

## PRIME LABELING OF DESARGUES GRAPHS

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### ABSTRACT

A graph with vertex set  $V$  is said to have a prime labeling if its vertices are labeled with distinct integers  $1, 2, \dots, |V|$  such that for edge  $xy$  the labels assigned to  $x$  and  $y$  are relatively prime. A graph which admits prime labeling is called a prime graph. In this paper I investigate prime labeling for some special graphs namely Desargues graph. I also discuss prime labeling in the context of graph operations namely duplication and switching in Desargues graph.

**Keywords:** Prime labeling, Desargues graph, Duplication, switching.

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### I. INTRODUCTION

In this paper, I consider only finite simple undirected graph. The graph  $G$  has vertex set  $V = V(G)$  and edge set  $E = E(G)$ . The set of vertices adjacent to a vertex  $u$  of  $G$  is denoted by  $N(u)$ . For notations and terminology we refer to Bondy and Murthy [1].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout [6]. Two integers  $a$  and  $b$  are said to be relatively prime if their greatest common divisor is 1. Many researchers have studied prime graph. Fu.H [3] has proved that the path  $P_n$  on  $n$  vertices is a prime graph. Deretsky *et al.* [2] have proved that the cycle  $C_n$  on  $n$  vertices is a prime graph. Around 1980 Roger Entringer conjectured that all trees have prime labeling which is not settled today.

In [7] S.K. Vaidhya and K.K. Kanmani have proved that the graphs obtained by identifying any two vertices duplicating arbitrary vertex and switching of any vertex in cycle  $C_n$  admit prime labeling. In [5] Meena and Vaithilingam have proved the Prime labeling for some helm related graphs.

In this paper I proved that the Desargues graph is a prime graph.

**Definition 1.1:** Let  $G = (V(G), E(G))$  be a graph with  $p$  vertices. A bijection  $f: V(G) \rightarrow \{1, 2, \dots, p\}$  is called a prime labeling if for each edge  $e = uv$ ,  $\gcd\{f(u), f(v)\} = 1$ . A graph which admits prime labeling is called a prime graph.

**Definition 1.2:** Duplication of a vertex  $v$  of graph  $G$  produces a new graph  $G'$  by adding a new vertex  $v'$  such that  $N(v') = N(v)$ . In other words a vertex  $v'$  is said to be duplication of  $v$  if all the vertices which are adjacent to  $v$  in  $G$  are also adjacent to  $v'$  in  $G'$ .

**Definition 1.3:** Duplication of a vertex  $v_k$  by a new edge  $e = v_k' v_k''$  in a graph  $G$  produces a new graph  $G'$  such that  $N(v_k') = \{v_k, v_k''\}$  and  $N(v_k'') = \{v_k, v_k'\}$ .

**Definition 1.4:** Duplication of an edge  $e = uv$  by a new vertex  $w$  in a graph  $G$  produces a new graph  $G'$  such that  $N(w) = \{u, v\}$ .

**Definition 1.5:** Duplication of an edge  $e = uv$  of a graph  $G$  produces a new graph  $G'$  by adding an edge  $e' = u'v'$  such that  $N(u') = N(u) \cup \{v'\} - \{v\}$  and  $N(v') = N(v) \cup \{u'\} - \{u\}$ .

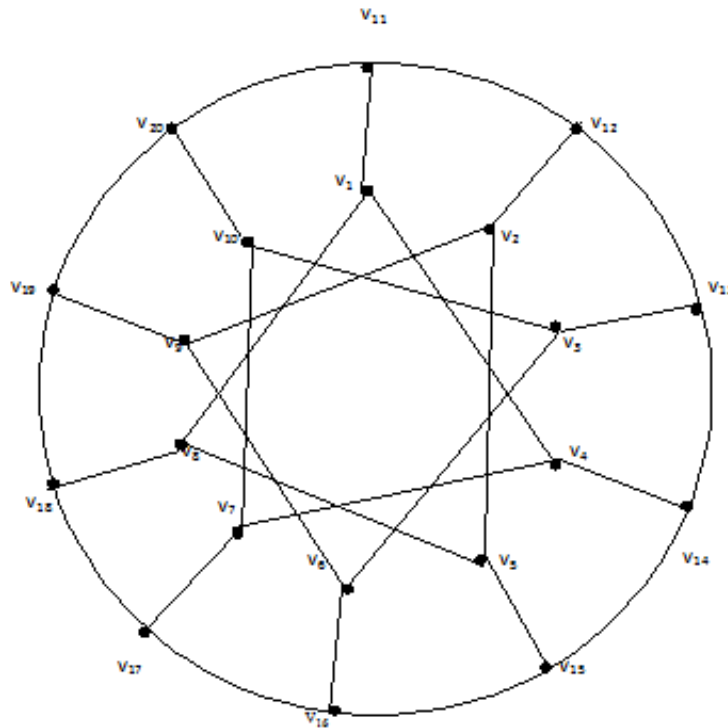
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**Definition 1.6:** A vertex switching  $G_v$  of a graph  $G$  is obtained by taking a vertex  $v$  of  $G$ , removing the entire edges incident with  $v$  and adding edges joining  $v$  to every vertex which are not adjacent to  $v$  in  $G$ .

**Definition 1.7:** The Desargues Graph is a distance transitive Cubic graph with 20 vertices and 30 edges.

**Example:**



### III. MAIN RESULTS

**Theorem 3.1:** The Desargues Graph is a prime graph.

**Proof:** Let  $G$  be a Desargues graph with 20 vertices and 30 edges.

Then  $|V(G)| = 20$  and  $|E(G)| = 30$ .

The edge set  $E(G) = \{v_i v_{i+3} / 1 \leq i \leq 7\} \cup \{v_i v_{i+7} / 1 \leq i \leq 3\} \cup \{v_i v_{i+1} / 11 \leq i \leq 19\} \cup \{v_{11} v_{20}\} \cup \{v_i v_{i+3} / i = 8,9,10\} \cup \{v_i v_{i+13} / 1 \leq i \leq 7\}$ .

Define a labeling  $f: V(G) \rightarrow \{1,2, \dots, \dots, 20\}$  by

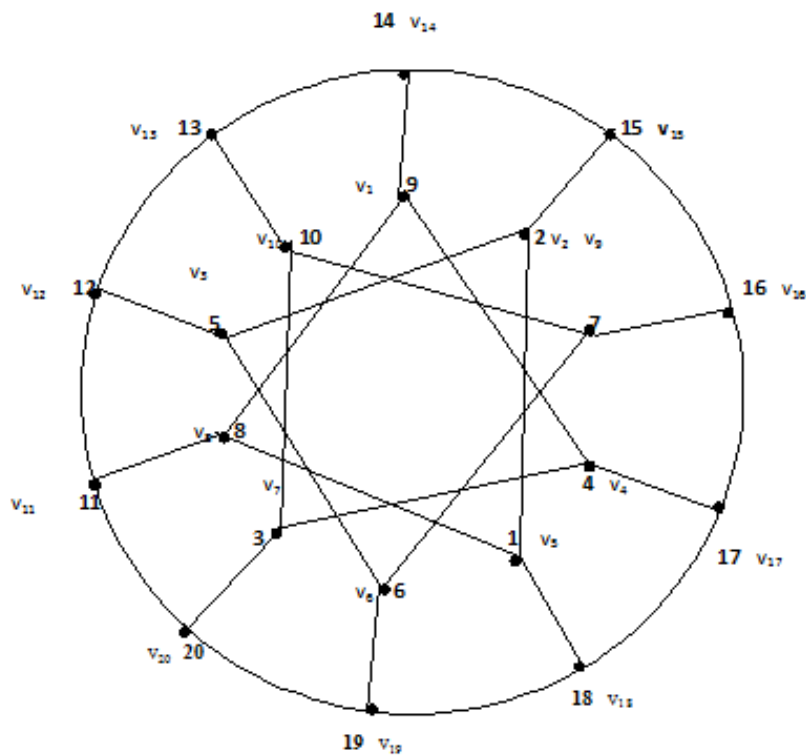
$$\begin{aligned} f(v_1) &= 9 \\ f(v_3) &= 7 \\ f(v_5) &= 1 \\ f(v_7) &= 3 \\ f(v_9) &= 5 \\ f(v_{2i}) &= 2i, \quad i = 1,2,3,4,5 \\ f(v_i) &= i, \quad 11 \leq i \leq 20 \end{aligned}$$

clearly,  $g.c.d \{f(v_i), f(v_{i+1})\} = 1, \quad i = 11,12, \dots, 19$   
 $g.c.d \{f(v_i), f(v_{i+3})\} = 1, \quad i = 1,2,3,4,5,6,7$   
 $g.c.d \{f(v_{11}), f(v_{20})\} = 1$   
 $g.c.d \{f(v_i), f(v_{13+i})\} = 1, \quad i = 1,2, \dots, 7$   
 $g.c.d \{f(v_i), f(v_{i+3})\} = 1, \quad i = 8,9,10$

Then  $f$  admits prime labeling.

Hence  $G$  is a prime graph.

Illustration of theorem 3.1:



Prime labeling of Desargues graph

**Theorem 3.2:** The graph obtained by duplication of a vertex in the Desargues graph is a prime graph.

**Proof:** Let  $G$  be the Desargues graph. Then  $|V(G)| = 20$  and  $|E(G)| = 30$ . Let  $G^*$  be the graph obtained by duplication of the vertex  $v_{12}$  by a new vertex  $v_k$ .

Then  $|V(G^*)| = 21$  and  $|E(G^*)| = 33$ .

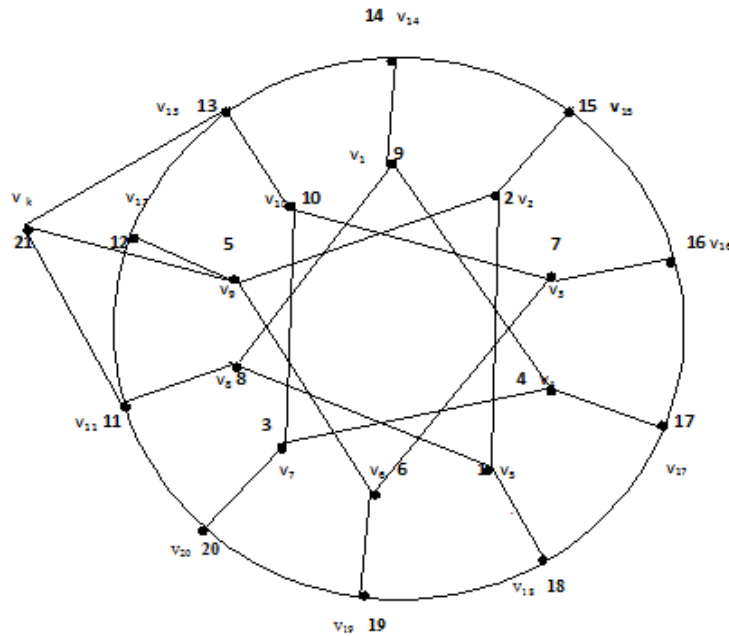
Then the edge set  $E(G^*) = E(G) \cup \{v_k v_{13}, v_k v_{11}, v_k v_5\}$ .

Define a labeling  $f: V(G^*) \rightarrow \{1, 2, \dots, 21\}$  by

$$\begin{aligned} f(v_1) &= 9 \\ f(v_3) &= 7 \\ f(v_5) &= 1 \\ f(v_7) &= 3 \\ f(v_9) &= 5 \\ f(v_{2i}) &= 2i, \quad i = 1, 2, 3, 4, 5 \\ f(v_i) &= i, \quad 11 \leq i \leq 20 \\ f(v_k) &= 21 \end{aligned}$$

Then  $f$  admits a prime labeling for  $G^*$ . Hence  $G^*$  is a prime graph.

**Illustration of theorem 3.2:**



Prime labeling of duplication of a vertex  $v_{12}$  by a new vertex  $v_k$  in the Desargues graph.

**Theorem 3.3:** The graph obtained by duplication of a vertex by an edge in the Desargues graph is a prime graph.

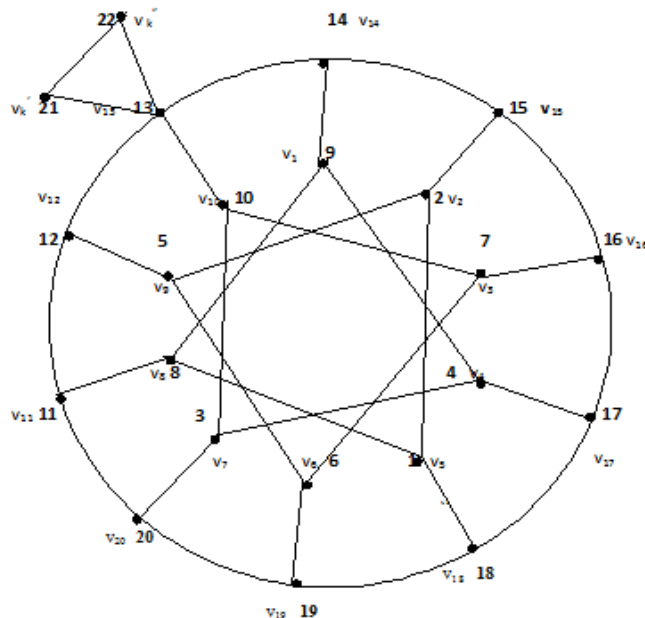
**Proof:** Let  $G$  be the Desargues graph. Then  $|V(G)| = 20$  and  $|E(G)| = 30$ . Let  $G^*$  be the graph obtained by duplication of a vertex  $v_{13}$  by an edge  $v_k v_k''$  in the Desargues graph.

Then  $|V(G^*)| = 22$  and  $|E(G^*)| = 33$ . The edge set  $E(G^*) = E(G) \cup \{v_{13} v_k', v_{13} v_k'', v_k' v_k''\}$ .

Define  $f: V(G^*) \rightarrow \{1, 2, \dots, 22\}$  by  
 $f(v_1) = 9; f(v_3) = 7; f(v_5) = 1; f(v_7) = 3; f(v_9) = 5;$   
 $f(v_{2i}) = 2i, i = 1, 2, 3, 4, 5$   
 $f(v_i) = i, 11 \leq i \leq 20; f(v_k') = 21; f(v_k'') = 22.$

Then  $f$  admits a prime labeling for  $G^*$ . Hence  $G^*$  is a prime graph.

**Illustration of theorem 3.3:**



Prime labeling of duplication of a vertex  $v_{13}$  by an edge  $v_k' v_k''$  in Desargues graph

**Theorem 3.4:** The graph obtained by duplication of an edge by a vertex in Desargues graph is a prime graph.

**Proof:** Let  $G$  be the Desargues graph. Then  $|V(G)| = 20$  and  $E(G) = 30$ .

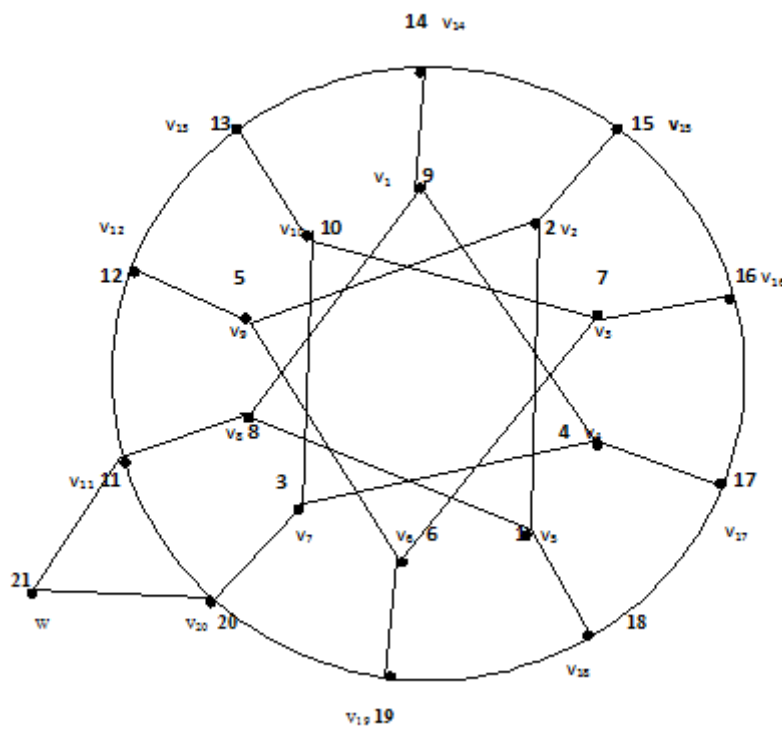
Let  $G^*$  be the graph obtained by Duplication of an edge  $v_{11}v_{20}$  by a vertex  $w$  in Desargues graph

Then  $|V(G^*)| = 21$  and  $|E(G^*)| = 32$ . The edge set  $E(G^*) = E(G) \cup \{v_{11}v_w, v_{20}v_w\}$ .

Define a labeling  $f: V(G) \rightarrow f \{1, 2, \dots, 21\}$  by  
 $f(v_1) = 9; f(v_3) = 7; f(v_5) = 1; f(v_7) = 3; f(v_9) = 5;$   
 $f(v_{2i}) = 2i, i = 1, 2, 3, 4, 5;$   
 $f(v_i) = i, 11 \leq i \leq 20; f(w) = 21.$

Then  $G^*$  admits a prime labeling .Hence  $G^*$  is a prime graph.

**Illustration of theorem 3.4:**



Prime labeling of duplication of an edge  $v_{11}v_{20}$  by a vertex  $w$  in Desargues graph.

**Theorem 3.5:** The graph obtained by duplication of an edge in the Desargues graph is a prime graph.

**Proof:** Let  $G$  be the Desargues graph .Then  $|V(G)| = 20$  and  $E(G) = 30$ .

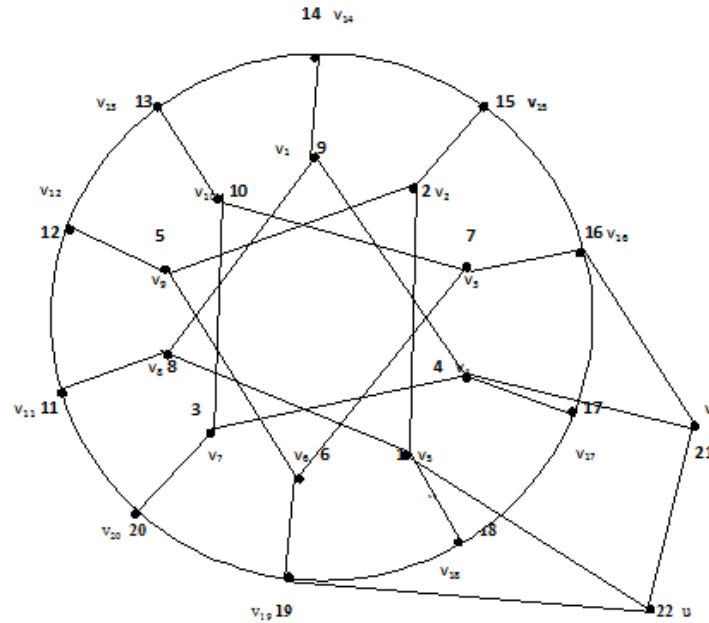
Let  $G^*$  be the graph obtained by duplication of an edge  $v_{17}v_{18}$  by a new edge  $uv$  in Desargues graph

Then  $|V(G^*)| = 22$  and  $|E(G^*)| = 35$ . The edge set  $E(G^*) = E(G) \cup \{v_{19}u, v_{16}v, v_4v, v_5u, uv\}$ .

Define a labeling  $f: V(G) \rightarrow f \{1, 2, \dots, 22\}$  by  
 $f(v_1) = 9; f(v_3) = 7; f(v_5) = 1; f(v_7) = 3; f(v_9) = 5;$   
 $f(v_{2i}) = 2i, i = 1, 2, 3, 4, 5;$   
 $f(v_i) = i, 11 \leq i \leq 20; f(v) = 21; f(u) = 22.$

Then  $G^*$  admits a prime labeling .Hence  $G^*$  is a prime graph.

**Illustration of theorem 3.5:**



Prime labeling of duplication of an edge  $v_{17}v_{18}$  by an edge  $uv$  in Desargues graph.

**Theorem 3.6:** The graph  $G_k$  obtained by switching a vertex in the Desargues graph is a prime graph.

**Proof:** Let  $G$  be the Desargues graph. Then  $|V(G)| = 20$  and  $|E(G)| = 30$ .

Let  $G_k$  be the graph obtained by switching the vertex  $v_5$  in the Desargues graph.

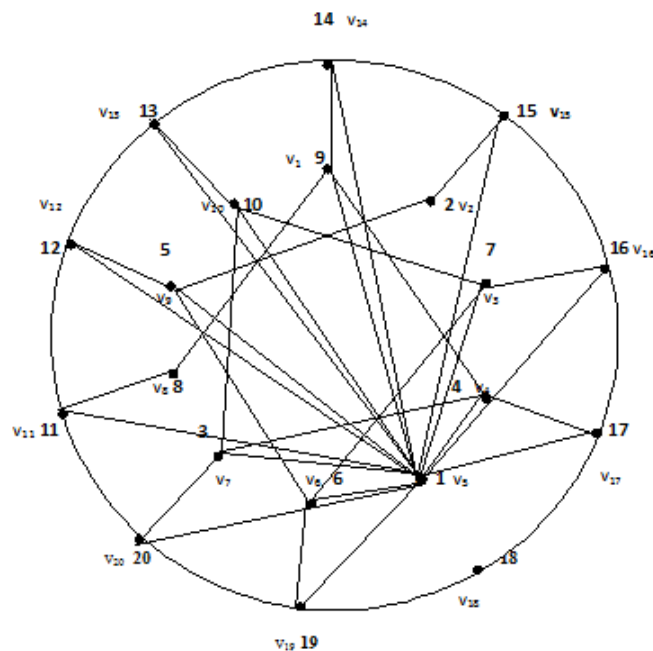
Then  $|V(G_k)| = 20$  and the edge set  $E(G_k) = E(G) \cup \{v_1v_i \mid 3 \leq i \leq 20; i \neq 8,18\}$ .

Define a labeling  $f : V(G_k) \rightarrow \{1,2, \dots, 20\}$  as follows.

$$f(v_1) = 9; f(v_3) = 7; f(v_5) = 1; f(v_7) = 3; f(v_9) = 5; \\ f(v_i) = i, i = 2,4,6,8,10,11,12,13,14,15,16,17,18,19,20.$$

Then  $f$  admits a prime labeling. Hence  $G_k$  is a prime graph.

**Illustration of theorem 3.6:**



Prime labeling of switching a vertex  $v_5$  in Desargues graph.

**Theorem 3.7:** The graph obtained by adjoining a vertex into each vertex in the outer cycle of the Desargues graph is a prime graph.

**Proof:** Let  $G$  be the Desargues graph .Then  $|V(G)| = 20$  and  $|E(G)| = 30$ .

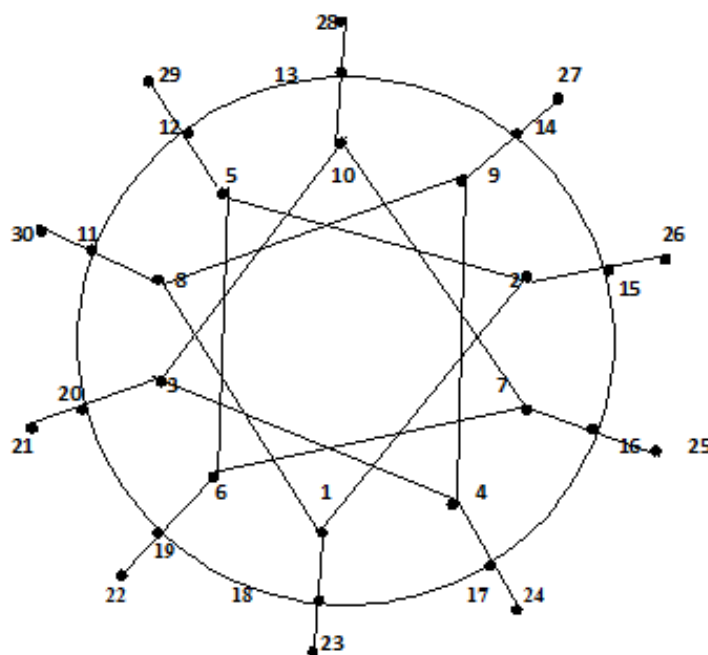
Let  $G^*$  be a graph obtained by adjoining vertices  $\{w_1, w_2, \dots \dots \dots w_{10}\}$  into vertices  $\{v_{20}, v_{19}, \dots \dots \dots, v_{11}\}$  in the outer cycle of Desargues graph respectively.

Then  $|V(G^*)| = 30$  and  $|E(G^*)| = 40$ .The edge set  $E(G^*) = E(G) \cup \{v_i w_j \ / 11 \leq i \leq 20; 1 \leq j \leq 10\}$ .

Define a labeling  $f : V(G^*) \rightarrow \{1, 2, \dots \dots \dots 30\}$  by  
 $f(v_1) = 9; f(v_3) = 7; f(v_5) = 1; f(v_7) = 3; f(v_9) = 5;$   
 $f(v_{2i}) = 2i, \ i = 1, 2, 3, 4, 5; f(v_i) = i, \ 11 \leq i \leq 20;$   
 $f(w_i) = 20 + i, 1 \leq i \leq 10.$

Then  $f$  admits a prime labeling. Hence  $G^*$  is a prime graph.

**Illustration of theorem 3.7:**



**IV.CONCLUSION**

I have investigated some results on prime labeling for the special graphs namely Desargues graph .Extending the study to other families of graph is an open area of research.

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