 \pmod 4$
$e_f(0) = e_f(1) + 1$ for $n \equiv 2 \pmod 4$

Therefore, path P_n satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, the path P_n is a relaxed absolute divisor cordial graph.

Example 3.2: Consider the graph P_6

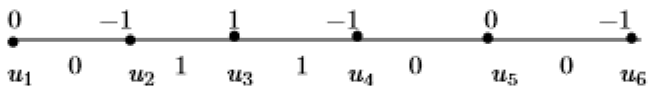

Figure 3.2

Here, $e_f(0) = 3$ and $e_f(1) = 2$

Therefore, path P_6 satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, the path P_6 is relaxed absolute divisor cordial graph.

Theorem 3.3: Cycle C_n is a relaxed absolute divisor cordial graph except when $n \equiv 2 \pmod 4$.

Proof: Let $V(C_n) = \{u_i : 1 \le i \le n\}$ and $E(C_n) = \{u_iu_{i+1} : 1 \le i \le n-1\} \cup \{u_1u_n\}$. Then $|V(C_n)| = n$ and $|E(C_n)| = n$.

Define $f: V(C_n) \to \{-1, 0, 1\}$ by

$$f(u_i) = \begin{cases} 0 & i \equiv 1 \pmod 4 \\ -1 & i \equiv 0 \pmod 2 \\ 1 & i \equiv 3 \pmod 4 \end{cases} \qquad 1 \le i \le n$$

The induced edge labeling are,

$f^*(u_1u_n) = 0$ and

$$f^*(u_iu_{i+1}) = \begin{cases} 0 & i \equiv 0,1 \pmod 4 \\ 1 & i \equiv 2,3 \pmod 2 \end{cases} \qquad 1 \le i \le n$$

Here, $e_f(0) = e_f(1) + 1$ for $n \equiv 1, 3 \pmod 4$
$e_f(0) = e_f(1) + 1$ for $n \equiv 0, 2 \pmod 4$

Therefore, cycle C_n satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, the cycle C_n is relaxed absolute divisor cordial graph except when $n \equiv 2 \pmod 4$.

Example 3.4: Consider the graph C_5.

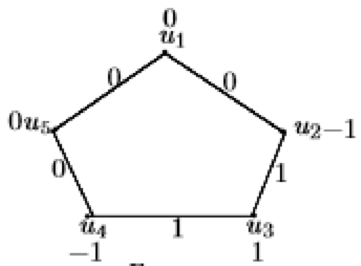

Figure 3.4

Here, $e_f(0) = 3$, $e_f(1) = 2$

Therefore, cycle C_5 satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, the cycle C_5 is a relaxed absolute divisor cordial graph except when $n \equiv 2(\text{mod}4)$.

Theorem 3.5: Cycle C_n is not a relaxed absolute divisor cordial graph for $n \equiv 2(\text{mod}4)$.

Proof: Let $V(C_n) = \{u_i : 1 \leq i \leq n\}$ and $(C_n) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_1u_n\}$. Then $|V(C_n)| = n$ and $|E(C_n)| = n$.

Define f: $V(C_n) \rightarrow \{-1, 0, 1\}$ by

$$f(u_i) = \begin{cases} 0 & i \equiv 1(\text{mod}\,4) \\ -1 & i \equiv 0(\text{mod}\,2) \\ 1 & i \equiv 3(\text{mod}\,4) \end{cases} \qquad 1 \leq i \leq n$$

The induced edge labeling are,

$$f^*(u_1u_n) = 0 \text{ and } f^*(u_iu_{i+1}) = \begin{cases} 0 & i \equiv 0,1(\text{mod}\,4) \\ 1 & i \equiv 2,3(\text{mod}\,2) \end{cases} \qquad 1 \leq i \leq n$$

Here, $e_f(0) = \frac{n}{2} + 1$ and $e_f(1) = \frac{n}{2} - 1$

Thus, $|e_f(0) - e_f(1)| = |\frac{n}{2} + 1 - \frac{n}{2} + 1| = 2 \nleq 1$.

Therefore, cycle C_n does not satisfy the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, the Cycle C_n is not a relaxed absolute divisor cordial graph $n \equiv 2(\text{mod}4)$.

Example 3.6: Consider the graph C_6.

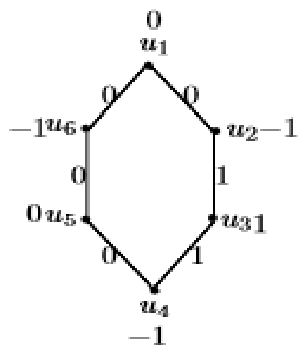

Figure 3.6

Here, $e_f(0) = 4$ and $e_f(1) = 2$

Therefore, cycle C_6 does not satisfy the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, the cycle C_5 is not a relaxed absolute divisor cordial graph for $n \equiv 2(\text{mod}4)$.

Theorem 3.7: Wheel W_n is a relaxed absolute divisor cordial graph when n is even.

Proof: Let $V(W_n) = \{u, u_i : 1 \le i \le n\}$ and $E(W_n) = \{uu_i : 1 \le i \le n\} \cup \{u_iu_{i+1} : 1 \le i \le n-1\} \cup \{u_nu_1\}$.

Then

$|V(W_n)| = n + 1$ and $|E(W_n)| = 2n$.

Define $f: V(W_n) \rightarrow \{-1, 0, 1\}$ by

$$f(u) = 0 \text{ and } f(u_i) = \begin{cases} -1 & i \equiv 1 \pmod 2 \\ 1 & i \equiv 0 \pmod 2 \end{cases} \quad 1 \le i \le n$$

The induced edge labeling are

$f^*(uu_i) = 0 \quad 1 \le i \le n$

$f^*(u_iu_{i+1}) = 1 \quad 1 \le i \le n-1$ and $f^*(u_1u_n) = 1$

Here, $e_f(0) = e_f(1)$, for all n

Therefore, wheel W_n satisfies the condition $|e_f(0) - e_f(1)| \le 1$ when n is even.

Hence, the Wheel W_n is a relaxed absolute divisor cordial graph when n is even.

Example 3.8: Consider the graph W_6.

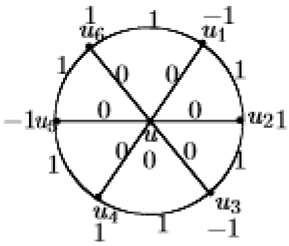

Figure 3.8

Here, $e_f(0) = 6$ and $e_f(1) = 6$

Therefore, wheel W_6 satisfies the condition $|e_f(0) - e_f(1)| \le 1$ when n is even.

Hence, the wheel W_6 is a relaxed absolute divisor cordial graph when n is even.

Theorem 3.9: Wheel W_n is not a relaxed absolute divisor cordial graph when n is odd.

Proof: Let $V(W_n) = \{u, u_i : 1 \le i \le n\}$ and $E(W_n) = \{uu_i : 1 \le i \le n\} \cup \{u_iu_{i+1} : 1 \le i \le n-1\} \cup \{u_nu_1\}$.

Then $|V(W_n)| = n + 1$ and $|E(W_n)| = 2n$.

Define $f: V(W_n) \rightarrow \{-1, 0, 1\}$ by

$f(u) = 0$

$$f(u_i) = \begin{cases} -1 & i \equiv 1 \pmod 2 \\ 1 & i \equiv 0 \pmod 2 \end{cases} \quad 1 \le i \le n$$

The induced edge labeling are

$f^*(uu_i) = 0 \quad 1 \le i \le n$

$f^*(u_iu_{i+1}) = 1 \quad 1 \le i \le n-1$ and $f^*(u_1u_n) = 1$

Here, $e_f(0) = n + 1$ and $e_f(1) = n - 1$

Thus, $|e_f(0) - e_f(1)| = |n + 1 - n + 1| = 2 \nleq 1$.

Therefore, wheel W_n does not satisfy the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, the wheel W_n is not a relaxed absolute divisor cordial graph when n is odd.

Example 3.10: Consider the graph W_3.

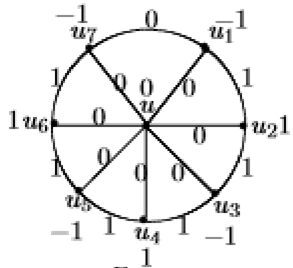

Figure 3.10

Here, $e_f(0) = 4$ and $e_f(1) = 2$

Therefore, wheel W_7 does not satisfy the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, the wheel W_7 is not a relaxed absolute divisor cordial graph when n is odd.

Theorem 3.11: Star $K_{1,n}$ is a relaxed absolute divisor cordial graph.

Proof: Let $V(K_{1,n}) = \{u, u_i : 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{uu_i : 1 \leq i \leq n\}$. Then $|V(K_{1,n})| = n + 1$ and $|E(K_{1,n})| = n$.

Define $f: V(K_{1,n}) \rightarrow \{-1, 0, 1\}$ by

$f(u) = 1$

$$f(u_i) = \begin{cases} 0 & i \equiv 1 \pmod 4 \\ -1 & i \equiv 0 \pmod 2 \\ 1 & i \equiv 3 \pmod 4 \end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are

$$f^*(uu_i) = \begin{cases} 0 & i \equiv 1 \pmod 2 \\ 1 & i \equiv 0 \pmod 2 \end{cases} \qquad 1 \leq i \leq n$$

Here, $e_f(0) = e_f(1)$ for $n \equiv 0 \pmod 2$
$e_f(0) = e_f(1) + 1$ for $n \equiv 1 \pmod 2$

Therefore, star $K_{1,n}$ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, the star $K_{1,n}$ is a relaxed absolute divisor cordial graph.

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Example 3.12: Consider the graph $K_{1,7}$.

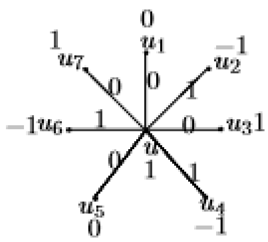

Figure 3.12

Here, $e_f(0) = 4$ and $e_f(1) = 3$.

Therefore, star $K_{1,7}$ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, the star $K_{1,7}$ is a relaxed absolute divisor cordial graph.

Theorem 3.13: Bistar $B_{m,n}$ is a relaxed absolute divisor cordial graph.

Proof: Let $V(B_{m,n}) = \{u, v, u_i, v_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(B_{m,n}) = \{uu_i : 1 \leq i \leq m\} \cup \{vv_j : 1 \leq j \leq n\}$.

Then $|V(B_{m,n})| = m + n + 2$ and $|E(B_{m,n})| = m + n + 1$.

Define $f: V(B_{m,n}) \to \{-1, 0, 1\}$ by

$f(u) = f(v) = 1$ and

$$f(u_i) = \begin{cases} -1 & i \equiv 0 (\mathrm{mod}\, 2) \\ 1 & i \equiv 1 (\mathrm{mod}\, 2) \end{cases} \quad 1 \leq i \leq m$$

$$f(v_j) = \begin{cases} 0 & j \equiv 0 (\mathrm{mod}\, 2) \\ 1 & j \equiv 1 (\mathrm{mod}\, 2) \end{cases} \quad 1 \leq j \leq n$$

The induced edge labeling are

$$f^*(uu_i) = \begin{cases} 0 & i \equiv 0 (\mathrm{mod}\, 2) \\ 1 & i \equiv 1 (\mathrm{mod}\, 2) \end{cases} \quad 1 \leq i \leq m$$

$$f^*(vv_j) = \begin{cases} 0 & j \equiv 0 (\mathrm{mod}\, 2) \\ 1 & j \equiv 1 (\mathrm{mod}\, 2) \end{cases} \quad 1 \leq j \leq n$$

Here, $e_f(0) = e_f(1)$ for all n

Therefore, bistar $B_{m,n}$ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, the bistar $B_{m,n}$ is a relaxed absolute divisor cordial graph.

Example 3.14: Consider the graph $B_{4,5}$.

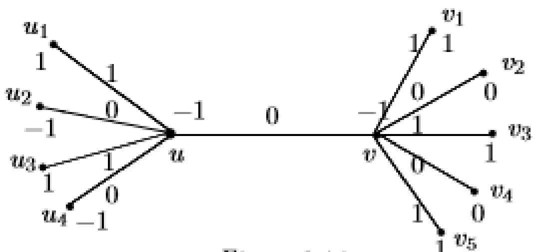

Figure 3.14

Here, $e_f(0) = 5$ and $e_f(1) = 5$

Therefore, bistar $B_{4,5}$ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, the bistar $B_{4,5}$ is a relaxed absolute divisor cordial graph.

Observation 3.15: Bistar $B_{n,n}$ is a relaxed absolute divisor cordial graph.

For, the vertex labeling and edge labeling are defined by the above **theorem - 3.12.**

Here, $e_f(0) = e_f(1) + 1$ for $i \equiv 0(\mathrm{mod}2)$ and
$e_f(1) = e_f(0) + 1$ for $i \equiv 1(\mathrm{mod}2)$

Therefore, bistar $B_{n,n}$ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, the bistar $B_{n,n}$ is a relaxed absolute divisor cordial graph.

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4. International Journal of scientific Engineering and Applied Science (IJSEAS) Volume-1 Dr.A.Nellai Murugan and R.Megala.

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