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FASTER CONVERGENT SERIES USING CORRECTION FUNCTIONS

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ABSTRACT

Here we shall deduce a series which is rapidly convergent than a given series, by applying a correction function to the series. The correction function and the corresponding error function are analysed. By this method, the rate of convergence of the new series can be increased.

Key Words: *Correction term, alternating series, Madhava series, rate of convergence, faster convergent series, error function.*

INTRODUCTION

The approximation of an alternating series can be done using remainder term of the series. This method was introduced by Madhava, an illusturious mathematician of 14th century. The absolute value of the remainder term is the correction function. The correction function plays a vital role in series approximation. It gives a better approximation for the series. The correction function and the corresponding error function are studied and analysed. We can also extract some rapidly convergent series using correction function and the error functions. The new series so extracted increases the rate of convergence of the series.

I. PRELIMINARY DEFINITIONS

Definition 1: An **alternating series** is an infinite series of the form $\sum_{n=1}^{\infty}(-)^{n-1}\, a_n$ where the terms $a_n > 0$.

Definition 2: The **remainder term** for an alternating series $\sum_{n=1}^{\infty}(-)^{n-1}\, a_n$ is the sum of the series after n terms. It is denoted by R_n.
ie $R_n = \sum_{k=n+1}^{\infty}(-)^{k-1}\, a_k$

If S denote the sum of the series and S_n denote the sequence of partial sums of the series, then $R_n = S - S_n$

Definition 3: The **correction function** to an alternating series $\sum_{n=1}^{\infty}(-)^{n-1}\, a_n$ is denoted by G_n and it is defined as the absolute value of the remainder term.

If R_n denotes the remainder term of the series, then $R_n = (-1)^n\, G_n$ where G_n is the correction function.
i.e. $G_n = \sum_{k=1}^{\infty}(-1)^{k-1} a_{n+k}$

If $\{a_n\}$ is monotonically decreasing, then $G_n = |\, S - S_n \,|$

Definition 4: An alternating series $\sum_{n=1}^{\infty}(-1)^{n-1} c_n$ is said to be **rapidly convergent** than the series $\sum_{n=1}^{\infty}(-1)^{n-1} d_n$ if the ratio $\frac{c_n}{d_n} \to 0$ as $n \to \infty$.

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II. CORRECTION FUNCTION FOR ALTERNATING HARMONIC SERIES

The Alternating Harmonic series (simply denote it as AHS) is convergent and converges to log2.

Thus log2 = $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+$......................... $+\frac{(-1)^{n-1}}{n}+$……………….

Proposition 1: The correction function for Alternating Harmonic series is $G_n = \frac{1}{2n+1}$

Proof: We have Alternating Harmonic series is convergent and converges to log2.

If G_n denotes the correction function after n terms of A H S, then we have $G_n + G_{n+1} = \frac{1}{n+1}$

Now we define, the error function as $E_n = G_n + G_{n+1} - \frac{1}{n+1}$

We may choose G_n in such a way that $|E_n|$ is a minimum .

For a fixed n and for $r \in$ R, let $G_n(r) = \frac{1}{(2n+2)-r}$.

Then $|E_n(r)|$ is minimum for $r = 1$.

For $|r| > 1$, the magnitude of the error function increases.

Hence for r=1, E_n and G_n are functions of a single variable n .

The minimum value of $|E_n| = \frac{1}{4n^3+12n^2+11n+3}$

Hence the correction function after n terms of AHS is $G_n = \frac{1}{2n+1}$ and the corresponding error function is

$$|E_n| = \frac{1}{4n^3+12n^2+11n+3}$$

Hence the proof.

III. RAPIDLY CONVERGENT SERIES FROM ALTERNATING HARMONIC SERIES

We have log2 = $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+$......................... $+\frac{(-1)^{n-1}}{n}+$……………….

Let $\partial_n = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+$......................... $+\frac{(-1)^{n-1}}{n} + (-1)^n G_n$

Let the error $\epsilon_n = \partial_{n+1} - \partial_n$

$\partial_{n+1} = \partial_n + \epsilon_n$

Put n = 1, 2, 3, …..n-1 in succession in the place of n and add to get

$\partial_n = \partial_1 + \epsilon_1 + \epsilon_2 + \epsilon_3 + \ldots + \epsilon_{n-1}$

$= 1 - G_1 + \epsilon_1 + \epsilon_2 + \epsilon_3 + \ldots + \epsilon_{n-1}$, since $\partial_1 = 1 - G_1$

$\lim_{n\to\infty} \partial_n = 1 - G_1 + \epsilon_1 + \epsilon_2 + \epsilon_3 + \ldots\ldots$

But $\lim_{n\to\infty} \partial_n$ = log2.

Hence log2 = $1 - G_1 + \epsilon_1 + \epsilon_2 + \epsilon_3 + \ldots\ldots\epsilon_n$

Case 1: For $G_n = \frac{1}{2n+1}$, $E_n = \frac{1}{4n^3+12n^2+11n+3}$

We have $\epsilon_n = (-1)^{n+1} E_n = \epsilon_n = \frac{(-1)^{n+1}}{4p^3-p}$ where p= n+1 .

The new deduced series is log2 = 1-G_1+ ϵ_1+ϵ_2+ϵ_3+.........

$$= \frac{2}{3} + \frac{1}{4.2^3-2} - \frac{1}{4.3^3-3} + \frac{1}{4.4^3-4} - \ldots\ldots$$

$$= \frac{2}{4.1^3-1} + \frac{1}{4.2^3-2} - \frac{1}{4.3^3-3} + \frac{1}{4.4^3-4} - \ldots\ldots$$

If c_n denotes the n^{th} term of the Alternating Harmonic series and if d_n denotes the n^{th} term of the new deduced series, then $c_n = \frac{(-1)^{n-1}}{n}$, $d_n = \epsilon_n$

It is clear that $\frac{dn}{cn} = \; \to 0$ as n $\to \infty$.

Hence the deduced series is rapidly convergent than the original series.

Hence the rate of convergence of new series is increased.

Thus the deduced series is a rapidly convergent series.

IV. APPLICATION

1. We have ln2 = 0.6931471806, using a calculator.

Number of terms(n)	S_n	$S_n + (-1)^n \; G_n$
10	0.6456349206	0.6932530476
100	0.6881721793	0.6931473037
1000	0.6926474306	0.6931471807
10000	0.6930971831	0.6931471806
100000	0.6931421806	0.6931471806

2. If S_n denotes the sequence of partial sum of the original series and if S_n' denotes the sequence of partial sums of the deduced series , then the rapidity of convergence is shown in the following table.

Number of terms (n)	S_n	S_n'
10	0.**6**456349206	**0.693**2530683

3. For a given accuracy, the number of terms required is shown below.

Accuracy	Number of terms required from the original series .	Number of terms required from the deduced series
0.**693**0971831	10000	10

V. CONCLUSION

The correction functions and the corresponding error functions play a vital role in series approximation. We can deduce new series which are rapidly convergent than the original series.

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