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On τ_p^+ Generalized Closed Sets, τ_p^+ g regular and τ_p^+ g normal spaces

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ABSTRACT

In this paper we introduce two classes of spaces called τ_p^+ g regular and τ_p^+ g normal space.These classes arise as a combination of simple extension topology and pre open sets in (X, τ).In the light of τ_p^+ g closed sets and τ_p^+ g open sets we study some of the properties of the newly introduced sets.

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***Key words:** τ_p^+ g regular, τ_p^+ g $-T_o$, τ_p^+ g $-R_o$, τ^+-T_2 space, τ_p^+ g irresolute, τ_p^+ g normal.*

INTRODUCTION:

In 1963 Levine [2], started the study of generalized open sets with the introduction of semi-open sets. With this notion, the concept of g-regular and g-normal spaces were introduced and studied by Munshi [7]. Futher Noiri and Popa [8] investigated the concepts introduced by Munshi[7].In 2010 M.E.Abd El Monsef [1] have defined the notion of Bg –closed sets, gB –continuity and gB-irresolute map. In this paper we define a τ_p^+ generalized closed set and study its regularity and normality.

1. PRELIMINARIES:

Throughout this paper (X, τ) and (Y,σ) represent non-empty topological spaces on which no separation axioms are assumed unless explicitly stated and they are simply written as X and Y respectively .For a subset A of (X, τ), the closure of A, the interior of A with respect to τ are denoted by cl(A) and int(A) respectively. The complement of A is denoted by A^c.

Before entering into our work we recall the following definitions.

Definition 1.1: A subset A of a topological space (X, τ) is called pre-open [6] if A ⊆ intcl(A).The complement of pre-open set is called pre-closed.

Definition 1.2: A subset of a topological space (X, τ) is called g-closed [4] if cl(A) ⊆ G whenever A⊆ G and G is open in (X, τ). The complement of g-closed is called g-open.

Definition 1.3: Levine [3], in 1963 defined $\tau^+ (B) = \{O \cup (O' \cap B) / O, O' \in \tau\}$ and called it the simple expansion of τ by B where $B \notin \tau$.

Definition 1.4: A map f: X→Y is called gc-irresolute [5] if $f^{-1}(F)$ is g-closed in X for every g closed set F in Y.

2. τ_p^+ GENERALIZED CLOSED SET:

Definition 2.1: A subset A of a topological space (X, τ) is said to be a τ_p^+ generalized closed (τ_p^+ g closed) if τ^+ cl(A)⊆ U whenever A ⊆ U and U is pre-open in (X, τ). Also $\tau^+ cl(A) = \cap \{S \subseteq X / A \subseteq S$ and S is closed in $\tau^+(B)\}$.The complement of τ_p^+ generalized closed is known as τ_p^+ generalized open in (X, τ).

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Example 2.2: Consider X= {a, b, c} τ = {X, φ {a}, {b}, {a, b}}. Let B={c} Here the τ_p^+ generalized closed sets are {X, φ,, {a},{b},{c},{a, b},{a, c}{b, c}}.

Theorem 2.3: The union of two τ_p^+ g closed set is τ_p^+ g closed.

Proof: Let A and B be two τ_p^+ g closed sets.

Let U be a pre open set such that A∪B⊆U .This implies A⊆Uand B⊆U.
Since A is τ_p^+ g closed set, we have τ^+ cl(A)⊆ U also if B is τ_p^+ g closed set, we have τ^+ cl(B)⊆ U

ie) τ^+ cl(A) ∪ τ^+ cl(B) ⊆ U

ie) τ^+ cl(A∪B) ⊆ U whenever A∪B⊆U where U is pre open.

Hence union of two τ_p^+ g closed set is τ_p^+ g closed.

Definition 2.4: A subset A of a topological space (X, τ) is said to be a τ^+ generalized closed (τ^+ g closed) if τ^+ cl(A)⊆ U whenever A ⊆ U and U is open in (X, τ).

Example 2.5: Consider X= {a, b, c} τ = {X, φ {b}, {c}, {b, c}}.Let B= {a, c} Here the τ^+ generalized closed sets are {X, φ, {a}, {b}, {a, b}{a, c}}.

Proposition 2.6: Every closed set in τ^+(B) is τ_p^+ g closed set.

Proof: This is true by the definition of τ_p^+ g closed set.

Theorem 2.7: Every τ_p^+ g closed set is τ^+ g closed.

Proof: Obvious

Remark 2.8:
(i) Every τ^+ g closed set need not be τ_p^+ g closed
(ii) Every τ_p^+ g closed set need not be τ^+(B) closed.

Proof: Follows from the following example.

Example 2.9: Consider X= {a, b, c} τ = {X, φ, {a}, {a, b}}. Let B={c} Here τ_p^+ generalized closed sets are {X, φ, {b}, {c}, {b, c}{a, b}.

The τ^+ generalized closed sets are {X, φ, {b}, {c}, {a, b}{a, c},{b, c}}.

Here {a, c} is τ^+ g closed but neither τ^+(B) closed nor τ_p^+ g closed

3: τ_p^+ g REGULAR SPACES:

Definition 3.1: A subset A of a space is regular τ^+ -clopen if A is both τ^+ open and τ^+ closed.

Definition 3.2: A space (X, τ) is said to be τ_p^+ generalized regular (τ_p^+ g regular) if for every τ_p^+ g closed set F and a point x ∉ F, there exist disjoint τ^+ open sets U and V such that F⊆U and x ∈ V.

Theorem 3.3: For a topological space ,the following are equivalent.
(i) (X, τ) is τ_p^+ g regular.
(ii) Every τ_p^+ g open set U is a union of τ^+ regular sets.
(iii) Every τ_p^+ g closed set A is a intersection of τ^+ regular sets.

Proof:
T.P (i) ⇨ (ii)
Let (X, τ) be τ_p^+ g regular .Let U be a τ_p^+ g open set and let x ∈ U. If A= X\ U, then A is τ_p^+ g closed. By assumption there exists disjoint τ^+ open subsets W_1 & W2 of X such that x ∈ W_1 and A ⊆ W_2.

If V = τ^+ cl(W_1) ,then V is τ^+ closed and V ∩ A ⊆ V ∩ W_2 = φ .It follows that x ∈ V ⊆ U.

Thus U is the union of τ^+ regular sets.

Hence (i) ⇨ (ii) and (ii) ⇨(iii) is obvious.

T.P (iii) ⇨ (i) Let A be τ_p^+ g closed and $x \in A$ .By assumption there exists a τ^+ regular set V such that $A \subseteq V$ and $x \notin V$.

If U= X\V then U is τ^+ open set containing x and $U \cap V = \varphi$ .Thus (X, τ) is τ_p^+ g regular. Hence the proof.

Now τ_p^+ g open sets give rise to various separation properties of which we have the following.

Definition 3.4: A topological space ((X, τ) is said to be
(i) τ_p^+ g - T_0 if for each pair of distinct points, there exists τ_p^+ g open set containing one point but not the other.
(ii) τ_p^+ g - R_0 space if $\tau^+ cl\{x\} \subseteq U$ whenever U is τ_p^+ g open and $x \in U$.

Definition 3.5: A topological space is said to be τ^+ -T_2 if for each pair of the distinct points x and y in X, there exist disjoint τ^+ open sets U and V in X such that $x \in U$ and $y \in V$.

Theorem 3.6: Every τ_p^+ g regular space is both τ^+ -T_2 and τ_p^+ g - R_0.

Proof: Let (X, τ) be τ_p^+ g regular space and let $x,y \in X$ such that $x \neq y$, By theorem 3.3 {x} is either τ^+ open or τ^+ closed.

Since every space is τ^+ -T_2.If {x} is τ^+ open, hence τ_p^+ g open.

Thus{x} and X\{x} are separately τ^+ open sets. If {x}is τ^+ closed then X\{x} is τ^+ open and by theorem 3.3 is the union of τ^+ regular sets. Hence there is a τ^+ regular set $V \subseteq X\backslash\{x\}$ containing y. This proves that (X, τ) is τ^+ -T_2. By theorem 3.3 it follows immediately, that (X, τ) is also τ_p^+ g - R_0.

Definition 3.7: The intersection of all τ_p^+ g closed set containing A is called the τ_p^+ g closure of A and denoted as τ_p^+ g cl (A). The characterization of τ_p^+ g cl (A) are as follows

Theorem 3.8: Let A be a subset of a space X and $x \in X$, then the following properties hold for of τ_p^+ g cl (A):

(i) $x \in \tau_p^+$ g cl(A) iff $A \cap U \neq \varphi$, for every $U \in \tau^+ O(X)$, containing x.
(ii) A is τ_p^+ g closed iff A= τ_p^+ gcl(A)
(iii) τ_p^+ g cl(A) is τ^+ g closed.
(iv) τ_p^+ g cl(A) $\subseteq \tau_p^+$ g cl(B) if $A \subseteq B$
(v)τ_p^+ g(τ_p^+ g cl(A)) = τ_p^+ g cl(A)

Proof: obvious.

Definition 3.9: A subset N of X is called τ_p^+ generalized neighbourhood (τ_p^+ g nbh) of a point $x \in X$, if there exists a τ_p^+ g open set U such that $x \in U \subseteq N$.

Theorem 3.10: Suppose that $B \subseteq A \subseteq X$, B is τ_p^+ g closed set relative to A and that A is open and τ_p^+ g closed in (X τ). Then B is τ_p^+ g closed in(X, τ).

Theorem 3.11: If (X, τ) is a τ_p^+ g regular space and Y is an open and τ_p^+ g closed subset of (X, τ),then the subspace Y is τ_p^+ g regular.

Proof: Let F be any τ_p^+ g closed subset of Y and $y \in F^c$. By above theorem 3.10 ,F is τ_p^+ g closed (X, τ).Since (X, τ) is τ_p^+ g regular, there exists disjoint τ^+ open sets U and V of (X, τ) such that $y \in U$ and $F \subseteq V$ .Since Y is open and hence τ^+ open we get $U \cap Y$ and $V \cap Y$ are disjoint τ^+ open sets of the subspace Y such that $y \in U \cap Y$ and $F \subseteq V \cap Y$ .Hence the subspace Y is τ_p^+ g regular.

Theorem 3.12: Let (X, τ) be a topological space .Then the following statements are equivalent.
(i) (X, τ) is τ_p^+ g regular

(ii)For each point $x\in X$ and for each $\tau_p{}^+$ g open nbh Wof x , there exists a τ^+ open set U of X such that $\tau^+cl(U) \subseteq W$.

(iii)For each point $x\in X$ and for each $\tau_p{}^+$ g closed set F not containing x, there exists a τ^+ open set V of X such that $\tau^+cl(V) \cap F = \varphi$.

Proof: To prove (i) ⇨ (ii).Let W be any $\tau_p{}^+$ g open nbh of x. Then there exists a $\tau_p{}^+$ g open set G such that $x\in G\subseteq W$. Since G^c is $\tau_p{}^+$ g closed and $x\notin G^c$ by hypothesis, there exists τ^+ open sets U and V such that $G^c\in U$, $x\in V$ and $U \cap V = \varphi$ and so $V \subseteq U^c$. Now $\tau^+cl(V) \subseteq \tau^+cl(U^c) = U^c$ and $G^c\subseteq U$ implies $U^c \subseteq G\subseteq W$. Thus $\tau^+cl(U) \subseteq W$.

To prove (ii) ⇨ (i).Let F be any $\tau_p{}^+$ g closed set and $x\notin F$. Then $x\in F^c$ and F^c is a $\tau_p{}^+$ g open set and so F^c is an $\tau_p{}^+$ g open nbh of x. By hypothesis, there exists τ^+ open set V of x such that $x\in V$ and $\tau^+cl(V) \subseteq F^c$ which implies $F\subseteq(\tau^+cl(V))^c$.

Then $(\tau^+cl(V))^c$ is τ^+ open containing F and $V \cap (\tau^+cl(V))^c = \varphi$.Therefore X is $\tau_p{}^+$ g regular.

To prove (ii) ⇨ (iii).Let $x\in X$ and F be a $\tau_p{}^+$ g closed set such that $x \notin F$. Then F^c is a $\tau_p{}^+$ g nbhd of x and by hypothesis, there exists a τ^+ open set V of X such that $\tau^+cl(V) \subseteq F^c$ and hence $\tau^+cl(V) \cap F= \varphi$.

To prove (iii) ⇨ (ii).Let $x\in X$ and W be a $\tau_p{}^+$ g nbhd of x. Then there exists a $\tau_p{}^+$ g open set G such that $x\in G\subseteq W$. Since G^c is $\tau_p{}^+$ g closed and $x\notin G^c$ by hypothesis, there exists τ^+ open set U of x such that $\tau^+cl(U) \cap G^c = \varphi$. Therefore $\tau^+cl(U) \subseteq G \subseteq W$.

Theorem 3.13: A topological space (X, τ) is $\tau_p{}^+$ g regular if and only if for each $\tau_p{}^+$ g closed set F of (X, τ) and each $x\in F^c$, there exists τ^+ open sets U and V of (X, τ) such that $x\in U$ and $F\subseteq V$ and $\tau^+cl(U) \cap \tau^+cl(V) = \varphi$.

Proof: Let F be any $\tau_p{}^+$ g closed set and $x\notin F$. Then there exists a τ^+ open sets U_x and V such that $x\in U_x$, $F\subseteq V$ and $U_x\cap V = \varphi$, which implies that $U_x \cap \tau^+cl(V) = \varphi$. Since (X, τ) is $\tau_p{}^+$ g regular,there exists τ^+ open sets G and H of (X, τ) such that $x\in G$, $\tau^+cl(V) \subseteq H$ and $G\cap H = \varphi$. This implies $\tau^+cl(G) \cap H = \varphi$.

Now let $U= U_x \cap G$, then U and V are τ^+ open sets of (X, τ) such that $x\in U$ and $F\subseteq V$ and $\tau^+cl(U) \cap \tau^+cl(V) = \varphi$.
The converse is straight forward.

Definition 3.14: A map $f: X\rightarrow Y$ is called M τ^+ open if f (V) is τ^+ open set in Y for every τ^+ open set V of X.

Definition 3.15: A map $f: X\rightarrow Y$ is called $\tau_p{}^+$ g irresolute (resp. τ^+ irresolute) if $f^{-1}(V)$ is $\tau_p{}^+$ g open(resp. τ^+ open) set in X for every $\tau_p{}^+$ g open (resp. τ^+ open) set V of Y.

Theorem 3.16: If (X, τ) is $\tau_p{}^+$ g regular space and if $f: (X, \tau) \rightarrow(Y,\sigma)$ is bijective, $\tau_p{}^+$ g irresolute and M τ^+ open, then (Y,σ) is $\tau_p{}^+$ g regular.

Proof: Let $y\in Y$ and F be any $\tau_p{}^+$ g closed set of (Y,σ) with $y \notin F$. Since f is $\tau_p{}^+$ g irresolute, $f^{-1}(F)$ is $\tau_p{}^+$ g closed set in (X, τ). Since f is bijective let $f(x) =y$, then $x\neq f^{-1}(y)$.By hypothesis, there exists τ^+ open sets U and V such that $x\in U$ and $f^{-1}(F) \subseteq V$. Since f is M τ^+ open and bijective we have $y\in f(U)$and $F\subseteq f(V)$ and $f(U)$ $f(V) = f(U\cap V) = \varphi$.
Hence (Y,σ) is $\tau_p{}^+$ g regular space.

Theorem 3.17: If $f: (X, \tau) \rightarrow(Y,\sigma)$ is gc –irresolute , M τ^+closed and A is a $\tau_p{}^+$ g closed subset of (X, τ) then f(A) is $\tau_p{}^+$ g closed.

Theorem 3.18: If $f: (X, \tau) \rightarrow(Y,\sigma)$ is gc –irresolute, M τ^+closed, injective and (Y,σ) is $\tau_p{}^+$ g regular then(X, τ) is $\tau_p{}^+$ g regular.

Proof: Let F be any $\tau_p{}^+$ g closed set of (X, τ) and $x\notin F$. Since f is gc irresolute, M τ^+closed by theorem 3.17, f(F) is τ^+ closed in Y and $f(x) \notin f(F)$.Since (Y,σ) is $\tau_p{}^+$ g regular and so there exists disjoint τ^+ open sets U and V in (Y,σ) such that $f(x) \in U$ and $f(F) \subseteq V$. By hypothesis, $f^{-1}(U)$ and $f^{-1}(V) \in \tau^+O(X)$, such that $x \in f^{-1} (U)$ and $F\subseteq f^{-1} (V)$ and $f^{-1} (U) \cap f^{-1}(V) = \varphi$. Therefore (X, τ) is $\tau_p{}^+$ g regular.

4. $\tau_p{}^+$ g NORMAL SPACES:

Here we introduce a weak form of normality called $\tau_p{}^+$ g normality in a topological space.

Definition 4.1:A topological space (X, τ) is said to be $\tau_p{}^+$ generalized normal ($\tau_p{}^+$ g normal) if for any pair of disjoint $\tau_p{}^+$ g closed sets A and B , there exists disjoint τ^+ open sets U and V such that A ⊆ U and B⊆ V.

Theorem 4.2: If (X, τ) is $\tau_p{}^+$ g normal space and Y is an open and $\tau_p{}^+$ g closed subset of (X, τ), then the subspace Y is $\tau_p{}^+$ g normal.

Proof: Let A and B be any two disjoint $\tau_p{}^+$ g closed sets of Y. By Theorem 3.10, A and B are $\tau_p{}^+$ g closed in (X, τ). Since (X, τ) is $\tau_p{}^+$ g normal, there exists disjoint τ^+ open sets U and V of (X, τ) such that A ⊆ U and B⊆ V. Since Y is open and hence τ^+ open, U ∩ Y and V∩ Y are disjoint τ^+ open sets of the subspace Y. Hence the subspace Y is $\tau_p{}^+$ g normal.

Theorem 4.3: Let (X, τ) be a topological space, then the following statements are equivalent.

(1) (X, τ) is $\tau_p{}^+$ g normal

(2) For each $\tau_p{}^+$ g closed set F and for $\tau_p{}^+$ g open set U containing F, there exists a τ^+ open set V containing F such that $\tau^+cl(V) \subseteq U$.

(3) For each pair of disjoint $\tau_p{}^+$ g closed set A and B in (X, τ), there exists a τ^+ open set containing A such that $\tau^+cl\ (U) \cap B = \varphi$.

(4) For each pair of disjoint $\tau_p{}^+$ g closed set A and B in (X, τ) there exists τ^+ open sets U and V such that A ⊆ U and B⊆ V and $\tau^+cl\ (A) \cap \tau^+cl\ (B) = \varphi$.

To Prove: (1) => (2)

Let F be a $\tau_p{}^+$ g closed set U be a $\tau_p{}^+$ g open set such that F ⊆ U. Then $F \cap U^c = \varphi$, by assumption, there exists τ^+ open set V and W such that F ⊆ U and $U^c \subseteq W$ and $V \cap W = \varphi \Rightarrow \tau^+cl\ (V) \cap W = \varphi$. Now $\tau^+cl\ (V) \cap U^c \subseteq \tau^+cl\ (V) \cap W = \varphi$ and so $\tau^+cl\ (V) \subseteq U$.

To Prove: (2) => (3)

Let A and B be disjoint $\tau_p{}^+$ g closed sets of (X, τ). Since $A\cap B = \varphi$; $A \subseteq B^c$ and B^c is $\tau_p{}^+$ g open. By assumption, there exists τ^+ open sets U containing A such that $\tau^+cl\ (U) \subseteq B^c$ and so $\tau^+\ cl\ (U) \cap B = \varphi$.

To Prove: (3) => (4)

Let A and B be disjoint $\tau_p{}^+$ g closed sets of (X, τ). Then by assumption, there exists τ^+ open sets U containing A such that $\tau^+cl(U) \cap B = \varphi$.

Since τ^+ cl (A) is τ^+ closed, it is $\tau_p{}^+$ g closed and so B and τ^+ cl (A) are disjoint $\tau_p{}^+$ g closed sets in (X, τ).

Hence by assumption, there exists a τ^+ open sets V containing B such that $\tau^+\ cl\ (A) \cap \tau^+\ cl\ (B) = \varphi$.

To Prove: (4) => (1)

Let A and B be disjoint $\tau_p{}^+$ g closed sets of (X, τ). By assumption, there exists τ^+ open sets U and V such that A ⊆ U and B ⊆ V and $\tau^+cl\ (U) \cap \tau^+\ cl\ (V) = \varphi$. We have $U \cap V = \varphi$ and thus (X, τ) is $\tau_p{}^+$ g normal.

Theorem 4.4: If f: (X, τ) →(Y,σ) is $\tau_p{}^+$ g irresolute, bijective, $M\tau^+$ open mapping and (X, τ) is $\tau_p{}^+$ g normal, then (Y,σ) is $\tau_p{}^+$ g normal.

Proof: Let A and B be any two disjoint τ_p^+ g closed sets of (Y,σ). Since f is τ_p^+ g irresolute, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint τ_p^+ g closed sets of (X, τ). As (X, τ) is τ_p^+ g normal, there exists disjoint τ^+ open sets U and V such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Since f is bijective and Mτ^+ open we have f (U) and f(V) are τ^+ open sets in (Y,σ) such that $A \subseteq f(U)$ and $B \subseteq f(V)$ and $f(U) \cap f(V) = \varphi$. Therefore (Y,σ) is τ_p^+ g normal.

Theorem 4.5: If f: (X, τ) → (Y,σ) is gc irresolute, Mτ^+ closed and τ^+ irresolute injection and (Y,σ) is τ_p^+ g normal, then (X, τ) is τ_p^+ g normal.

Proof: Let A and B be any two disjoint τ_p^+ g closed sets of (X, τ). Since f is gc irresolute and M τ^+ closed. f (A) and f (B) are disjoint τ_p^+ g closed sets of (Y,σ).

By Theorem 3.17 since (Y,σ) is τ_p^+ g normal , there exists disjoint τ^+ open sets U and V such that $f(A) \subseteq U$ and $f(B) \subseteq V$ i.e.) $A \subseteq f^{-1}(U)$ and $B \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \varphi$.

Since f is τ^+ irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are τ^+ open sets in (X, τ) ,we have (X, τ) is τ_p^+ g normal.

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