International Journal of Mathematical Archive-8(3), 2017, 138-143
Available online through www.ijma.info ISSN 2229 – 5046

A STUDY ON SOFT PRE-OPEN SETS

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(Received On: 01-03-17; Revised & Accepted On: 27-03-17)

ABSTRACT

The objective of this paper is to describe the concepts of soft pre-open sets and soft pre-closed sets in soft topological spaces by studying the basics of soft set theory by D. Molodstov's description.

Keywords: *soft set, soft topology, soft open sets and soft closed sets.*

Subject Classification: *54A10, 54A05.*

1. INTRODUCTION

Several researchers followed Molodtsov [4] 1999, after his introduction of Soft set theory as a common mathematical application in dealing with the vagueness of not well defined objects, many researchers followed him. When several proved mathematical applications have more clarity while applying on formal modeling, reasoning and computing, but few of the engineering sciences, life sciences, social sciences and ecological sciences does not have clarity many times. Shabir and Naz [10] described the soft topological spaces and its basic notations in detail.

The equality of two soft sets, subset and superset of Soft set, complement of a soft set, null soft set and absolute soft set with examples were well defined by Maji [7]. As the researchers now have several latest sophisticated techniques, they have now applied them in operations research, Riemans integration, Game theory, theory of probability and researched several basic notations of soft set theory. Naim *et al.*, [8] presented the foundations of the theory of soft topological spaces to open the door for experiments for soft mathematical concepts and structures that are based on soft set-theoretic operations. Recently Soft topology has been studied in depth in the papers [1, 2, 3, 5, 6, 9, 11, 12, 13, 14]. In this paper, we make a theoretical study of the new set called soft pre-open set and soft pre-closed sets in soft topological space.

2. PRELIMINARIES

The following definitions are essential for the development of the paper.

Definition 2.1 [4]: Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U. The pair (F, E) or simply F_E, is called a *soft set* over U, where F is a mapping given by $F: E \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe. For $e \in U$, $F(e)$ may be considered as the set of e-approximate elements of the soft set F. The collection of all soft sets over U and E is denoted by $S(U)$. If $A \subseteq E$, then the pair (F, A) or simply F_A, is called a *soft set* over U, where F is a mapping $F: A \to P(U)$. Note that for $e \notin A$, $F(e) = \emptyset$.

Definition 2.2 [11]: The *union of two soft sets* of F_B and G_C over the common universe U is the soft set H_D, where B and C are subsets of the parameter set E, $D = B \cup C$ and for all $e \in D$, $H(e) = F(e)$ if $e \in B - C$, $H(e) = G(e)$ if $e \in C - B$ and $H(e) = F(e) \cup G(e)$ if $e \in B \cap C$, we write $F_B \tilde{\cup} G_C = H_D$.

Definition 2.3 [11]: The *intersection of two soft sets* of F_B and G_C over the common universe U is the soft set H_D, where $D = B \cap C$ and for all $e \in D$, $H(e) = F(e) \cap G(e)$ if $D = B \cap C$. We write $F_B \tilde{\cap} G_C = H_D$.

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Definition 2.4 [11]: Let F_B and G_C be soft sets over a common universe set U and $B, C \subseteq E$. Then F_B is a *soft subset* of G_C, denoted by $F_B \tilde{\subseteq} G_C$, if (i) $B \subseteq C$ and (ii) for all $e \in B$, $F(e) = G(e)$. Also G_C, is called the soft super set of F_B and is denoted by $F_B \tilde{\supseteq} G_C$.

Definition 2.5 [11]: The soft sets F_B and G_C over a common universe set U are said to be *soft equal*, if $F_B \tilde{\subseteq} G_C$, and $F_B \tilde{\supseteq} G_C$. Then we write $F_B = G_C$.

Definition 2.6 [11]: A soft set F_B over U is called a *null soft set* denoted by F_ϕ, if for all $e \in B$, $F(e) = \phi$.

Definition 2.7 [10]: The *relative complement* of a soft set F_A, denoted by F_A^c, is defined by the approximate function $f_{A^c}(e) = f_A^c(e)$, where $f_A^c(e)$ is the complement of the set $f_A(e)$, that is $f_A^c(e) = U - f_A(e)$ for all $e \in E$. It is easy to see that $(F_A^c)^c = F_A$, $F_\phi^c = F_E$ and $F_E^c = F_\phi$.

Definition 2.8 [11]: Let U be an initial universe and E be a set of parameters. If $B \subseteq E$, the soft set F_B over U is called an *absolute soft set*, if for all $e \in B$, $F(e) = U$.

The following is the definition of soft topology used by various authors.

Definition 2.9 [1]: Let U be an initial universe and E be a set of parameters. Let τ be a subcollection of $S(U)$, the collection of soft sets defined on U. Then τ is a *soft topology* if it satisfies the following conditions.

(i) $F_\phi, F_E \in \tau$
(ii) The union of any number of soft sets in τ belongs to τ.
(iii) The intersection of any two soft sets in τ belongs to τ.

If this definition is considered for further development, then the results are similar to that of results in topological spaces. Therefore, throughout the paper the following definition of soft topology is used.

Definition 2.10 [10]: Let U be an initial universe and E be a set of parameters. Let $F_A \in S(U)$. A *soft topology* on F_A, denoted by $\tilde{\tau}$, is a collection of soft subsets of F_A having the following properties:

(i) F_ϕ, $F_A \in \tilde{\tau}$
(ii) $\{F_{A_i} \tilde{\subseteq} F_A : i \in I\} \tilde{\subseteq} \tilde{\tau} \Rightarrow \tilde{\bigcup}_{i \in I} F_{A_i} \in \tilde{\tau}$
(iii) $\{F_{A_i} \tilde{\subseteq} F_A : 1 \leq i \leq n,\ n \in \mathrm{N}\} \tilde{\subseteq} \tilde{\tau} \Rightarrow \tilde{\bigcap}_{i=1}^{n} F_{A_i} \in \tilde{\tau}$.
(iv) The pair $(F_A, \tilde{\tau})$ is called a soft topological spaces.

Definition 2.11 [9]: Let $(F_A, \tilde{\tau})$ be a soft topological space in F_A. Elements of $\tilde{\tau}$ are called *soft open* sets. A soft set F_B in F_A is said to be a *soft closed* set in F_A, if its relative complement F_B^c, belongs to $\tilde{\tau}$.

Definition 2.12 [12]: Let $(F_A, \tilde{\tau})$ be a soft topological space and F_B be a soft subset of F_A.

(i) The *soft interior* of F_B is the soft set *int*(F_B) = $\tilde{\bigcup} \{F_C : F_C$ is soft open and $F_C \tilde{\subseteq} F_B\}$.
(ii) The *soft closure* of F_B is the soft set cl(F_B) = $\tilde{\bigcap} \{F_C : F_C$ is soft closed and $F_B \tilde{\subseteq} F_C\}$.

Its clearly int(F_B) is the largest soft open set contained in F_B and cl(F_B) is the smallest soft closed set containing F_B.

Lemma 2.13 [12]: Let $(F_A, \tilde{\tau})$ be a soft topological space and F_B and F_C be a soft subsets of F_A. Then the following holds.

(i) int(int(F_B)) = int(F_B).
(ii) $F_B \tilde{\subseteq} F_C$ implies int(F_B) $\tilde{\subseteq}$ int(F_C).
(iii) int(F_B) $\tilde{\cap}$ int(F_C) = int($F_B \tilde{\cap} F_C$).
(iv) int(F_B) $\tilde{\cup}$ int(F_C) $\tilde{\subseteq}$ int($F_B \tilde{\cup} F_C$).
(v) F_B is soft open set if and only if F_B = int(F_B).

Lemma 2.14 [12]: Let $(F_A, \tilde{\tau})$ be a soft topological space and F_B and F_C be a soft set in F_A. Then the following hold.

(i) cl(cl(F_B)) = cl(F_B).
(ii) $F_B \tilde{\subseteq} F_C$ implies cl(F_B) $\tilde{\subseteq}$ cl(F_C).
(iii) cl(F_B) $\tilde{\cap}$ cl(F_C) $\tilde{\supseteq}$ cl($F_B \tilde{\cap} F_C$).
(iv) cl(F_B) $\tilde{\cup}$ cl(F_C) = cl($F_B \tilde{\cup} F_C$).
(v) F_B is soft closed set if and only if F_B = cl(F_B).

Lemma 2.15 [9]: Arbitrary union of soft open sets is soft open and finite intersection of soft closed sets is soft closed set.

Proposition 2.16 [2]: Let $(F_A, \tilde{\tau})$ be a soft topological space over F_A. Then the following hold.

(i) $F_\emptyset, F_A$ are soft closed sets in F_A.
(ii) The union of any two soft closed sets is a soft closed set in F_A.
(iii) The intersection of any two soft closed sets is a soft closed set in F_A.

Proposition 2.17 [12]: If $\{F_{B_\alpha} | \alpha \in I\}$ is a collection of soft sets, then the following hold.

(i) $\tilde{\cup}$ int $(F_{B_\alpha}) \tilde{\subseteq}$ int $(\tilde{\cup} F_{B_\alpha})$
(ii) $\tilde{\cup}$ cl$(F_{B_\alpha}) \tilde{\subseteq}$ cl$(\tilde{\cup} F_{B_\alpha})$.

Lemma 2.18 [13]:

(i) For every soft open set F_B in a soft topological space $(F_A, \tilde{\tau})$ and every soft set F_C, we have cl(F_C) $\tilde{\cap}$ $F_B \tilde{\subseteq}$ cl$(F_C \tilde{\cap} F_B)$.
(ii) For every soft closed set in F_B a soft topological space $(F_A, \tilde{\tau})$ and every soft set F_C, we have int$(F_B \tilde{\cup} F_C) \tilde{\subseteq}$ int$(F_B) \tilde{\cup} F_C$.

Proposition 2.19 [7]: If F_B and F_C are any two soft sets in $(F_A, \tilde{\tau})$, then the following hold
$F_A - (F_B - F_C) = (F_A - F_B) \tilde{\cup} (F_A - F_C)$.

3. SOFT PRE-OPEN SET AND SOFT PRE-CLOSED SET

This section is devoted to the study of soft pre-open sets and soft pre- closed sets.

Definition 3.1: Let $(F_A, \tilde{\tau})$ be a soft topological space and F_B be a soft subset of F_A. F_B is said to be a *soft pre-open set*, if $F_B \tilde{\subseteq}$ *int(cl(*F_B*))*.

Example 3.2: Let $U = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2\} \tilde{\subseteq} E$.

$F_A = \{(e_1, \{a, b\}), (e_2, \{b, c\})\}$, $F_1 = \{(e_1, \{a\})\}$, $F_2 = \{(e_1, \{b\})\}$ $F_3 = \{(e_1, \{a, b\})\}$,

$F_4 = \{(e_2, \{b\})\}$, $F_5 = \{(e_1, \{c\})\}$, $F_6 = \{(e_2, \{b, c\})\}$, $F_7 = \{(e_1, \{a\}), (e_2, \{b\})\}$,

$F_8 = \{(e_1, \{a\}), (e_2, \{c\})\}$, $F_9 = \{(e_1, \{a\}) (e_2, \{b, c\})\}$, $F_{10} = \{(e_1, \{b\}) (e_2, \{b\})\}$, $F_{11} = \{(e1, \{b\}) (e_2, \{c\})\}$,

$F_{12} = \{(e_1, \{b\}) (e_2, \{b, c\})\}$, $F_{13} = \{(e1, \{a, b\}), (e_2, \{b\})\}$,

$F_{14} = \{(e_1, \{a, b\}), (e_2, \{c\})\}$, $F_{15} = F_A$ and $F_{16} = F_\emptyset$.

Let $\tilde{\tau} = \{F_\emptyset, F_A, F_1, F_3, F_7, F_9, F_{13}\}$. Then $(F_A, \tilde{\tau})$ is a soft topological space. The family of all soft closed sets is $\{F_\emptyset, F_A, F_{12}, F_6, F_{11}, F_2, F_5\}$. The family of soft pre-open sets is $\{F_A, F_\emptyset, F_1, F_3, F_4, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14}\}$. The family of soft pre-closed sets is $\{F_A, F_\emptyset, F_1, F_2, F_4, F_5, F_6, F_8, F_{10}, F_{11}, F_{12}, F_{14}\}$.

Theorem 3.3: Every soft open set in a soft topological space $(F_A, \tilde{\tau})$ is a soft pre-open set.

Proof: The proof follows from the Definition 3.1.

The following Example 3.4 shows that the converse implication of Theorem 3.3 is not true.

Example 3.4: Consider the soft topological space of Example 3.2. Here F_4, F_8, F_{10}, F_{12} and F_{14} are soft pre-open sets but not soft open sets, since $F_4, F_8, F_{10}, F_{12}, F_{14} \notin \tilde{\tau}$.

Remark 3.5: $F_\emptyset$ and F_A are always soft pre-closed sets and soft pre-open sets.

Theorem 3.6: Arbitrary union of soft pre-open sets is a soft pre-open set.

Remark 3.7: Arbitrary intersection of soft pre-closed sets is a soft pre-closed set.

The following Example 3.8 shows that the finite intersection of soft pre-open sets need not be a soft pre-open set.

Example 3.8: In Example 3.2, $F_B = F_3 = \{(e_1, \{a, b\})\}$ and $F_C = F_{10} = \{(e_1, \{b\}), (e_2, \{b\})\}$ are soft pre-open sets. But $F_B \tilde{\cap} F_C = F_2 = \{(e_1, \{b\})\}$ is not a soft pre-open set.

Theorem 3.9: If F_B is a soft pre-open set such that $F_C \widetilde{\subseteq} F_B \widetilde{\subseteq}$ cl(F_C), then F_C is also a soft pre-open set.

Proof: $F_B \widetilde{\subseteq}$ cl(F_C) implies that cl(F_B)$\widetilde{\subseteq}$ cl(F_C) and so $F_B \widetilde{\subseteq}$ *int(cl(F_B))* $\widetilde{\subseteq}$ *int(cl(F_C))* which implies that $F_C \widetilde{\subseteq}$ int(cl(F_C)). Therefore, F_C is a soft pre-open set.

Theorem 3.10: Let F_B be a soft subset of a soft topological space (F_A, $\tilde{\tau}$). Then F_B is soft pre-closed set if and only if cl(int(F_B)) $\widetilde{\subseteq} F_B$.

Proof: Let F_B be a soft pre-closed set. Then $(F_B)^c$ is a soft pre-open set. So $(F_B)^c \widetilde{\subseteq}$ *int(cl($(F_B)^c$))* $\Rightarrow$ $(F_B)^c \widetilde{\subseteq}$ *[cl(int(F_B))]^c* which implies that (F_B)$\widetilde{\supseteq}$ cl(int(F_C)). Conversely, *cl(int(F_B))* $\widetilde{\subseteq} F_B \Rightarrow$*[cl(int($F_B$))] ^{c}* $\widetilde{\supseteq}$ $(F_B)^c$ which implies that int(cl(F_B) ^{c})] $\widetilde{\supseteq}$ $(F_B)^c$. Therefore, F_B^c is a soft pre-open set and so F_B is a soft pre-closed set.

Theorem 3.11: Let (F_A, $\tilde{\tau}$) be a soft topological space in which every soft subset is pre-open if and only if every soft open set in (F_A, $\tilde{\tau}$) is soft closed set.

Proof: Suppose that every soft open set is a soft closed set. Let F_B be a soft set. Let $F_C = F_B^c$ and so, $F_B = F_C^c$. Since int(F_B) is a soft open set, by hypothesis, *int(F_B)* is a soft closed set and so *cl(int(F_B))=int(F_B)* $\widetilde{\subseteq} F_B = F_C^c$. Then $F_C \widetilde{\supseteq}$*(cl(int($F_B$)))^c = int(cl($F_B^c$)) = int(cl($F_C$))*. Every soft subset is soft pre-open set. Conversely, let F_B be a soft open set. Since F_B^c is a soft set, by hypothesis, $F_B^c \widetilde{\subseteq}$*int(cl($F_B^c$))* and so *cl(int($F_B$))* $\widetilde{\subseteq} F_B$ which implies that *cl*(F_B)$\widetilde{\subseteq} F_B$. Therefore, every soft open set is a soft closed set. This complete the proof.

Definition 3.12: Let (F_A, $\tilde{\tau}$) be a soft topological space and F_B be a soft subset of F_A.

(i) The *soft pre-interior* of.F_B is the soft set *p-int(F_B)* =$\tilde{\cup}$ {F_C: F_C is soft pre-open set and $F_C \widetilde{\subseteq} F_B$}.

(ii) The *soft pre-closure* of F_B is the soft set *p-cl(F_B)* =$\tilde{\cap}$ {F_C: F_C is soft pre-closed set and $F_B \widetilde{\subseteq} F_C$}.

Its clearly, *p-int(F_B)* is the largest soft pre-open set contained in F_B and *p-cl(F_B)* is the smallest soft pre-closed set containing F_B.

Theorem 3.13: Let (F_A, $\tilde{\tau}$) be a soft topological space and F_B be a soft subset of F_A. Then the following holds.

(i) (p-cl(F_B))^c = p-int(F_B^c)
(ii) (p-int(F_B))^c = p-cl(F_B^c)
(iii) (p-int(F_B))^c = p-cl(F_B^c)

Proof:
(i) (p-cl(F_B))^c= ($\tilde{\cap}${F_C: F_C is soft pre-closed set and $F_C \widetilde{\subseteq} F_B$})^c
= $\tilde{\cup}${F_C^c: F_C^c is soft pre-open set and $F_C^c \widetilde{\subseteq} F_B^c$}
= $\tilde{\cup}${F_C^c: F_C^c is soft pre-open set and $F_C^c \widetilde{\subseteq} F_B^c$}= p-int($F_B^c$).
(ii) (s-int(F_B))^c = ($\tilde{\cup}${F_C: F_C is soft pre-open set and $F_C \widetilde{\subseteq} F_B$})^c
= $\tilde{\cap}${F_C^c : F_C^c is soft pre-open set and $F_B^c \widetilde{\subseteq} F_C^c$ }
= $\tilde{\cap}${F_C^c: F_C^c is soft pre-closed set and $F_B^c \widetilde{\subseteq} F_C^c$ }= p-cl(F_B^c).

(iii) The proof follows from (ii). □

Theorem 3.14: Let (F_A, $\tilde{\tau}$) be a soft topological space and let F_B and F_C be soft subsets of F_A. Then the following holds.

(i) p-cl(F_ϕ) = F_ϕ and p-cl(F_A) = F_A.
(ii) F_B is a soft pre-closed set if and only if F_B=p-cl(F_B).
(iii) p-cl(p-cl(F_B)) = p-cl(F_B).
(iv) $F_B \widetilde{\subseteq} F_C$ implies p-cl(F_B) $\widetilde{\subseteq}$ p-cl(F_C).
(v) p-cl($F_B \tilde{\cap} F_C$) $\widetilde{\subseteq}$ p-cl(F_B) $\tilde{\cap}$ p-cl(F_C).
(vi) p-cl($F_B \tilde{\cup} F_C$) = p-cl(F_B) $\tilde{\cup}$ p-cl(F_C).

Proof:

(i) is obvious.
(ii) If F_B is soft pre-closed set, then F_B is itself a soft pre-closed set in F_A which contains F_B . Since p-cl(F_B) is the smallest soft pre-closed set containing F_B, and F_B = *p-cl(F_B)*. Conversely, suppose that F_B = p-cl(F_B). Since p-cl(F_B) being the intersection of soft pre-closed set is soft pre-closed set, F_B is soft pre-closed sets in F_A.
(iii) Since p-cl(F_B) is a soft pre-closed set, by part (ii). Therefore, *p-cl(p-cl(F_B)) = p-cl(F_B)*.
(iv) Suppose that $F_B \widetilde{\subseteq} F_C$. *p-cl*(F_B)= ($\tilde{\cap}${F_D : $F_B \widetilde{\subseteq} F_D$ and F_D belongs to soft pre-closed set in F_A} and *p-cl*(F_C)= ($\tilde{\cap}${F_E: $F_C \widetilde{\subseteq} F_E$ and F_E belongs to soft pre-closed set in F_A}. Since $F_B \widetilde{\subseteq}$ p-cl(F_B) and $F_C \widetilde{\subseteq}$ p-cl(F_C) $\Rightarrow F_B \widetilde{\subseteq} F_C \widetilde{\subseteq}$ p-cl(F_C) $\Rightarrow F_B \widetilde{\subseteq}$ p-cl(F_C). But *p-cl(F_B)* is the smallest soft pre-closed set containing F_B. Therefore, *p-cl(F_B)* $\widetilde{\subseteq}$*p-cl(F_C)*.

(v) Since $F_B \tilde{\cap} F_C \tilde{\subseteq} F_B$ and $F_B \tilde{\cap} F_C \tilde{\subseteq} F_C$, by part (iv), *p-cl*$(F_B \tilde{\cap} F_C) \tilde{\subseteq}$ *p-cl*(F_B) and *p-cl*$(F_B \tilde{\cap} F_C) \tilde{\subseteq}$ *p-cl*(F_C) . Thus *p-cl*$(F_B \tilde{\cap} F_C) \tilde{\subseteq}$ *p-cl*$(F_B) \tilde{\cap}$ *p-cl*(F_C).

(vii) Since $F_B \tilde{\subseteq} F_B \tilde{\cup} F_C$ and $F_C \tilde{\subseteq} F_B \tilde{\cup} F_C$, by part (iv). Then *p-cl*$(F_B) \tilde{\subseteq}$ *p-cl*$(F_B \tilde{\cup} F_C)$ and *p-cl*$(F_C) \tilde{\subseteq}$ *p-cl*$(F_B \tilde{\cup} F_C)$, which implies that *p-cl*$(F_B) \tilde{\cup}$ *p-cl*$(F_C) \tilde{\subseteq}$ *p-cl*$(F_B \tilde{\cup} F_C)$. Now, *p-cl*(F_B) and *p-cl*(F_C) are soft pre-closed sets in F_A, which implies that *p-cl*$(F_B) \tilde{\cup}$ *p-cl*(F_C) is a soft pre-closed set in F_A. Then $F_B \tilde{\subseteq}$ p-cl(F_B) and $F_C \tilde{\subseteq}$ p-cl(F_C) imply that $F_B \tilde{\cup} F_C \tilde{\subseteq}$ p-cl$(F_B) \tilde{\cup}$ *p-cl*(F_C). That is *p-cl*$(F_B) \tilde{\cup}$ *p-cl*(F_C) is a soft pre-closed set containing $F_B \tilde{\cup} F_C$. But *p-cl*$(F_B \tilde{\cup} F_C)$ is the smallest soft pre-closed set containing $F_B \tilde{\cup} F_C$.
Hence *p-cl*$(F_B \tilde{\cup} F_C) \tilde{\subseteq}$ *p-cl*$(F_B) \tilde{\cup}$ *p-cl*(F_C). Therefore, *p-cl*$(F_B \tilde{\cup} F_C)$ = *p-cl*$(F_B) \tilde{\cup}$ *p-cl*(F_C). □

Theorem 3.15: Let $(F_A, \tilde{\tau})$ be a soft topological space and let F_B and F_C be soft subsets of F_A. Then the following holds.

(i) *p-int*$(F_\phi) = F_\phi$ *and p-int*$(F_A) = F_A$.
(ii) F_B is a soft pre-open set if and only if F_B = *p-int*(F_B).
(iii) *p-int(p-int*(F_B)*)=p-int*(F_B).
(iv) $F_B \tilde{\subseteq} F_C$ implies *p-int*$(F_B) \tilde{\subseteq}$ *p-int*(F_C).
(v) *p-int*$(F_B) \tilde{\cap}$ *p-int*$(F_C) \tilde{\subseteq}$ *p-int*$(F_B \tilde{\cap} F_C)$.
(vi) *p-int*$(F_B \tilde{\cup} F_C)$ = *p-int*$(F_B) \tilde{\cup}$ *p-int*(F_C).

Proof:

(i) is obvious.

(ii) If F_B is soft pre-open set, then F_B is itself a soft pre-open set in F_A which contains F_B. So p-int(F_B) is the largest soft pre-open set contained in F_B and F_B = *s-int*(F_B). Conversely, suppose that F_B = *p-int*(F_B). Since *p-int*(F_B) being the union of soft pre-open sets is soft pre-open sets, so *p-int*(F_B) be a soft pre-open set of F_A which implies that F_B is soft pre-open set in F_A.

(iii) Since *p-int*(F_B) is a soft pre-open set, by part (ii). Therefore, *p-int(p-int*(F_B)*)* = *p-int*(F_B).

(iv) Suppose that $F_B \tilde{\subseteq} F_C$. *p-int*$(F_B)$ = $\tilde{\cup}\{F_D : F_D \tilde{\subseteq} F_B$ and F_D be a soft pre-open set in $F_A\}$ and *p-int*(F_C) = $\tilde{\cup}\{F_E: F_E \tilde{\subseteq} F_C$ and F_E be a soft pre-open set in $F_A\}$. Since *p-int*$(F_B) \tilde{\subseteq} F_B \tilde{\subseteq} F_C$, *p-int*$(F_B) \tilde{\subseteq} F_C$. Since *p-int*$(F_C)$ is the largest soft pre-open set contained in F_C, *p-int*$(F_B) \tilde{\subseteq}$ *p-int*(F_C).

(v) Since $F_B \tilde{\cap} F_C \tilde{\subseteq} F_B$ and $F_B \tilde{\cap} F_C \tilde{\subseteq} F_C$, by part (iv), $F_B \tilde{\subseteq} F_C$ which implies that *p-int*$(F_B) \tilde{\subseteq}$ *p-int*(F_C). Since *p-int*$(F_B \tilde{\cap} F_C) \tilde{\subseteq}$ *p-int*(F_B) and *p-int*$(F_B \tilde{\cap} F_C) \tilde{\subseteq}$ *p-int*(F_C) which implies that *p-int*$(F_B \tilde{\cap} F_C) \tilde{\subseteq}$ *p-int*$(F_B) \tilde{\cap}$*p-int*(F_C). Now, *p-int*(F_B) and *p-int*(F_C) are soft pre-open sets in F_A. So that *p-int*$(F_B) \tilde{\cap}$ *p-int*(F_C) be a soft pre-open set in F_A.Then p-int$(F_B) \tilde{\subseteq} F_B$ and p-int$(F_C) \tilde{\subseteq} F_C$ which implies that p-int$(F_B) \tilde{\cap}$ p-int$(F_C) \tilde{\subseteq}$ *p-int*$(F_B \tilde{\cap} F_C)$. That is p-int$(F_B) \tilde{\cap}$ p-int(F_C) is a soft pre-open set contained in $F_B \tilde{\cap} F_C$. But, *p-int*$(F_B \tilde{\cap} F_C)$ is the largest soft pre-open set contained in $F_B \tilde{\cap} F_C$. Hence *p-int*$(F_B) \tilde{\cap}$ *p-int*$(F_C) \tilde{\subseteq}$ *p-int*$(F_B \tilde{\cap} F_C)$. Therefore, *p-int*$(F_B \tilde{\cap} F_C)$= *p-int*$(F_B) \tilde{\cap}$ *p-int*(F_C).

(vi) Since $F_B \tilde{\subseteq} F_B \tilde{\cup} F_C$ and $F_C \tilde{\subseteq} F_B \tilde{\cup} F_C$, by part (iv), $F_B \tilde{\subseteq} F_C$ which implies that *p-int*$(F_B) \tilde{\subseteq}$ *p-int*(F_C). Then *p-int*$(F_B) \tilde{\subseteq}$ *p-int*$(F_B \tilde{\cup} F_C)$ and *p-int*$(F_C) \tilde{\subseteq}$ *p-int*$(F_B \tilde{\cup} F_C)$, which implies that p-int$(F_B) \tilde{\cup}$ *p-int*$(F_C) \tilde{\subseteq}$ *p-int*$(F_B \tilde{\cup} F_C)$. Now, *p-int*(F_B) and *p-int*(F_C) are soft pre-open sets in F_A which implies that p-int$(F_B) \tilde{\cup}$ p-int(F_C) is a soft pre-open set in F_A. Then $F_B \tilde{\subseteq}$ p-int(F_B) and $F_C \tilde{\subseteq}$ *p-int*(F_C) imply that $F_B \tilde{\cup} F_C \tilde{\subseteq}$ *p-int*$(F_B) \tilde{\cup}$ *p-int*(F_C). That is p-int$(F_B) \tilde{\cup}$ s-int(F_C) is a soft pre-open set containing $F_B \tilde{\cup} F_C$. Hence *p-int*$(F_B \tilde{\cup} F_C) \tilde{\subseteq}$ *p-int*$(F_B) \tilde{\cup}$ *p-int*(F_C). Therefore, *p-int*$(F_B \tilde{\cup} F_C)$=*p-int*$(F_B) \tilde{\cup}$ *p- int*(F_C). □

Theorem 3.16: Let $(F_A, \tilde{\tau})$ be a soft topological space and let F_B be soft subset of F_A. Then the following are equivalent.

(i) F_B is a soft pre-closed set
(ii) *int*$(cl(F_B)) \tilde{\subseteq} F_B$
(iii) *cl*$(int(F_B^c)) \tilde{\supseteq} F_B^c$
(iv) F_B^c is a soft pre-open set.

Proof:

(i) ⇒ (ii): If F_B is a soft pre-closed set, then there exist soft closed set F_C such that *int*$(F_C) \tilde{\subseteq} F_B \tilde{\subseteq} F_C \Rightarrow$ *int*$(F_C) \tilde{\subseteq} F_B \tilde{\subseteq}$ *cl*$(F_B) \tilde{\subseteq} F_C$. By the property of interior, we then have *int*$(cl(F_B)) \tilde{\subseteq}$ *int*$(F_C) \tilde{\subseteq} F_B$.

(ii)⇒(iii): *int*$(cl(F_B)) \tilde{\subseteq} F_B \Rightarrow F_B^c \tilde{\subseteq}$ *int*$(cl(F_B))^c$ =*cl*$(int(F_B^c))$.

(iii)⇒(iv): *int*(F_B^c) is an soft open set such that *int*$(F_B^c) \tilde{\subseteq} F_B^c \tilde{\subseteq}$ *cl*$(int(F_B^c))$. Hence F_B^c is a soft pre-open set.

(iv)⇒(i): Given that F_B^c is a soft pre-open set. Therefore, $(F_B^c)^c = F_B$ is a soft pre-closed set. □

Theorem 3.17: Let F_B be a soft subset of a soft topological space $(F_A, \tilde{\tau})$. Then the following holds.

(i) p-cl(F_B)= $F_B \tilde{\cup}$ cl(int(F_B))
(ii) p-int(F_B)= $F_B \tilde{\cap}$ int(cl(F_B)).

Proof:

(i) $cl[int[F_B \tilde{\cup}\ cl(int(F_B))]] \tilde{\subseteq} cl[int(F_B) \tilde{\cup}\ cl(int(F_B))] = cl(int(F_B)) \tilde{\subseteq} F_B \tilde{\cup}\ cl(int(F_B))$ by Lemma 2.18. Hence $F_B \tilde{\cup}\ cl(int(F_B))$ is a soft pre-closed set and thus $p\text{-}cl(F_B) \tilde{\subseteq} F_B \tilde{\cup}\ cl(int(F_B))$. On the other hand, since p-cl(F_B) is a soft pre-closed set, we have $cl(int(F_B)) \tilde{\subseteq} cl(int(p\text{-}cl(F_B))) \tilde{\subseteq} p\text{-}cl(F_B)$. Hence $F_B \tilde{\cup}\ cl(int(F_B)) \tilde{\subseteq} p\text{-}cl(F_B)$ which implies that $p\text{-}cl(F_B) = F_B \tilde{\cup}\ cl(int(F_B))$.

(ii) is a consequence of (i). □

Theorem 3.18: Let F_B be a soft subset of a soft topological space $(F_A, \tilde{\tau})$. Then the following holds.

(i) $cl(p\text{-}int(F_B)) = cl(int(cl(F_B)))$

(ii) $p\text{-}cl(p\text{-}int(F_B)) = p\text{-}int(F_B) \tilde{\cup}\ cl(int(F_B))$.

Proof:

(i) Let $cl(p\text{-}int(F_B)) = cl[F_B \tilde{\cap}\ int(cl(F_B))]$ by Lemma 2.18. $\tilde{\subseteq} cl(F_B) \tilde{\cap}\ cl(int(cl(F_B))) \tilde{\subseteq} cl(int(cl(F_B)))$. Now, $cl(int(cl(F_B))) \tilde{\subseteq} cl[int(cl(F_B))) \tilde{\cap}\ cl(F_B)] \tilde{\subseteq} cl(cl(int(cl(F_B))) \tilde{\cap}\ F_B) \tilde{\subseteq} cl(int(cl(F_B)) \tilde{\cap}\ F_B)$ [by Lemma 2.18.] $\tilde{\subseteq} cl(p\text{-}int(F_B))$. *Hence* $cl(int(cl(F_B))) = cl(p\text{-}int(F_B))$.

(ii) Since soft open set is a soft pre-open set, we have $intl(F_B) \tilde{\subseteq} p\text{-}int(F_B) \tilde{\subseteq} F_B$. Therefore, $int(p\text{-}int(F_B)) = int(F_B)$. Now, by Theorem 3.17(ii) $p\text{-}cl(p\text{-}int(F_B)) = p\text{-}int(F_B) \tilde{\cup}\ cl(int(p\text{-}int(F_B)))\ p\text{-}int(F_B) \tilde{\cup}\ cl(int(F_B))$.

Theorem 3.19: Let $(F_A, \tilde{\tau})$ be a soft topological space. Then for any $F_B \tilde{\subseteq} F_A$, $p\text{-}int(F_B) = F_A - p\text{-}cl(F_A - F_B)$.

Proof: Let $F_A - p\text{-}int(F_B) = F_A - [F_B \tilde{\cap}\ int(cl(F_B))] = [F_A - F_B] \tilde{\cup} [F_A - int(cl(F_B))] = [F_A - F_B] \tilde{\cup}\ cl[F_A - cl(F_B)] = [F_A - F_B] \tilde{\cup}\ cl(int(F_A - F_B)] = p\text{-}cl[F_A - F_B]$, by Proposition 2.19 and Theorem 3.17. Therefore, $p\text{-}int(F_B) = F_A - p\text{-}cl(F_A - F_B)$. □

REFERENCES

1. Arockiarani and A. Arokia Lancy, 'Generalized soft $g\beta$ closed sets and soft $gs\beta$ closed sets in soft topological spaces', International Journal of Mathematical Archive-4(2), (2013), 17-23.
2. Bin Chen, 'Some Local Properties of soft semi open sets', Hindawi Publishing Corporation Discrete Dynamics in Nature and Society, 57(2009), 1547-1553.
3. B.V.S.T. Sai and V. Srinivasa kumar, 'On soft semi-open sets and soft semi-topology', International Journal of Mathematical Archive-4(4), (2013), 114-117.
4. D. Molodtsov, 'Soft set theory-first results', Computers and Mathematics with Applications, 37(4-5), (1999), 19-31.
5. J. Krishnaveni and C. Sekar, 'Soft semi connected and soft locally semi connected properties in soft topological spaces', International Journal of Mathematics and soft Computing, 3(3), (2013), 85-91.
6. Keun Min. W., 'A Note on Soft Topological Spaces', Comp. Math. Appl., 62, (2011), 3524-3528.
7. Li.F., ' Notes on the soft operations', Arpn Journal of system and software, 66, (2011), 205-208.
8. M. Shabir and M. Naz, 'Some properties of soft topological spaces', Computers and Mathematics with Applications, 62, (2011), 4058-4067.
9. M. Shabir and M. Naz, 'On soft topological spaces', Computers and Mathematics with Applications, 61(7), (2011), 1786-1799.
10. Naim Çağman, Serkan Karataş, and Serdar Enginoglu, 'Soft topology', Computers and Mathematics with Applications, 62, 351-358, doi: 10.1016/j.camwa.2011.05.016, 2011.
11. P.K. Maji, R.Biswas, and A.R. Roy, 'Soft set theory', Computers and Mathematics with Applications, 45 (4-5), (2003), 555-562.
12. I.Zorlutuna, M.Akdag, W.K.Min, and S.Atmaca, 'Remarks on soft topological spaces', Annals of Fuzzy Mathematics and Informatics, 3(2), (2012), 171-185.
13. D.Sivaraj and V.E.Sasikala, 'A Study on Soft $\alpha-$ open sets', IOSR Journal of Mathematics, 12, (2016), 70-74 .
14. V.E. Sasikala and D. Sivaraj, 'On Soft Semi-Open Sets', International Journal of Scientific & Engineering Research, 7(12), 2016), 31-35.

Source of support: Nil, Conflict of interest: None Declared.