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PERTURBATION TECHNIQUE TO MHD Free Convection Flow of Kuvshinshiki Fluid with Heat and Mass Transfer Past a Vertical Porous Plate

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ABSTRACT

The objective of this paper is to study the effect of Kuvshinshki fluid on MHD free convection flow past a vertical porous plate with heat and mass transfer taking Visco-elastic and Darcy resistance terms into account and the constant permeability of the medium numerically and neglecting induced magnetic field in comparison to applied magnetic field. The velocity, temperature and concentration distributions are derived, discussed numerically and shown in figures 1, 2,3and 4 respectively. It is observed that velocity increases with increase in G_m, K and but it decreases with the increase in M and λ. It is observed that increase in Prandtl number P_r causes decreases in temperature. It is observed that increase in Schmidt number S_c leads to decreases in concentration. It is also noticed that skin friction increases with increase in G_m, K and but it decreases with the increase in M and λ.

Keywords: *Heat and mass transfer, Free convection, MHD, Porous medium, Vertical plate, Kuvshinshki fluid.*

INTRODUCTION

The convection problem in a porous medium has important applications in geothermal reservoirs and geothermal extractions. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear rector, chemical process industries and many engineering applications in which the fluid is the working medium. The wide range of technological and industrial applications has stimulated considerable amount of interest in the study of heat and mass transfer in convection flows. Free convective flow past a vertical plate has been studied extensively by Ostrach (1953). Siegel (1958) investigated the transient free convection from a vertical flat plate. Cheng and Lau (1977) and Cheng and Teckchandani (1977) obtained numerical solutions for the convective flow in a porous medium bounded by two isothermal parallel plates in the presence of the withdrawal of the fluid. In all the above mentioned studies, the effect of porosity, permeability and the thermal resistance of the medium is ignored or treated as constant. However, porosity measurements by Benenati and Broselow (1962) show that porosity is not constant but varies from the surface of the plate to its interior to which as a result permeability also varies. In case of unsteady free convective flow, Soundalgekar (1972) studied the effects of viscous dissipation on the flow past an infinite vertical porous plate. The combined effect of buoyancy forces from thermal and mass diffusion on forced convection was studied by Chep. *et al.* (1980). The free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient was studied by Ramanaiah and Malarvizhi (1991). Bejan and Khair (1985) have investigated the vertical free convective boundary layer flow embedded in a porous medium resulting from the combined heat and mass transfer. Lin and Wu (1995) analyzed the problem of simultaneous heat and mass transfer with the entire range of buoyancy ratio for most practical and chemical species in dilute and aqueous solutions. Rushi Kumar and Nagarajan (2007) studied the mass transfer effects of MHD free convection flow of incompressible viscous dissipative fluid past an infinite vertical plate. Mass transfer effects on free convection flow of an incompressible viscous dissipative fluid have been studied by Manohar and Nagaragan (2001).

In this study we consider the work of Sivaiah et al (2009) with Kuvshinshki fluid. The aim of present investigation is to study the effect of Kuvshinshki fluid on MHD free convection flow past a vertical porous plate with heat and mass transfer.

MATHEMATICAL ANALYSIS

We study the two-dimensional free convection and mass transfer flow of an incompressible visco-elastic Kuvshinshki type fluid past an infinite vertical porous plate under the following assumptions:

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- The plate temperature is constant.
- Visco-elastic and Darcy's resistance terms are taken into account with constant permeability of the medium.
- Boussinesq's approximation is valid.
- Visco elastic Kuvshinshki type fluid
- The suction velocity normal to the plate is a constant and can be written as,

$$v^1 = -U_0$$

A system of rectangular co-ordinates $O(x^1, y^1, z^1)$ is taken, such that $y^1 = 0$ on the plate and z^1 axis is along its leading edge. All the fluid properties considered constant except that the influence of the density variation with temperature is considered. The influence of the density variation in other terms of the momentum and the energy equation and the variation of the expansion coefficient with temperature is considered negligible. This is the well-known Boussinesq approximation.

Under these conditions, the problem is governed by the following system of Equations:
Equation of continuity:

$$\frac{\partial v^1}{\partial y^1} = 0 \tag{1}$$

Equation of Momentum:

$$\left(1+\lambda^1 \frac{\partial}{\partial t^1}\right)\frac{\partial u^1}{\partial t^1} + v^1 \frac{\partial v^1}{\partial y^1} = g\beta(T^1 - T^1_\infty) + g\beta^1(C^1 - C^1_\infty) + \nu \frac{\partial^2 v^1}{\partial y^{1^2}} - \left(1+\lambda^1 \frac{\partial}{\partial t^1}\right)\left(\frac{\nu}{K^1} + \frac{\sigma B_0^2}{\rho}\right)u^1 \tag{2}$$

Equation of Energy:

$$\frac{\partial T^1}{\partial t^1} + v^1 \frac{\partial T^1}{\partial y^1} = \alpha\left(\frac{\partial^2 T^1}{\partial y^{1^2}}\right) \tag{3}$$

Equation of Concentration:

$$\frac{\partial C^1}{\partial t^1} + v^1 \frac{\partial C^1}{\partial y^1} = D\left(\frac{\partial^2 C^1}{\partial y^{1^2}}\right) \tag{4}$$

Where u^1, v^1 are the velocity components. T^1, C^1 are the temperature and concentration components, ν is the kinematic viscosity. ρ is the density, σ is the electric conductivity, B_0 is the magnetic induction, α is the thermal conductivity and D is the concentration diffusivity.

The boundary conditions for the velocity and temperature and concentration fields are:

$$\begin{aligned} &u^1 = 0, T^1 = T^1_w, C^1 = C^1_w \quad \text{at} \quad y^1 = 0 \\ &u^1 = 0, T^1 = T_\infty, C^1 = C_\infty \quad \text{at} \quad y^1 \to \infty \end{aligned} \tag{5}$$

Let us introduce the non-dimensional variables

$$u = \frac{u^1}{U_0},\quad t = \frac{t^1 U_0^2}{\nu},\quad y = \frac{y^1 U_0}{\nu},\quad \theta = \frac{T^1 - T^1_\infty}{T^1_w - T^1_\infty},\quad C = \frac{C^1 - C^1_\infty}{C^1_w - C^1_\infty}$$

$$K = \frac{K^1 U_0^2}{\nu^2},\quad P_r = \frac{\nu}{\alpha},\quad S_c = \frac{\nu}{D},\quad M = \frac{\sigma B_0^2 \nu}{\rho U_0^2},$$

$$N = \frac{\beta^1 (C_w^1 - C_\infty^1)}{\beta (T_w^1 - T_\infty^1)}, \; G_r = \frac{\nu g \beta (T_w^1 - T_\infty^1)}{U_0^3}, \; \lambda = \frac{\lambda^1 U_0^2}{\nu}$$

Where P_r is the Prandtl number, G_r is the Grashof number, N is the buoyancy ratio, S_c is the Schmidt number, M is the magnetic parameter, K is the permeability parameter, β is the thermal expansion coefficient, β^1 is the concentration expansion coefficient and, λ is the visco-elastic parameter. Other physical variables have their usual meaning. Introducing the non-dimensional quantities describes above, the governing equation reduce to

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = G_r(\theta + NC) + \frac{\partial^2 u}{\partial y^2} - \left(1+\lambda\frac{\partial}{\partial t}\right)\left(M + \frac{1}{K}\right)u \tag{7}$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} \tag{8}$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \tag{9}$$

and the corresponding boundary conditions are

$$\begin{aligned} &u = 0, \theta = 1, C = 1 \text{ at } y = 0 \\ &u = 0, \theta = 0, C = 0 \text{ at } y \to \infty \end{aligned} \tag{10}$$

METHOD OF SOLUTION:

We assume the solution of eq. (7), (8), (9) as

$$\begin{aligned} &u(y,t) = u_0(y)e^{-nt}, \\ &\theta(y,t) = \theta_0(y)e^{-nt}, \\ &C(y,t) = C_0(y)e^{-nt} \end{aligned} \tag{11}$$

Using eq.(11) in eq. (7), (8), (9) and we get

$$u_0'' + u_0' - \left[\left(M + \frac{1}{K} - n\right)(1-\lambda n)\right]u_0 = -G_r\theta_0 - G_r N C_0 \tag{12}$$

$$\theta_0'' + P_r\theta_0' + P_r n\theta_0 = 0 \tag{13}$$

$$C_0'' + S_c C_0' + S_c n C_0 = 0 \tag{14}$$

Now the corresponding boundary conditions are

$$\begin{aligned} &u_0 = 0, \theta_0 = 1, C_0 = 1 \text{ at } y = 0 \\ &u_0 = 0, \theta_0 = 0, C_0 = 0 \text{ at } y \to \infty \end{aligned} \tag{15}$$

On (12) to (14) which are ordinary linear differential equations in, u_0, θ_0 and C_0 with boundary conditions (16) , we get

$$u_0 = (A_1 + A_2)e^{-m_3 y} - A_1 e^{-m_1 y} - A_2 e^{-m_2 y} \tag{16}$$

$$\theta_0 = e^{-m_1 y} \tag{17}$$

$$C_0 = e^{-m_2 y} \tag{18}$$

Where
$$m_1 = \frac{P_r + \sqrt{P_r^2 - 4P_r n}}{2}$$
$$m_2 = \frac{S_c + \sqrt{S_c^2 - 4S_c n}}{2}$$
$$m_3 = \frac{1 + \sqrt{1 + 4\left\{\left(M + \frac{1}{K} - n\right)(1 - \lambda n)\right\}}}{2}$$

$$A_1 = \frac{G_r}{\left[m_1^2 - m_1 - \left\{\left(M + \frac{1}{K} - n\right)(1 - \lambda n)\right\}\right]}$$
$$A_2 = \frac{G_r N}{\left[m_2^2 - m_2 - \left\{\left(M + \frac{1}{K} - n\right)(1 - \lambda n)\right\}\right]}$$

Hence, The equations for u, θ and C will be as follows

$$u = \left[(A_1 + A_2)e^{-m_3 y} - A_1 e^{-m_1 y} - A_2 e^{-m_2 y}\right]e^{-nt} \tag{19}$$
$$\theta = e^{-m_1 y}e^{-nt} \tag{20}$$
$$C = e^{-m_2 y}e^{-nt} \tag{21}$$

Skin Friction: The skin friction coefficient at y = 0 is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left[-m_3(A_1 + A_2) + m_1 A_1 + m_2 A_2\right]e^{-nt} \tag{22}$$

RESULT AND DISCUSSION:

Fluid velocity profile of fluid flow is tabulated in table -1 and plotted in figure-1 having five graphs P_r=0.71, S_c=1.5, n=0.1, t=0.1, **N=1.5** for following different value of M, K, G_r and λ.

	G_r	M	1/K	λ
For Graph-1	1	0.02	0.01	0.5
For Graph-2	2	0.02	0.01	0.5
For Graph-3	1	0.04	0.01	0.5
For Graph-4	1	0.02	0.02	0.5
For Graph-5	1	0.02	0.01	1.0

It is observed from figure -1 that all velocity distribution graphs are increasing sharply up to y=1.2 after that velocity in each graphs begins to decrease and tends to zero with the increasing in y. it is also observed from figure -1 that velocity increases with increase in G_m ,K and but it decreases with the increase in M and λ.

It is observed from figure -2 that all temperature distribution graphs decreases with increase in y. It is also noticed that graphs decreases with the value of P_r increases.

It is observed from figure -3 that all concentration distribution graphs decreases with increase in y. It is also noticed that graphs decreases with the value of S_c increases.

The skin friction profile of fluid flow is tabulated in table -4 and plotted in figure-4 having five graphs at P_r=0.71, S_c=1.5, n=0.1, **N=1.5** for the different value of M, K, G_r and λ. It is noticed that skin friction decreases gradually with increasing time t. It is also observed from figure -4 that skin friction increases with increase in G_m ,K and but it decreases with the increase in M and λ.

Table-1: Value of velocity u for Fig-1 at P_r = 0.71, S_c = 1.5, n =0.1, t =0.1, N = 1.5 and different values of G_r, M, K and λ.

y	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5
0	0	0	0	0	0
1	8.6913	17.39267	7.1803991	7.869426	8.328
2	9.8185	19.64547	8.1474984	8.908914	9.432315
3	8.8024	17.61141	7.3217261	7.99578	8.470656
4	7.3747	14.75448	6.1402537	6.701737	7.103717
5	6.0419	12.0878	5.0314632	5.490759	5.822786

Table-2: Value of temperature t for Fig-2 at n =0.1, t = 0.1 and different values of P_r .

y	Graph 1	Graph 2	Graph 3
0	0.99005	0.990049834	0.990049834
1	0.549045	0.40766956	0.148907914
2	0.304481	0.069121116	0.022396415
3	0.168854	0.069121116	0.003368521
4	0.09364	0.028461774	0.000506641

Table-3: Value of concentration C for Fig-3 at n =0.1, t = 0.1 and different values of S_c.

y	Graph 1	Graph 2	Graph 3
0	0.99005	0.990049834	0.990049834
1	0.810584	0.616803316	0.500152257
2	0.66365	0.38426988	0.252666353
3	0.543351	0.239401016	0.127641703
4	0.444858	0.149147382	0.064481891

Table-4: Value of skin friction τ for Fig-4 at P_r=0.71, S_c=1.5, n =0.1, **N=1.5** for the different value of M, K, G_r and λ.

t	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5
0	28.77961	57.55923	13.64591	14.97859	15.86586
1	26.04087	52.08174	12.34733	13.55319	14.35602
2	23.56275	47.12551	11.17233	12.26344	12.98987
3	21.32046	42.64092	10.10914	11.09642	11.75372
4	19.29155	38.5831	9.14713	10.04045	10.6352
5	17.45572	34.91144	8.276665	9.084977	9.62313

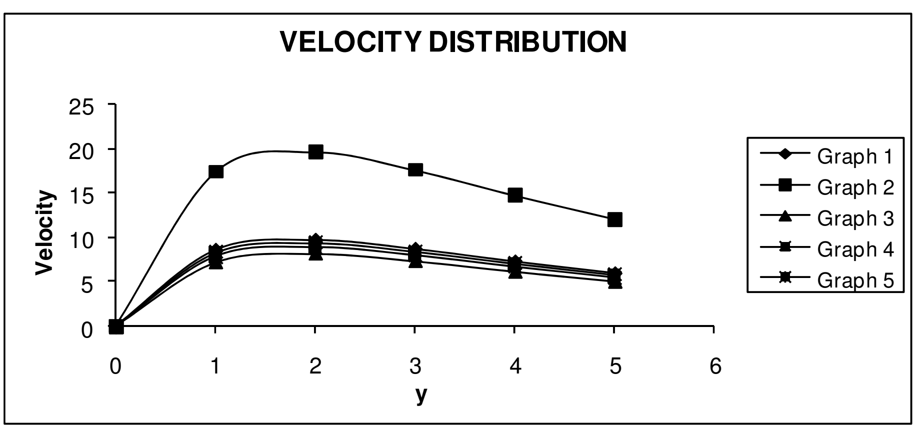

Figure-1

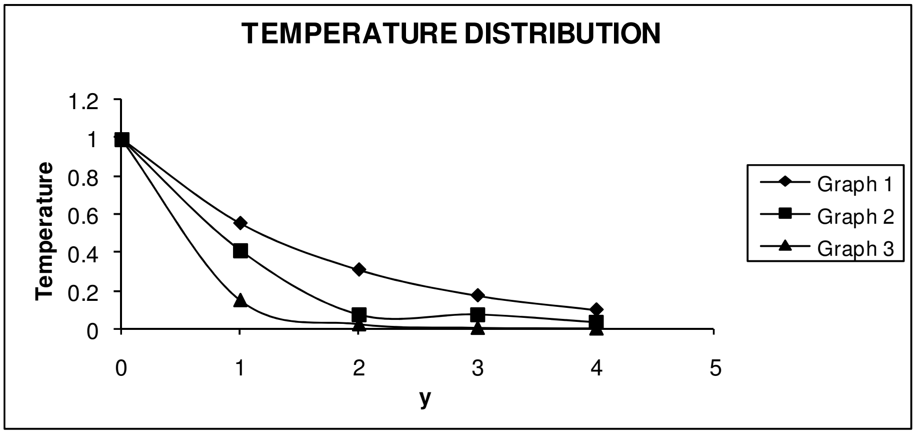

Figure-2

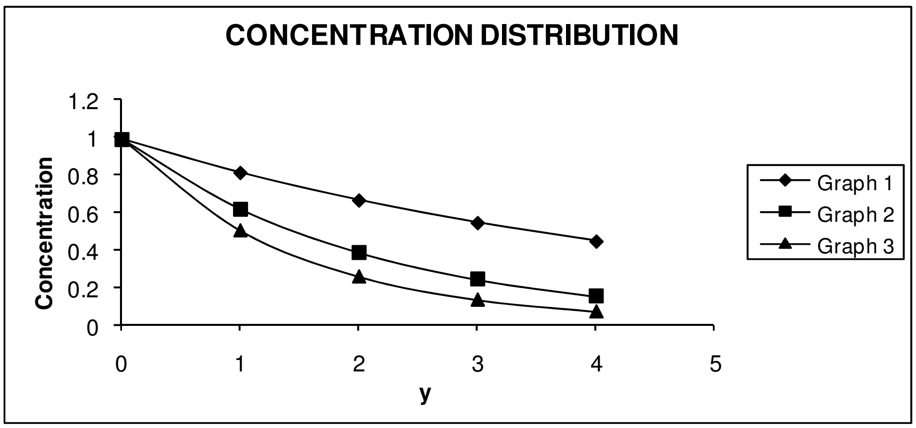

Figure-3

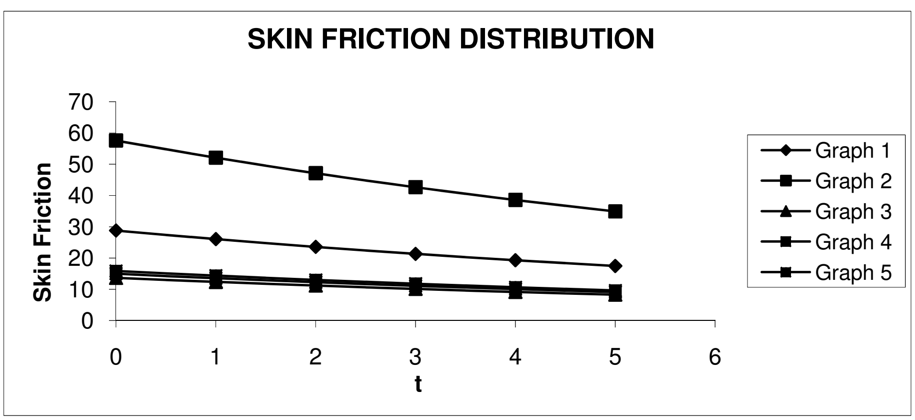

Figure-4

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