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# A STUDY ON SOFT PRE-OPEN SETS

**V. E. SASIKALA*, D. SIVARAJ**

**Meenakshi Academy of Higher Education and Research,
Meenakshi University, Chennai, Tamil Nadu, India.**

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***ABSTRACT***

*The objective of this paper is to describe the concepts of soft pre-open sets and soft pre-closed sets in soft topological spaces by studying the basics of soft set theory by D. Molodstov's description.*

***Keywords:*** *soft set, soft topology, soft open sets and soft closed sets.*

***Subject Classification:*** *54A10, 54A05.*

## 1. INTRODUCTION

Several researchers followed Molodtsov [4] 1999, after his introduction of Soft set theory as a common mathematical application in dealing with the vagueness of not well defined objects, many researchers followed him. When several proved mathematical applications have more clarity while applying on formal modeling, reasoning and computing, but few of the engineering sciences, life sciences, social sciences and ecological sciences does not have clarity many times. Shabir and Naz [10] described the soft topological spaces and its basic notations in detail.

The equality of two soft sets, subset and superset of Soft set, complement of a soft set, null soft set and absolute soft set with examples were well defined by Maji [7]. As the researchers now have several latest sophisticated techniques, they have now applied them in operations research, Riemans integration, Game theory, theory of probability and researched several basic notations of soft set theory. Naim *et al.*, [8] presented the foundations of the theory of soft topological spaces to open the door for experiments for soft mathematical concepts and structures that are based on soft set-theoretic operations. Recently Soft topology has been studied in depth in the papers [1, 2, 3, 5, 6, 9, 11, 12, 13, 14]. In this paper, we make a theoretical study of the new set called soft pre-open set and soft pre-closed sets in soft topological space.

## 2. PRELIMINARIES

The following definitions are essential for the development of the paper.

**Definition 2.1 [4]:** Let $U$ be an initial universe and $E$ be a set of parameters. Let $P(U)$ denote the power set of $U$. The pair $(F, E)$ or simply $F_E$, is called a *soft set* over $U$, where $F$ is a mapping given by $F: E \to P(U)$. In other words, a soft set over $U$ is a parameterized family of subsets of the universe. For $e \in U$, $F(e)$ may be considered as the set of e-approximate elements of the soft set $F$. The collection of all soft sets over $U$ and $E$ is denoted by $S(U)$. If $A \subseteq E$, then the pair $(F, A)$ or simply $F_A$, is called a *soft set* over $U$, where $F$ is a mapping $F: A \to P(U)$. Note that for $e \notin A$, $F(e) = \emptyset$.

**Definition 2.2 [11]:** The *union of two soft sets* of $F_B$ and $G_C$ over the common universe $U$ is the soft set $H_D$, where $B$ and $C$ are subsets of the parameter set $E$, $D = B \cup C$ and for all $e \in D$, $H(e) = F(e)$ if $e \in B - C$, $H(e) = G(e)$ if $e \in C - B$ and $H(e) = F(e) \cup G(e)$ if $e \in B \cap C$, we write $F_B \tilde{\cup} G_C = H_D$.

**Definition 2.3 [11]:** The *intersection of two soft sets* of $F_B$ and $G_C$ over the common universe $U$ is the soft set $H_D$, where $D = B \cap C$ and for all $e \in D$, $H(e) = F(e) \cap G(e)$ if $D = B \cap C$. We write $F_B \tilde{\cap} G_C = H_D$.

***Corresponding Author: V. E. Sasikala*[1]***
***[1]No: 10-A, Rainbow Avenue, 2nd street, Keelkattalai, Chennai-600117, Tamil Nadu, India.***

**Definition 2.4 [11]:** Let $F_B$ and $G_C$ be soft sets over a common universe set $U$ and $B, C \subseteq E$. Then $F_B$ is a *soft subset* of $G_C$, denoted by $F_B \tilde{\subseteq} G_C$, if (i) $B \subseteq C$ and (ii) for all $e \in B$, $F(e) = G(e)$. Also $G_C$, is called the soft super set of $F_B$ and is denoted by $F_B \tilde{\supseteq} G_C$.

**Definition 2.5 [11]:** The soft sets $F_B$ and $G_C$ over a common universe set $U$ are said to be *soft equal*, if $F_B \tilde{\subseteq} G_C$, and $F_B \tilde{\supseteq} G_C$. Then we write $F_B = G_C$.

**Definition 2.6 [11]:** A soft set $F_B$ over $U$ is called a *null soft set* denoted by $F_\phi$, if for all $e \in B$, $F(e) = \phi$.

**Definition 2.7 [10]:** The *relative complement* of a soft set $F_A$, denoted by $F_A^c$, is defined by the approximate function $f_{A^c}(e) = f_A^c(e)$, where $f_A^c(e)$ is the complement of the set $f_A(e)$, that is $f_A^c(e) = U - f_A(e)$ for all $e \in E$. It is easy to see that $(F_A^c)^c = F_A$, $F_\phi^c = F_E$ and $F_E^c = F_\phi$.

**Definition 2.8 [11]:** Let $U$ be an initial universe and $E$ be a set of parameters. If $B \subseteq E$, the soft set $F_B$ over $U$ is called an *absolute soft set*, if for all $e \in B$, $F(e) = U$.

The following is the definition of soft topology used by various authors.

**Definition 2.9 [1]:** Let $U$ be an initial universe and $E$ be a set of parameters. Let $\tau$ be a subcollection of $S(U)$, the collection of soft sets defined on $U$. Then $\tau$ is a *soft topology* if it satisfies the following conditions.

(i) $F_\phi, F_E \in \tau$
(ii) The union of any number of soft sets in $\tau$ belongs to $\tau$.
(iii) The intersection of any two soft sets in $\tau$ belongs to $\tau$.

If this definition is considered for further development, then the results are similar to that of results in topological spaces. Therefore, throughout the paper the following definition of soft topology is used.

**Definition 2.10 [10]:** Let $U$ be an initial universe and $E$ be a set of parameters. Let $F_A \in S(U)$. A *soft topology* on $F_A$, denoted by $\tilde{\tau}$, is a collection of soft subsets of $F_A$ having the following properties:

(i) $F_\phi,\ F_A \in \tilde{\tau}$
(ii) $\{F_{A_i} \tilde{\subseteq} F_A : i \in I\} \tilde{\subseteq} \tilde{\tau} \Rightarrow \tilde{\cup}_{i \in I} F_{A_i} \in \tilde{\tau}$
(iii) $\{F_{A_i} \tilde{\subseteq} F_A : 1 \le i \le n,\ n \in \mathrm{N}\} \tilde{\subseteq} \tilde{\tau} \Rightarrow \tilde{\cap}_{i=1}^{n} F_{A_i} \in \tilde{\tau}$.
(iv) The pair $(F_A, \tilde{\tau})$ is called a soft topological spaces.

**Definition 2.11 [9]:** Let $(F_A, \tilde{\tau})$ be a soft topological space in $F_A$. Elements of $\tilde{\tau}$ are called *soft open* sets. A soft set $F_B$ in $F_A$ is said to be a *soft closed* set in $F_A$, if its relative complement $F_B^c$, belongs to $\tilde{\tau}$.

**Definition 2.12 [12]:** Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B$ be a soft subset of $F_A$.

(i) The *soft interior* of $F_B$ is the soft set *int*($F_B$) = $\tilde{\cup}$ $\{F_C : F_C$ is soft open and $F_C \tilde{\subseteq} F_B\}$.
(ii) The *soft closure* of $F_B$ is the soft set cl($F_B$) = $\tilde{\cap}$ $\{F_C : F_C$ is soft closed and $F_B \tilde{\subseteq} F_C\}$.

Its clearly int($F_B$) is the largest soft open set contained in $F_B$ and cl($F_B$) is the smallest soft closed set containing $F_B$.

**Lemma 2.13 [12]:** Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B$ and $F_C$ be a soft subsets of $F_A$. Then the following holds.

*(i) int(int($F_B$)) = int($F_B$).*
(ii) $F_B \tilde{\subseteq} F_C$ implies int($F_B$) $\tilde{\subseteq}$ int($F_C$).
*(iii) int($F_B$) $\tilde{\cap}$ int($F_C$) = int($F_B \tilde{\cap} F_C$).*
*(iv) int($F_B$) $\tilde{\cup}$ int($F_C$) $\tilde{\subseteq}$ int($F_B \tilde{\cup} F_C$).*
(v) $F_B$ is soft open set if and only if $F_B$ = int($F_B$).

**Lemma 2.14 [12]:** Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B$ and $F_C$ be a soft set in $F_A$. Then the following hold.

*(i) cl(cl($F_B$)) = cl($F_B$).*
(ii) $F_B \tilde{\subseteq} F_C$ implies cl($F_B$) $\tilde{\subseteq}$ cl($F_C$).
*(iii) cl($F_B$) $\tilde{\cap}$ cl($F_C$) $\tilde{\supseteq}$ cl($F_B \tilde{\cap} F_C$).*
*(iv) cl($F_B$) $\tilde{\cup}$ cl($F_C$) = cl($F_B \tilde{\cup} F_C$).*
(v) $F_B$ is soft closed set if and only if $F_B$ = cl($F_B$).

**Lemma 2.15 [9]:** Arbitrary union of soft open sets is soft open and finite intersection of soft closed sets is soft closed set.

 

**Proposition 2.16 [2]:** Let $(F_A, \tilde{\tau})$ be a soft topological space over $F_A$. Then the following hold.

(i) $F_\emptyset, F_A$ are soft closed sets in $F_A$.
(ii) The union of any two soft closed sets is a soft closed set in $F_A$.
(iii) The intersection of any two soft closed sets is a soft closed set in $F_A$.

**Proposition 2.17 [12]:** If $\{F_{B_\alpha} | \alpha \in I\}$ is a collection of soft sets, then the following hold.

(i) $\tilde{\cup}\ \mathrm{int}\ (F_{B_\alpha}) \tilde{\subseteq} \mathrm{int}\ (\tilde{\cup} F_{B_\alpha})$
(ii) $\tilde{\cup}\ \mathrm{cl}(F_{B_\alpha}) \tilde{\subseteq} \mathrm{cl}(\tilde{\cup} F_{B_\alpha})$.

**Lemma 2.18 [13]:**

(i) For every soft open set $F_B$ in a soft topological space $(F_A, \tilde{\tau})$ and every soft set $F_C$, we have $\mathrm{cl}(F_C)\ \tilde{\cap}\ F_B \tilde{\subseteq}\ \mathrm{cl}(F_C \tilde{\cap} F_B)$.
(ii) For every soft closed set in $F_B$ a soft topological space $(F_A, \tilde{\tau})$ and every soft set $F_C$, we have $\mathrm{int}(F_B \tilde{\cup} F_C) \tilde{\subseteq}\ \mathrm{int}(F_B) \tilde{\cup} F_C$.

**Proposition 2.19 [7]:** If $F_B$ and $F_C$ are any two soft sets in $(F_A, \tilde{\tau})$, then the following hold
$F_A - (F_B - F_C) = (F_A - F_B) \tilde{\cup} (F_A - F_C)$.

## 3. SOFT PRE-OPEN SET AND SOFT PRE-CLOSED SET

This section is devoted to the study of soft pre-open sets and soft pre- closed sets.

**Definition 3.1:** Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B$ be a soft subset of $F_A$. $F_B$ is said to be a *soft pre-open set*, if $F_B \tilde{\subseteq}$ *int(cl($F_B$)).*

**Example 3.2:** Let $U=\{a,b,c\}$, $E=\{e_1,e_2,e_3\}$, $A=\{e_1,e_2\} \tilde{\subseteq} E$.

$F_A=\{(e_1,\{a,b\}),(e_2,\{b,c\})\}$, $F_1=\{(e_1,\{a\})\}$, $F_2=\{(e_1,\{b\})\}$ $F_3=\{(e_1,\{a,b\})\}$,

$F_4=\{(e_2,\{b\})\}$, $F_5=\{(e_1,\{c\})\}$, $F_6=\{(e_2,\{b,c\})\}$, $F_7=\{(e_1,\{a\}),(e_2,\{b\})\}$,

$F_8=\{(e_1,\{a\}),(e_2,\{c\})\}$, $F_9=\{(e_1,\{a\})(e_2,\{b,c\})\}$, $F_{10}=\{(e_1,\{b\})(e_2,\{b\})\}$, $F_{11}=\{(e1,\{b\})(e_2,\{c\})\}$,

$F_{12}=\{(e_1,\{b\})(e_2,\{b,c\})\}$, $F_{13}=\{(e1,\{a,b\}),(e_2,\{b\})\}$,

$F_{14}=\{(e_1,\{a,b\}),(e_2,\{c\})\}$, $F_{15}=F_A$ and $F_{16}=F_\emptyset$.

Let $\tilde{\tau}=\{F_\emptyset, F_A, F_1, F_3, F_7, F_9, F_{13}\}$. Then $(F_A, \tilde{\tau})$ is a soft topological space. The family of all soft closed sets is $\{F_\emptyset, F_A, F_{12}, F_6, F_{11}, F_2, F_5\}$. The family of soft pre-open sets is $\{F_A, F_\emptyset, F_1, F_3, F_4, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14}\}$. The family of soft pre-closed sets is $\{F_A, F_\emptyset, F_1, F_2, F_4, F_5, F_6, F_8, F_{10}, F_{11}, F_{12}, F_{14}\}$.

**Theorem 3.3:** Every soft open set in a soft topological space $(F_A, \tilde{\tau})$ is a soft pre-open set.

**Proof:** The proof follows from the Definition 3.1.

The following Example 3.4 shows that the converse implication of Theorem 3.3 is not true.

**Example 3.4:** Consider the soft topological space of Example 3.2. Here $F_4$, $F_8$, $F_{10}$, $F_{12}$ and $F_{14}$ are soft pre-open sets but not soft open sets, since $F_4, F_8, F_{10}, F_{12}, F_{14} \notin \tilde{\tau}$.

**Remark 3.5:** $F_\emptyset$ and $F_A$ are always soft pre-closed sets and soft pre-open sets.

**Theorem 3.6:** Arbitrary union of soft pre-open sets is a soft pre-open set.

**Remark 3.7:** Arbitrary intersection of soft pre-closed sets is a soft pre-closed set.

The following Example 3.8 shows that the finite intersection of soft pre-open sets need not be a soft pre-open set.

**Example 3.8:** In Example 3.2, $F_B = F_3 = \{(e_1, \{a,b\})\}$ and $F_C = F_{10} = \{(e_1, \{b\}), (e_2, \{b\})\}$ are soft pre-open sets. But $F_B \tilde{\cap} F_C = F_2 = \{(e_1, \{b\})\}$ is not a soft pre-open set.

**Theorem 3.9:** If $F_B$ is a soft pre-open set such that $F_C \tilde{\subseteq} F_B \tilde{\subseteq}$ cl($F_C$), then $F_C$ is also a soft pre-open set.

**Proof:** $F_B \tilde{\subseteq}$ cl($F_C$) implies that cl($F_B$)$\tilde{\subseteq}$ cl($F_C$) and so $F_B \tilde{\subseteq}$ *int(cl($F_B$))* $\tilde{\subseteq}$ *int(cl($F_C$))* which implies that $F_C \tilde{\subseteq}$ int(cl($F_C$)). Therefore, $F_C$ is a soft pre-open set.

**Theorem 3.10:** Let $F_B$ be a soft subset of a soft topological space $(F_A, \tilde{\tau})$. Then $F_B$ is soft pre-closed set if and only if cl(int($F_B$)) $\tilde{\subseteq} F_B$.

**Proof:** Let $F_B$ be a soft pre-closed set. Then $(F_B)^c$ is a soft pre-open set. So $(F_B)^c \tilde{\subseteq}$ *int(cl($(F_B)^c$))* $\Rightarrow$ $(F_B)^c \tilde{\subseteq}$ *[cl(int($F_B$))]$^c$* which implies that $(F_B) \tilde{\supseteq}$ cl(int($F_C$)). Conversely, *cl(int($F_B$))* $\tilde{\subseteq} F_B \Rightarrow$*[cl(int($F_B$))]$^c$* $\tilde{\supseteq} (F_B)^c$ which implies that int(cl($F_B$)$^c$)] $\tilde{\supseteq} (F_B)^c$. Therefore, $F_B^c$ is a soft pre-open set and so $F_B$ is a soft pre-closed set.

**Theorem 3.11:** Let $(F_A, \tilde{\tau})$ be a soft topological space in which every soft subset is pre-open if and only if every soft open set in $(F_A, \tilde{\tau})$ is soft closed set.

**Proof:** Suppose that every soft open set is a soft closed set. Let $F_B$ be a soft set. Let $F_C = F_B^c$ and so, $F_B = F_C^c$. Since int($F_B$) is a soft open set, by hypothesis, *int($F_B$)* is a soft closed set and so *cl(int($F_B$))=int($F_B$)* $\tilde{\subseteq} F_B = F_C^c$. Then $F_C \tilde{\subseteq}$*(cl(int($F_B$)))$^c$ = int(cl($F_B^c$)) = int(cl($F_C$))*. Every soft subset is soft pre-open set. Conversely, let $F_B$ be a soft open set. Since $F_B^c$ is a soft set, by hypothesis, $F_B^c \tilde{\subseteq}$*int(cl($F_B^c$))* and so *cl(int($F_B$))* $\tilde{\subseteq} F_B$ which implies that *cl*($F_B$)$\tilde{\subseteq} F_B$. Therefore, every soft open set is a soft closed set. This complete the proof.

**Definition 3.12:** Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B$ be a soft subset of $F_A$.

(i) The *soft pre-interior* of.$F_B$ is the soft set *p-int($F_B$)* =$\tilde{\cup}$ {$F_C$: $F_C$ is soft pre-open set and $F_C \tilde{\subseteq} F_B$}.

(ii) The *soft pre-closure* of $F_B$ is the soft set *p-cl($F_B$)* =$\tilde{\cap}$ {$F_C$: $F_C$ is soft pre-closed set and $F_B \tilde{\subseteq} F_C$}.

Its clearly, *p-int($F_B$)* is the largest soft pre-open set contained in $F_B$ and *p-cl($F_B$)* is the smallest soft pre-closed set containing $F_B$.

**Theorem 3.13:** Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B$ be a soft subset of $F_A$. Then the following holds.

*(i) (p-cl($F_B$))$^c$ = p-int($F_B^c$)*
*(ii) (p-int($F_B$))$^c$ = p-cl($F_B^c$)*
*(iii) (p-int($F_B$))$^c$ = p-cl($F_B^c$)*

**Proof:**

(i) ($p$-cl($F_B$))$^c$= ($\tilde{\cap}${$F_C$: $F_C$ is soft pre-closed set and $F_C \tilde{\subseteq} F_B$})$^c$
= $\tilde{\cup}${$F_C^c$: $F_C^c$ is soft pre-open set and $F_C^c \tilde{\subseteq} F_B^c$}
= $\tilde{\cup}${$F_C^c$: $F_C^c$ is soft pre-open set and $F_C^c \tilde{\subseteq} F_B^c$}= p-int($F_B^c$).

(ii) ($s$-int($F_B$))$^c$ = ($\tilde{\cup}${$F_C$: $F_C$ is soft pre-open set and $F_C \tilde{\subseteq} F_B$})$^c$
= $\tilde{\cap}${$F_C^c$ : $F_C^c$ is soft pre-open set and $F_B^c \tilde{\subseteq} F_C^c$ }
= $\tilde{\cap}${$F_C^c$: $F_C^c$ is soft pre-closed set and $F_B^c \tilde{\subseteq} F_C^c$ }= p-cl($F_B^c$).

(iii) The proof follows from (ii). □

**Theorem 3.14:** Let $(F_A, \tilde{\tau})$ be a soft topological space and let $F_B$ and $F_C$ be soft subsets of $F_A$. Then the following holds.

(i) $p$-cl($F_\phi$ ) = $F_\phi$ and $p$-cl($F_A$ ) = $F_A$.
(ii) $F_B$ is a soft pre-closed set if and only if $F_B$=p-cl($F_B$).
(iii) $p$-cl($p$-cl($F_B$)) = $p$-cl($F_B$).
(iv) $F_B \tilde{\subseteq} F_C$ implies $p$-cl($F_B$) $\tilde{\subseteq}$ $p$-cl($F_C$).
(v) $p$-cl($F_B \tilde{\cap} F_C$) $\tilde{\subseteq}$ $p$-cl($F_B$) $\tilde{\cap}$ $p$-cl($F_C$).
(vi) $p$-cl($F_B \tilde{\cup} F_C$) = $p$-cl($F_B$) $\tilde{\cup}$ $p$-cl($F_C$).

**Proof:**

(i) is obvious.
(ii) If $F_B$ is soft pre-closed set, then $F_B$ is itself a soft pre-closed set in $F_A$ which contains $F_B$ . Since p-cl($F_B$) is the smallest soft pre-closed set containing $F_B$, and $F_B$ = *p-cl($F_B$)*. Conversely, suppose that $F_B$ = p-cl($F_B$). Since p-cl($F_B$) being the intersection of soft pre-closed set is soft pre-closed set, $F_B$ is soft pre-closed sets in $F_A$.
(iii) Since p-cl($F_B$) is a soft pre-closed set, by part (ii). Therefore, *p-cl(p-cl($F_B$)) = p-cl($F_B$)*.
(iv) Suppose that $F_B \tilde{\subseteq} F_C$ . *p-cl*($F_B$)= ($\tilde{\cap}${$F_D$ : $F_B \tilde{\subseteq} F_D$ and $F_D$ belongs to soft pre-closed set in $F_A$} and *p-cl*($F_C$)= ($\tilde{\cap}${$F_E$: $F_C \tilde{\subseteq} F_E$ and $F_E$ belongs to soft pre-closed set in $F_A$}. Since $F_B \tilde{\subseteq}$ p-cl($F_B$) and $F_C \tilde{\subseteq}$ p-cl($F_C$) $\Rightarrow F_B \tilde{\subseteq} F_C \tilde{\subseteq}$ p-cl($F_C$) $\Rightarrow F_B \tilde{\subseteq}$ p-cl($F_C$). But *p-cl($F_B$)* is the smallest soft pre-closed set containing $F_B$. Therefore, *p-cl($F_B$)* $\tilde{\subseteq}$*p-cl($F_C$)*.

 

(v) Since $F_B \tilde{\cap} F_C \tilde{\subseteq} F_B$ and $F_B \tilde{\cap} F_C \tilde{\subseteq} F_C$, by part (iv), *p-cl*($F_B \tilde{\cap} F_C$) $\tilde{\subseteq}$ *p-cl*($F_B$) and *p-cl*($F_B \tilde{\cap} F_C$) $\tilde{\subseteq}$ *p-cl*($F_C$) . Thus *p-cl*($F_B \tilde{\cap} F_C$) $\tilde{\subseteq}$ *p-cl*($F_B$) $\tilde{\cap}$ *p-cl*($F_C$).

(vii) Since $F_B \tilde{\subseteq} F_B \tilde{\cup} F_C$ and $F_C \tilde{\subseteq} F_B \tilde{\cup} F_C$, by part (iv). Then *p-cl*($F_B$) $\tilde{\subseteq}$ *p-cl*($F_B \tilde{\cup} F_C$) and *p-cl*($F_C$) $\tilde{\subseteq}$ *p-cl*($F_B \tilde{\cup} F_C$), which implies that *p-cl*($F_B$) $\tilde{\cup}$ *p-cl*($F_C$) $\tilde{\subseteq}$ *p-cl*($F_B \tilde{\cup} F_C$). Now, *p-cl*($F_B$) and *p-cl*($F_C$) are soft pre-closed sets in $F_A$, which implies that *p-cl*($F_B$) $\tilde{\cup}$ *p-cl*($F_C$) is a soft pre-closed set in $F_A$. Then $F_B \tilde{\subseteq}$ p-cl($F_B$) and $F_C \tilde{\subseteq}$ p-cl($F_C$) imply that $F_B \tilde{\cup} F_C \tilde{\subseteq}$ p-cl($F_B$) $\tilde{\cup}$ *p-cl*($F_C$). That is *p-cl*($F_B$) $\tilde{\cup}$ *p-cl*($F_C$) is a soft pre-closed set containing $F_B \tilde{\cup} F_C$. But *p-cl*($F_B \tilde{\cup} F_C$) is the smallest soft pre-closed set containing $F_B \tilde{\cup} F_C$.
Hence *p-cl*($F_B \tilde{\cup} F_C$) $\tilde{\subseteq}$ *p-cl*($F_B$) $\tilde{\cup}$ *p-cl*($F_C$). Therefore, *p-cl*($F_B \tilde{\cup} F_C$) = *p-cl*($F_B$ ) $\tilde{\cup}$ *p-cl*($F_C$). □

**Theorem 3.15:** Let $(F_A, \tilde{\tau})$ be a soft topological space and let $F_B$ and $F_C$ be soft subsets of $F_A$. Then the following holds.

*(i) p-int*($F_\phi$ ) = $F_\phi$ *and p-int*($F_A$) = $F_A$.

(ii) $F_B$ is a soft pre-open set if and only if $F_B$ = *p-int*($F_B$).

(iii) *p-int*(*p-int*($F_B$))=*p-int*($F_B$).

(iv) $F_B \tilde{\subseteq} F_C$ implies *p-int*($F_B$) $\tilde{\subseteq}$ *p-int*($F_C$).

*(v) p-int*($F_B$)$\tilde{\cap}$*p-int*($F_C$) $\tilde{\subseteq}$ *p-int*($F_B \tilde{\cap} F_C$).

(vi) *p-int*($F_B \tilde{\cup} F_C$) = *p-int*($F_B$) $\tilde{\cup}$ *p-int*($F_C$).

**Proof:**

(i) is obvious.

(ii) If $F_B$ is soft pre-open set, then $F_B$ is itself a soft pre-open set in $F_A$ which contains $F_B$. So p-int($F_B$) is the largest soft pre-open set contained in $F_B$ and $F_B$ = *s-int*($F_B$). Conversely, suppose that $F_B$ = *p-int*($F_B$). Since *p-int*($F_B$) being the union of soft pre-open sets is soft pre-open sets, so *p-int*($F_B$) be a soft pre-open set of $F_A$ which implies that $F_B$ is soft pre-open set in $F_A$.

(iii) Since *p-int*($F_B$) is a soft pre-open set, by part (ii). Therefore, *p-int*(*p-int*($F_B$)) = *p-int*($F_B$).

(iv) Suppose that $F_B \tilde{\subseteq} F_C$. *p-int*($F_B$)= $\tilde{\cup}$\{$F_D$ : $F_D \tilde{\subseteq} F_B$ and $F_D$ be a soft pre-open set in $F_A$\} and *p-int*($F_C$)= $\tilde{\cup}$ \{$F_E$: $F_E \tilde{\subseteq} F_C$ and $F_E$ be a soft pre-open set in $F_A$\}. Since *p-int*($F_B$)$\tilde{\subseteq}$ $F_B \tilde{\subseteq} F_C$, *p-int*($F_B$) $\tilde{\subseteq} F_C$. Since *p-int*($F_C$) is the largest soft pre-open set contained in $F_C$, *p-int*($F_B$) $\tilde{\subseteq}$ *p-int*($F_C$).

(v) Since $F_B \tilde{\cap} F_C \tilde{\subseteq} F_B$ and $F_B \tilde{\cap} F_C \tilde{\subseteq} F_C$, by part (iv), $F_B \tilde{\subseteq} F_C$ which implies that *p-int*($F_B$) $\tilde{\subseteq}$ *p-int*($F_C$). Since *p-int*($F_B \tilde{\cap} F_C$) $\tilde{\subseteq}$ *p-int*($F_B$) and *p-int*($F_B \tilde{\cap} F_C$) $\tilde{\subseteq}$ *p-int*($F_C$) which implies that *p-int*($F_B \tilde{\cap} F_C$) $\tilde{\subseteq}$ *p-int*($F_B$)$\tilde{\cap}$*p-int*($F_C$). Now, *p-int*($F_B$) and *p-int*($F_C$) are soft pre-open sets in $F_A$. So that *p-int*($F_B$)$\tilde{\cap}$*p-int*($F_C$) be a soft pre-open set in $F_A$.Then p-int($F_B$) $\tilde{\subseteq} F_B$and p-int($F_C$) $\tilde{\subseteq} F_C$ which implies that p-int($F_B$)$\tilde{\cap}$p-int($F_C$) $\tilde{\subseteq}$ *p-int*($F_B \tilde{\cap} F_C$). That is p-int($F_B$)$\tilde{\cap}$p-int($F_C$) is a soft pre-open set contained in $F_B \tilde{\cap} F_C$. But, *p-int*($F_B \tilde{\cap} F_C$) is the largest soft pre-open set contained in $F_B \tilde{\cap} F_C$. Hence *p-int*($F_B$)$\tilde{\cap}$*p-int*($F_C$) $\tilde{\subseteq}$ *p-int*($F_B \tilde{\cap} F_C$). Therefore, *p-int*($F_B \tilde{\cap} F_C$)= *p-int*($F_B$)$\tilde{\cap}$*p-int*($F_C$).

(vi) Since $F_B \tilde{\subseteq} F_B \tilde{\cup} F_C$ and $F_C \tilde{\subseteq} F_B \tilde{\cup} F_C$, by part (iv), $F_B \tilde{\subseteq} F_C$ which implies that *p-int*($F_B$) $\tilde{\subseteq}$ *p-int*($F_C$). Then *p-int*($F_B$) $\tilde{\subseteq}$ *p-int*($F_B \tilde{\cup} F_C$) and *p-int*($F_C$) $\tilde{\subseteq}$ *p-int*($F_B \tilde{\cup} F_C$), which implies that p-int($F_B$) $\tilde{\cup}$ *p-int*($F_C$) $\tilde{\subseteq}$ *p-int*($F_B \tilde{\cup} F_C$). Now, *p-int*($F_B$) and *p-int*($F_C$) are soft pre-open sets in $F_A$ which implies that p-int($F_B$) $\tilde{\cup}$ p-int($F_C$) is a soft pre-open set in $F_A$. Then $F_B \tilde{\subseteq}$ p-int($F_B$) and $F_C \tilde{\subseteq}$ *p-int*($F_C$) imply that $F_B \tilde{\cup} F_C$ $\tilde{\subseteq}$ *p-int*($F_B$) $\tilde{\cup}$*p-int*($F_C$). That is p-int($F_B$) $\tilde{\cup}$ s-int($F_C$) is a soft pre-open set containing $F_B \tilde{\cup} F_C$. Hence *p-int*($F_B \tilde{\cup} F_C$) $\tilde{\subseteq}$ *p-int*($F_B$) $\tilde{\cup}$ *p-int*($F_C$). Therefore, *p-int*($F_B \tilde{\cup} F_C$)=*p-int*($F_B$) $\tilde{\cup}$ *p- int*($F_C$). □

**Theorem 3.16:** Let $(F_A, \tilde{\tau})$ be a soft topological space and let $F_B$ be soft subset of $F_A$. Then the following are equivalent.

(i) $F_B$ is a soft pre-closed set

*(ii) int*(*cl*($F_B$)) $\tilde{\subseteq} F_B$

*(iii) cl*(*int*($F_B^c$)) $\tilde{\supseteq} F_B^c$

(iv) $F_B^c$ is a soft pre-open set.

**Proof:**

**(i) ⇒ (ii):** If $F_B$ is a soft pre-closed set, then there exist soft closed set $F_C$ such that *int*($F_C$) $\tilde{\subseteq} F_B \tilde{\subseteq} F_C \Rightarrow$*int*($F_C$) $\tilde{\subseteq} F_B$ $\tilde{\subseteq}$*cl*($F_B$ ) $\tilde{\subseteq} F_C$. By the property of interior, we then have *int*(*cl*($F_B$)) $\tilde{\subseteq}$ *int*($F_C$) $\tilde{\subseteq} F_B$.

**(ii)⇒(iii):** *int*(*cl*($F_B$)) $\tilde{\subseteq} F_B \Rightarrow F_B^c \tilde{\subseteq}$ *int*(*cl*($F_B$))$^c$ =*cl*(*int*($F_B^c$)).

**(iii)⇒(iv):** *int*($F_B^c$) is an soft open set such that *int*($F_B^c$) $\tilde{\subseteq} F_B^c \tilde{\subseteq}$ *cl*(*int*($F_B^c$)). Hence $F_B^c$ is a soft pre-open set.

**(iv)⇒(i):** Given that $F_B^c$ is a soft pre-open set. Therefore, $(F_B^c)^c = F_B$ is a soft pre-closed set. □

**Theorem 3.17:** Let $F_B$ be a soft subset of a soft topological space $(F_A, \tilde{\tau})$. Then the following holds.

(i) p-cl($F_B$)= $F_B \tilde{\cup}$ cl(int($F_B$))

(ii) p-int($F_B$)= $F_B \tilde{\cap}$ int(cl($F_B$)).

**Proof:**

(i) $cl[int[F_B \tilde{\cup}\ cl(int(F_B))]] \tilde{\subseteq} cl[int(F_B) \tilde{\cup}\ cl(int(F_B))] = cl(int(F_B)) \tilde{\subseteq} F_B \tilde{\cup}\ cl(int(F_B))$ by Lemma 2.18. Hence $F_B \tilde{\cup}\ cl(int(F_B))$ is a soft pre-closed set and thus $p\text{-}cl(F_B) \tilde{\subseteq} F_B \tilde{\cup}\ cl(int(F_B))$. On the other hand, since p-cl($F_B$) is a soft pre-closed set, we have $cl(int(F_B)) \tilde{\subseteq} cl(int(p\text{-}cl(F_B))) \tilde{\subseteq} p\text{-}cl(F_B)$. Hence $F_B \tilde{\cup}\ cl(int(F_B)) \tilde{\subseteq} p\text{-}cl(F_B)$ which implies that $p\text{-}cl(F_B) = F_B \tilde{\cup}\ cl(int(F_B))$.

(ii) is a consequence of (i). □

**Theorem 3.18:** Let $F_B$ be a soft subset of a soft topological space $(F_A, \tilde{\tau})$. Then the following holds.

*(i)* $cl(p\text{-}int(F_B)) = cl(int(cl(F_B)))$

*(ii)* $p\text{-}cl(p\text{-}int(F_B)) = p\text{-}int(F_B) \tilde{\cup}\ cl(int(F_B))$.

**Proof:**

(i) Let $cl(p\text{-}int(F_B)) = cl[F_B \tilde{\cap}\ int(cl(F_B))]$ by Lemma 2.18. $\tilde{\subseteq} cl(F_B) \tilde{\cap}\ cl(int(cl(F_B))) \tilde{\subseteq} cl(int(cl(F_B)))$. Now, $cl(int(cl(F_B))) \tilde{\subseteq} cl[int(cl(F_B))) \tilde{\cap}\ cl(F_B)] \tilde{\subseteq} cl(cl(int(cl(F_B))) \tilde{\cap}\ F_B) \tilde{\subseteq} cl(int(cl(F_B)) \tilde{\cap}\ F_B)$ [by Lemma 2.18.] $\tilde{\subseteq} cl(p\text{-}int(F_B))$. *Hence* $cl(int(cl(F_B))) = cl(p\text{-}int(F_B))$.

*(ii)* Since soft open set is a soft pre-open set, we have $intl(F_B) \tilde{\subseteq} p\text{-}int(F_B) \tilde{\subseteq} F_B$. Therefore, $int(p\text{-}int(F_B)) = int(F_B)$. Now, by Theorem 3.17(ii) $p\text{-}cl(p\text{-}int(F_B)) = p\text{-}int(F_B) \tilde{\cup}\ cl(int(p\text{-}int(F_B)))\ p\text{-}int(F_B) \tilde{\cup}\ cl(int(F_B))$.

**Theorem 3.19:** Let $(F_A, \tilde{\tau})$ be a soft topological space. Then for any $F_B \tilde{\subseteq} F_A$, $p\text{-}int(F_B) = F_A - p\text{-}cl(F_A - F_B)$.

**Proof:** Let $F_A - p\text{-}int(F_B) = F_A - [F_B \tilde{\cap}\ int(cl(F_B))] = [F_A - F_B] \tilde{\cup} [F_A - int(cl(F_B))] = [F_A - F_B] \tilde{\cup}\ cl[F_A - cl(F_B)] = [F_A - F_B] \tilde{\cup}\ cl(int(F_A - F_B)] = p\text{-}cl[F_A - F_B]$, by Proposition 2.19 and Theorem 3.17. Therefore, $p\text{-}int(F_B) = F_A - p\text{-}cl(F_A - F_B)$. □

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