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AN INTRODUCTION TO FUZZY NEUTROSOPHIC TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce fuzzy neutrosophic topological spaces and its some properties. Also we provide fuzzy continuous and fuzzy compactness of fuzzy neutrosophic topological space and its some properties and examples.

Keywords: *fuzzy neutrosophic set, fuzzy neutrosophic topology, fuzzy neutrosophic topological spaces, fuzzy continuous and fuzzy compactness.*

1. INTRODUCTION

Fuzzy sets were introduced by Zadeh in 1965. The concepts of intuitionistic fuzzy sets by K. Atanassov several researches were conducted on the generalizations of the notion of intuitionistic fuzzy sets. Floretin Smarandache [5, 6] developed Neutrosophic set &logic ofA Generalization of the Intuitionistic Fuzzy Logic& set respectively. A.A.Salama & S.A.Alblowi [1] introduced and studied Neutrosophic Topological spaces and its continuous function in [2]. In this paper, we define thenotion of fuzzy neutrosophic topological spaces and investigate continuity and compactness by using Coker's intuitionistic topological spaces in [4]. We discuss New examples of FNTS.

2. PRELIMINARIES

Here we shall present the fundamental definitions. The following one is obviously inspired by Haibin Wang and Florentin Smarandache in [7] and A.A.Salama, S.S.Alblow in [1]. Smarandache introduced the neutrosophic set and neutrosophic components. The sets T, I, F are not necessarily intervals but may be any real sub-unitary subsets of $]^-0,1^+[$. The neutrosophic components T, I, F represents the truth value, indeterminacy value and falsehood value respectively.

Definition 2.1 [7]: Let X be a non-empty fixed set. A fuzzy neutrosophic set (FNS for short) A is an object having the form $A=\{\langle x,\mu_A(x),\sigma_A(x),\nu_A(x)\rangle : x\in X\}$ where the functions

$$\mu_A: X\to]^-0,1^+[,\qquad \sigma_A: X\to]^-0,1^+[,\qquad \nu_A: X\to]^-0,1^+[$$

denote the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy function (namely $\sigma_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) respectively of each element $x\in X$ to the set A and $^+0\le \mu_A(x)+\sigma_A(x)+\nu_A(x)\le 1^+$, for each $x\in X$.

Remark 2.2 [7]: Every fuzzy set A on a non-empty set X is obviously a FNS having the form

$$A=\{\langle x,\mu_A(x),\sigma_A(x),1-\mu_A(x)\rangle : x\in X\}$$

A fuzzy neutrosophic set $A=\{\langle x,\mu_A(x),\sigma_A(x),\nu_A(x)\rangle : x\in X\}$ can be identified to an ordered triple $\langle x,\mu_A,\sigma_A,\nu_A\rangle$ in $]^-0,1^+[$ on X.

Definition 2.3[1]: Let X be a non-empty set and the FNSs A and B be in the form $A=\{\langle x,\mu_A(x),\sigma_A(x),\nu_A(x)\rangle : x\in X$ and $B=x, \mu Bx, \sigma Bx, \nu Bx{:}x\in X$ on X and let $Ai{:}i\in J$ be an arbitrary family of FNS's in X, where $Ai=x, \mu Aix, \sigma Aix, \nu Aix{:}x\in X$e.

a) $A\subseteq B$ iff $\mu_A(x)\le\mu_B(x), \sigma_A(x)\le\sigma_B(x)$ and $\nu_A(x)\ge\nu_B(x)$ for all $x\in X$.
b) $A=B$ iff $A\subseteq B$ and $B\subseteq A$.
c) $\bar{A}=\{\langle x,\nu_A(x),1-\sigma_A(x),\mu_A(x)\rangle : x\in X\}$
d) $\cup A_i=\{\langle x,\vee_{i\in J}\mu_{A_i}(x),\vee_{i\in J}\sigma_{A_i}(x),\wedge_{i\in J}\nu_{A_i}(x)\rangle : x\in X\}$

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e) $\cap A_i = \{\langle x, \wedge_{i\in J} \mu_{A_i}(x), \wedge_{i\in J} \sigma_{A_i}(x), \vee_{i\in J} \nu_{A_i}(x)\rangle : x \in X\}$
f) $A - B = \{\langle x, \mu_A(x) \wedge \nu_B(x), \sigma_A(x) \wedge (1 - \sigma_B(x)), \nu_A(x) \vee \mu_B(x)\rangle : x \in X\}$
g) $[]A = \{\langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x)\rangle : x \in X\}$
h) $<> A = \{\langle x, 1 - \nu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$
i) $0_N = \langle x, 0, 0, 1\rangle$ and $1_N = \langle x, 1, 1, 0\rangle$

Example 2.4: Let $X = \{a, b, c\}$ and

$$A = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.2}, \frac{c}{0.3}\right), \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.2}\right)\rangle,$$
$$B = \langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.2}\right), \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.4}\right)\rangle,$$
$$C = \langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.3}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.2}\right)\rangle,$$
$$D = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.2}, \frac{c}{0.2}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right)\rangle$$

Here $\bar{A} = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.8}, \frac{c}{0.7}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}\right)\rangle$ and $A \subseteq C$ because $\mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$. Further,

$$A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x)\rangle$$
$$= \langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.3}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.2}\right)\rangle$$

$$A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x)\rangle$$
$$= \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.2}, \frac{c}{0.2}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right)\rangle$$

Corollary 2.5: Let A, B, C and A_i be FNS's in $X (i \in J)$. Then
a) $A_i \subseteq B$ for each $i \in J \Longrightarrow \cup A_i \subseteq B$
b) $B \subseteq A_i$ for each $i \in J \Longrightarrow B \subseteq \cap A_i$
c) $\overline{\cup A_i} = \cap \bar{A_i}$ and $\overline{\cap A_i} = \cup \bar{A_i}$
d) $A \subseteq B \Leftrightarrow \bar{B} \subseteq \bar{A}$
e) $\overline{(\bar{A})} = A$
f) $\overline{0_N} = 1_N$ and $\overline{1_N} = 0_N$

Now we shall define the image and preimage of FNS's. Let X, Y be two non-empty sets and $f: X \longrightarrow Y$ a function.

Definition 2.6[4]:
a) If $B = \{\langle y, \mu_B(y), \sigma_B(y), \nu_B(y)\rangle : y \in Y\}$ is an FNS in Y then the preimage of B under f, denoted by $f^{-1}(B)$, is the FNS in X defined by
$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\nu_B)(x)\rangle : x \in X\}$$
Where $f^{-1}(\mu_B)(x) = \mu_B f(x), f^{-1}(\sigma_B)(x) = \sigma_B f(x)$ and $f^{-1}(\nu_B)(x) = \nu_B f(x)$
b) If $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ is a FNS in X, then the image of A under f, denoted by $f(A)$, is the FNS in Y defined by
$$f(A) = \{\langle y, f(\mu_A)(y), f(\sigma_A)(y), (1 - f(1 - \nu_A))(y)\rangle : y \in Y\}$$
$$f(\mu_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & if\ f^{-1}(y) \neq \emptyset \\ 0 & otherwise \end{cases}$$
$$f(\sigma_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \sigma_A(x) & if\ f^{-1}(y) \neq \emptyset \\ 0 & otherwise \end{cases}$$
$$(1 - f(1 - \nu_A))(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \nu_A(x) & if\ f^{-1}(y) \neq \emptyset \\ 1 & otherwise \end{cases}$$
For the sake of simplicity, let us use the symbol $f\_(\nu_A)$ for $(1 - f(1 - \nu_A))$.

Corollary 2.7[4]: Let $A, A_i (i \in J)$ be FNS's in X, $B, B_j (j \in K)$ be FNS's in Y, and $f: X \longrightarrow Y$ a function.
a) $A_1 \subseteq A_2 \Longrightarrow f(A_1) \subseteq f(A_2), B_1 \subseteq B_2 \Longrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,
b) $A \subseteq f^{-1}(f(A))$, and if f is injective then $A = f^{-1}(f(A))$
c) $f^{-1}(f(B)) \subseteq B$, and if f is surjective, then $f^{-1}(f(B)) = B$
d) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j), f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$,
e) $f(\cup A_i) = \cup f(A_i), f(\cap A_i) \subseteq \cap f(A_i)$; and if f is injective, then $f(\cap A_i) = \cap f(A_i)$
f) $f^{-1}(1_N) = 1_N, f^{-1}(0_N) = 0_N$
g) $f(0_N) = 0_N, f(1_N) = 1_N$, if f is surjective

h) If f is surjective, then $\overline{f(A)} \subseteq f(\bar{A})$. If furthermore, f is injective, then $\overline{f(A)} = f(\bar{A})$

i) $f^{-1}(\bar{B}) = \overline{f^{-1}(B)}$

3. FUZZY NEUTROSOPHIC TOPOLOGICAL SPACES

Definition 3.1: A fuzzy neutrosophic topology (FNT for short) a non-empty set X is a family τ of fuzzy neutrosophic subsets in X satisfying the following axioms.

(NT1) $0_N, 1_N \in \tau$

(NT2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(NT3) $\cup\, G_i \in \tau, \forall \{G_i : i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called a Fuzzy neutrosophic topological space (FNTS for short) and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set (FNOS for short) in X. The elements of τ are called open fuzzy neutrosophic sets.

The complement of FNOS in the FNTS (X, τ) is called fuzzy neutrosophic closed set (FNCS for short).

Example 3.2: Let $X = \{a, b, c\}$ and consider the value of A, B, C, D are defined in Example 2.4, the family $\tau = \{0_N, 1_N, A, B, C, D\}$ of FNS in X. Then (X, τ) is FNTS on X.

Example 3.3: Let (X, τ) be a fuzzy topological space such that τ is not indiscrete. Suppose now that $\tau = \{0, 1\} \cup \{G_i : i \in J\}$. Then we can construct two FNT's on X as follows:

a) $\tau_1 = \{0_N, 1_N\} \cup \{\langle x, G_i, \sigma(x), 0\rangle : i \in J\}$, b) $\tau_2 = \{0_N, 1_N\} \cup \{\langle x, 0, \sigma(x), G_i\rangle : i \in J\}$

Proposition 3.4: Let (X, τ) be a FNTS on X. Then we can also construct several FNTS's on X in the following way $\tau_{0,1} = \{[\quad] G : G \in \tau\}$, b) $\tau_{0,2} = \{\langle\quad\rangle G : G \in \tau\}$

Definition 3.5: Let $(X, \tau_1), (X, \tau_2)$ be two FNTS's on X. Then τ_1 is said to be contained in τ_2 (in symbols, $\tau_1 \subseteq \tau_2$), if $G \in \tau_2$ for each $G \in \tau_1$. In this case, we also say that τ_1 is coarser than τ_2.

Proposition 3.6: Let $\{\tau_i : i \in J\}$ be a family of FNT's on X. Then $\bigcap_{i\in J} \tau_i$ is FNT on X. Furthermore, $\bigcap_{i\in J} \tau_i$ is the coarsest FNT on X containing all τ_i's.

Definition 3.7: Let (X, τ) be a FNTS on X.

a) A family $\beta \subseteq \tau$ is called a base for (X, τ) iff each member of τ can written as a union of elements of β.

b) A family $\gamma \subseteq \tau$ is called a subbase for (X, τ) iff the family of finite intersections of elements in γ forms a base for (X, τ). In this case the FNT τ is said to be generated by γ.

Definition 3.8: Let (X, τ) be a FNTS and $A = \langle x, \mu_A, \sigma_A, \nu_A\rangle$ be a FNS in X. Then the fuzzy interior and fuzzy closure of A are defined by

$$cl(A) = \cap \{K : K \text{is a FNCS in } X \text{ and } A \subseteq K\}, int(A) = \cup \{G : G \text{is a FNOS in } X \text{ and } G \subseteq A\}$$

Now that $cl(A)$ is a FNCS and $int(A)$ is a FNOS in X. Further,

a) A is a FNCS in X iff $cl(A) = A$

b) A is a FNOS in X iff $int(A) = A$

Example 3.9: Consider the FNTS (X, τ) is defined in Example 3.2. If

$$F = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.2}\right), \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.3}\right)\rangle$$

Then, $int(A) = D$ and $cl(A) = 1_N$.

Proposition 3.10: For any FNS A in (X, τ), we have $cl(\bar{A}) = \overline{int(A)}, int(\bar{A}) = \overline{cl(A)}$.

Proposition 3.11: Let (X, τ) be a FNTS and A, B be FNS's in X. Then the following properties hold:

a) $int(A) \subseteq A, A \subseteq cl(A)$

b) $A \subseteq B \Longrightarrow int(A) \subseteq int(B), A \subseteq B \Longrightarrow cl(A) \subseteq cl(B)$

c) $int\big(int(A)\big) = int(A), cl\big(cl(A)\big) = cl(A)$

d) $int(A \cap B) = int(A) \cap int(B), cl(A \cup B) = cl(A) \cup cl(B)$

e) $int(1_N) = 1_N, cl(0_N) = 0_N$

Proposition 3.12: Let (X,τ) be a FNTS. If $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$ be a FNS in X. Then we have

a) $int(A) \subseteq \langle x, int_{\tau_1}(\mu_A), int_{\tau_1}(\sigma_A), cl_{\tau_2}(\nu_A) \rangle \subseteq A$

b) $A \subseteq \langle x, cl_{\tau_2}(\mu_A), cl_{\tau_2}(\sigma_A), int_{\tau_1}(\nu_A) \rangle \subseteq cl(A)$
where τ_1 and $\tau_2$are FTS on X defined by $\tau_1 = \{\mu_A : G \in \tau\}$, $\tau_2 = \{1 - \nu_A : G \in \tau\}$

Corollary 3.13: Let $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$ be a FNS in (X,τ)

a) If A is a FNCS, then μ_A is fuzzy closed in (X,τ_2) and ν_A is fuzzy open in (X,τ_1).

b) If A is a FNOS, then μ_A is fuzzy open in (X,τ_1) and ν_A is fuzzy closed in (X,τ_2).

Example 3.14: Consider the FNTS (X,τ) is defined in Example 3.2. If

$$F = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.2}\right), \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.3}\right) \rangle$$

Then, $int(A) = D$. Noting that we have

$$\tau_1 = \left\{0, 1, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right)\right\}$$

$$\tau_2 = \left\{0, 1, \left(\frac{a}{0.7}, \frac{b}{0.8}, \frac{c}{0.8}\right), \left(\frac{a}{0.8}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.8}, \frac{b}{0.8}, \frac{c}{0.8}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\}$$

and $\quad int_{\tau_1}(\mu_A) = \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right), cl_{\tau_2}(\nu_A) = \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right)$

4. FUZZY NEUTROSOPHIC CONTINUITY

Definition 4.1: Let (X,τ) and (Y,φ) be two FNTSs and let $f: X \to Y$ be a function. Then f is said to be fuzzy continuous iff the preimage of each FNS in φ is a FNS in τ.

Definition 4.2: Let (X,τ) and (Y,φ) be two FNTSs and let $f: X \to Y$ be a function. Then f is said to be fuzzy open iff the image of each FNS in τ is a FNS in φ.

Example 4.3: Let (X,τ_0) and (Y,φ_0) be two fuzzy topological spaces.

a) If $f: X \to Y$ is fuzzy continuous in the usual sense, then in this case, f is fuzzy continuous iff the preimage of each FNS in φ_0 is a FNS in τ_0. Consider the FNTs on X and Y respectively, as follows:
$$\tau = \{\langle x, \mu_G, 0, 1-\mu_G \rangle : \mu_G \in \tau_0\}, \varphi = \{\langle y, \delta_H, 0, 1-\delta_H \rangle : \delta_H \in \varphi_0\}$$
In this case we have for each $\langle y, \delta_H, 0, 1-\delta_H \rangle \in \varphi, \delta_H \in \varphi_0$,
$$\begin{aligned} f^{-1}\langle y, \delta_H, 0, 1-\delta_H \rangle &= \langle x, f^{-1}(\delta_H), f^{-1}(0), f^{-1}(1-\delta_H) \rangle \\ &= \langle x, f^{-1}(\delta_H), f^{-1}(0), 1-f^{-1}(\delta_H) \rangle \in \tau \end{aligned}$$

b) Let $f: X \to Y$ be a fuzzy open in the usual sense, then f is fuzzy openaccording to Definition 4.2. In this case we have, for each $\langle x, \mu_G, 0, 1-\mu_G \rangle \in \tau, \mu_G \in \tau_0$,
$$\begin{aligned} f\langle x, \mu_G, 0, 1-\mu_G \rangle &= \langle x, f(\mu_G), 0, f\_(1-\mu_G) \rangle \\ &= \langle x, f(\mu_G), 0, 1-f\_(\mu_G) \rangle \in \varphi \end{aligned}$$

Proposition 4.4: $f: (X,\tau) \to (Y,\varphi)$ is fuzzy continuous iff the preimage of each FNCS in φ is a FNCS in τ.

Proposition 4.5: The following are equivalent to each other

a) $f: (X,\tau) \to (Y,\varphi)$ is fuzzy continuous

b) $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ for each FNS B in Y.

c) $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$for each FNS B in Y.

Example 4.6: Let (Y,φ) be a FNTS and X be a non-empty set and $f: X \to Y$ a function. In this case $\tau = \{f^{-1}(H) : H \in \varphi\}$ is a FNT on X. Indeed, t is the coarsest FNT on X which makes the function $f: X \to Y$ continuous. One may call the FNT t on X the initial FNT with respect to f.

5. FUZZY NEUTROSOPHIC COMPACTNESS

Definition 5.1: Let (X,τ) be a FNTS.

a) If a family $G_i = \left\{\langle x, \mu_{G_i}, \sigma_{G_i}, \nu_{G_i} \rangle : i \in J\right\}$ of FNOS is X satisfy the condition $\cup\, G_i = 1_N$, then it is called a fuzzy open cover of X. A finite subfamily of fuzzy open cover G_i of X, which is also a fuzzy open cover of X is called a finite subcover of G_i.

b) A family $K_i = \left\{\langle x, \mu_{K_i}, \sigma_{K_i}, \nu_{K_i} \rangle : i \in J\right\}$ of FNCSs in X satisfies the finite intersection property iff every finite subfamily $\left\{\langle x, \mu_{K_i}, \sigma_{K_i}, \nu_{K_i} \rangle : i = 1, 2, \dots, n\right\}$ of the family satisfies the condition $\bigcap_{i=1}^{n}\left\{\langle x, \mu_{K_i}, \sigma_{K_i}, \nu_{K_i} \rangle\right\} \neq 0_N$.

Definition 5.2: A FNTS (X,τ) is fuzzy compact iff every fuzzy open cover of X has a finite subcover.

Example 5.3: An FNTS (X,τ), where $X=\{1,2,3\}$,

$$G_n=\langle x,\left(\frac{1}{\frac{n}{n+1}},\frac{2}{\frac{n}{n+2}},\frac{3}{\frac{n}{n+3}}\right),\left(\frac{1}{\frac{n+1}{n+2}},\frac{2}{\frac{n+2}{n+3}},\frac{3}{\frac{n+3}{n+4}}\right),\left(\frac{1}{\frac{1}{n+2}},\frac{2}{\frac{1}{n+3}},\frac{3}{\frac{1}{n+4}}\right)\rangle$$

and $\tau=\{0_N,1_N\}\cup\{G_n:n\in\mathbb{N}\}$. Note that $\bigcup_{n\in\mathbb{N}}G_n$ is an open cover for X, but this cover has no finite subcover. Consider

$$G_1=\langle x,\left(\frac{1}{0.5},\frac{2}{0.3},\frac{3}{0.25}\right),\left(\frac{1}{0.7},\frac{2}{0.75},\frac{3}{0.8}\right),\left(\frac{1}{0.3},\frac{2}{0.25},\frac{3}{0.2}\right)\rangle$$
$$G_2=\langle x,\left(\frac{1}{0.7},\frac{2}{0.5},\frac{3}{0.4}\right),\left(\frac{1}{0.75},\frac{2}{0.8},\frac{3}{0.8}\right),\left(\frac{1}{0.25},\frac{2}{0.2},\frac{3}{0.2}\right)\rangle$$
$$G_3=\langle x,\left(\frac{1}{0.75},\frac{2}{0.6},\frac{3}{0.5}\right),\left(\frac{1}{0.8},\frac{2}{0.8},\frac{3}{0.9}\right),\left(\frac{1}{0.2},\frac{2}{0.2},\frac{3}{0.1}\right)\rangle$$

and observe that $G_1\cup G_2\cup G_3=G_3$. So, for any finite subcollection $\{G_{n_i}:i\in \mathrm{I},\text{where I is a finite subset of }\mathbb{N}\}$, $\bigcup_{n_i\in\mathrm{I}}G_{n_i}=G_m\neq 1_N$, where $m=max\{n_i:n_i\in\mathrm{I}\}$. therefore the FNTS (X,τ) is not compact.

Proposition 5.4: Let (X,τ) be a FNTS on X. Then (X,τ) is fuzzy compact iff the FNTS $(X,\tau_{0,1})$ is fuzzy compact.

Proof: Let (X,τ) be fuzzy compact and consider fuzzy open cover $\{[\quad]G_j:j\in K\}$ of X in $(X,\tau_{0,1})$. Since $\cup([\quad]G_j)=1_N$. We obtain $\vee\mu_{G_j}=1,\vee\sigma_{G_j}=1$ and hence, by $\nu_{G_j}\le 1-\mu_{G_j}\Rightarrow\wedge\nu_{G_j}\le 1-\vee\mu_{G_j}=0\Rightarrow\wedge\nu_{G_j}=0$, we deduce $\cup G_j=1_N$. Since (X,τ) is fuzzy compact there exist $G_1,G_2,G_3,\dots G_n$ such that $\bigcup_{i=1}^n G_i=1_N$ from which we obtain $\bigvee_{i=1}^n\mu_{G_i}=1,\bigvee_{i=1}^n\sigma_{G_i}=1$ and $\bigwedge_{i=1}^n(1-\mu_{G_i})=0$. That is, $(X,\tau_{0,1})$ is fuzzy compact.

Suppose that $(X,\tau_{0,1})$ is fuzzy compact and consider a fuzzy open cover $\{G_j:j\in K\}$ of X in (X,τ). Since $\cup G_j=1_N$, we obtain $\vee\mu_{G_j}=1,\vee\sigma_{G_j}=1$ and $1-\vee\mu_{G_j}=0$. Since $(X,\tau_{0,1})$ is fuzzy compact, there exist $G_1,G_2,G_3,\dots G_n$ such that $\bigcup_{i=1}^n([\quad]G_j)=1_N$, that is, $\bigvee_{i=1}^n\mu_{G_i}=1,\bigvee_{i=1}^n\sigma_{G_i}=1$ and $\bigwedge_{i=1}^n(1-\mu_{G_i})=0$. Hence $\mu_{G_i}\le 1-\nu_{G_i}\Rightarrow 1=\bigvee_{i=1}^n\mu_{G_i}\le 1-\bigwedge_{i=1}^n\nu_{G_i}=0.\Rightarrow\bigwedge_{i=1}^n\nu_{G_i}=0$. Hence $\bigcup_{i=1}^n G_i=1_N$. Therefore (X,τ) is fuzzy compact.

Corollary 5.5: Let (X,τ) and (Y,φ) be two FNTSs and let $f:X\to Y$ be a fuzzy continuous surjection. If (X,τ) is fuzzy compact, then so is (Y,φ).

Corollary 5.6: A FNTS (X,τ) is fuzzy compact iff every family $\{\langle x,\mu_{K_i},\sigma_{K_i},\nu_{K_i}\rangle:i\in J\}$ of FNCSs in X having the FNP has a non-empty intersection.

Definition 5.7:

a) Let (X,τ) be a FNTS and A a FNS in X. If a family $G_i=\{\langle x,\mu_{G_i},\sigma_{G_i},\nu_{G_i}\rangle:i\in J\}$ of FNOSs in X satisfies the condition $A\subseteq\cup G_i$, then it is called a fuzzy open cover of A. A finite subfamily of the fuzzy open cover G_i of A, which is also fuzzy open cover of A, is called a finite subcover of G_i.

b) A FNS $A=\langle x,\mu_A,\sigma_A,\nu_A\rangle$ in a FNTS (X,τ) is called fuzzy compact iff every fuzzy open cover of A has a finite subcover.

Corollary 5.8: A FNS $A=\langle x,\mu_A,\sigma_A,\nu_A\rangle$ in a FNTS (X,τ) is fuzzy compact iff for each family $\mathcal{G}=\{G_i:i\in J\}$ where $G_i=\{\langle x,\mu_{G_i},\sigma_{G_i},\nu_{G_i}\rangle:i\in J\}$, of FNOS in X with properties $\mu_A\le\bigvee_{i\in J}\mu_{G_i},\sigma_A\le\bigvee_{i\in J}\sigma_{G_i}$ and $1-\nu_A\le\bigvee_{i\in J}(1-\nu_{G_i})$ there exists a finite subfamily $\{G_i:1,2,\dots,n\}$ of $\mathcal{G}$ such that $\mu_A\le\bigvee_{i=1}^n\mu_{G_i},\sigma_A\le\bigvee_{i=1}^n\sigma_{G_i}$ and $1-\nu_A\le\bigvee_{i=1}^n(1-\nu_{G_i})$.

Example 5.9: Let $X=I$ and consider the FNSs $(G_n)_{n\epsilon Z_2}$, where $G_n=\langle x,\mu_{G_n},\sigma_{G_n},\nu_{G_n}\rangle,n=2,3,\dots.$ and $G=\langle x,\mu_G,\sigma_G,\nu_G\rangle$ defined by

$$\mu_{G_n}(x)=\begin{cases}0.7, if\ x=0\\ nx, if\ 0<x<\frac{1}{n}\\ 1, if\ \frac{1}{n}<x\le 1\end{cases},\sigma_{G_n}(x)=\begin{cases}0.8, & if\ x=0\\ (n+1)x, if\ 0<x<\frac{1}{n}\\ 1, & if\ \frac{1}{n}<x\le 1\end{cases},\quad \nu_{G_n}=\begin{cases}0.1, & if\ x=0\\ 1-nx, & if\ 0<x<\frac{1}{n}\\ 0, & if\ \frac{1}{n}<x\le 1\end{cases}$$

and $\mu_G=\begin{cases}0.7 & if\ x=0\\ 1, & otherwise\end{cases},\sigma_G=\begin{cases}0.8 & if\ x=0\\ 1, & otherwise\end{cases},$

$\nu_G=\begin{cases}0.1 & if\ x=0\\ 1, & otherwise\end{cases}.$

Then $\tau=\{0_N,1_N,G\}\cup\{G_n:n\in Z_2\}$ is a FNT on X, and consider the FNSs $C_{\alpha,\beta,\gamma}$ in (X,τ) defined by $C_{\alpha,\beta,\gamma}=\{\langle x,\alpha,\beta,\gamma\rangle:x\in X\}$, where $\alpha,\beta,\gamma\in I$ are arbitrary and $\alpha+\beta+\gamma\le 2$. Then the FNSs $C_{0.75,0.85,0.05},C_{0.65,0.75,0.15},C_{0.85,0.850,05}$ are all compact.

Corollary 5.10: Let (X, τ) and (Y, φ) be two FNTSs and let $f: X \to Y$ be a fuzzy continuous function. If A is fuzzy compact in(X, τ), then so is $f(A)$ in (Y, φ).

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