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TOTAL HOMO-CORDIAL LABELING OF GRAPHS

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ABSTRACT

Let G = (V, E) be a graph with p vertices and q edges. A Total Homo-Cordial Labeling of a graph G with vertex set V is a bijection from V to {0, 1} such that each uv is assigned the label 1 if f(u)=f(v) or 0 if f(u)≠f(v) with the condition that $|ev_f(0)-ev_f(1)|\leq 1$ where $ev_f(x)$ denotes the total number of vertices and edges labeled with x (x=0,1). The graph that admits a Total Homo-Cordial Labeling is called Total Homo-Cordial Graph. In this paper, we prove some graphs such as path, cycle, wheel, comp and fan are total homo- cordial labeling graphs.

Keywords: *Cordial labeling, Homo-cordial labeling, Homo-cordial graph.*

AMS Subject classification (2010): *05C78.*

1. INTRODUCTION

All graphs considered here are finite, simple and undirected. Gallian [1] has given a dynamic survey of graph labeling. The origin of graph labelings can be attributed to Rosa [2]. A Path Related Homo-Cordial Graph was introduced by Dr.A. Nellai Murugan and A.Mathubala [3]. A Total Mean Cordial Labeling of Graphs was introduced by R. Ponraj, S. Sathish Narayanan and A. M. S Ramasamy [4]. This definition motivates us to define a Total Homo-Cordial Labeling of a graph and we prove some graphs such as path, cycle, wheel, comp and fan are Total Homo-Cordial.

2. PRELIMINARIES

Definition 2.1: A labeling f of G where N={0,1} and the induced edge labeling $\bar{f}$ is given by $\bar{f}$(u, v) = |f(u) − f(v)|, $\bar{N}$={0, 1}. We call such a labeling cordial if the following condition is satisfied $|v_f(1)-v_f(0)|\leq 1$, $|e_f(1)-e_f(0)|\leq 1$, where $v_f(i)$ and $e_f(i)$, i={0,1}, is the number of vertices and edges of G respectively, with label i. A graph is cordial if it admits a cordial labeling.

Definition 2.2: Let G = (V, E) be a graph with p vertices and q edges. A Homo-Cordial Labeling of a graph G with vertex set V is a bijection from V to {0,1} such that each uv is assigned the label 1 if f(u)=f(v) or 0 if f(u)≠f(v) with the condition that $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$. The graph that admits a Homo-Cordial Labeling is called Homo-Cordial Graph.

3. MAIN RESULTS

Definition 3.1: Let G = (V, E) be a graph with p vertices and q edges. A Total Homo-Cordial Labeling of a graph G with vertex set V is a bijection from V to {0,1} such that each uv is assigned the label 1 if f(u)=f(v) or 0 if f(u)≠f(v) with the condition that $|ev_f(0)-ev_f(1)|\leq 1$, where $ev_f(x)$ denotes the total number of vertices and edges labeled with x (x = 0, 1). The graph that admits a Total Homo-Cordial Labeling is called Total Homo-Cordial Graph.

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Theorem 3.2: Path P_n is Total Homo-Cordial Graph.

Proof: Let $V(P_n) = \{u_i : 1 \le i \le n\}$ and $E(P_n)=\{(u_iu_{i+1}): 1 \le i \le n-1\}$.

Define $f: V(P_n)\rightarrow\{0,1\}$.

The vertex labeling are,

$$f(u_i) = \begin{cases} 1 & i \equiv 2,3 \bmod 4 \\ 0 & i \equiv 0,1 \bmod 4 \end{cases} \quad 1 \le i \le n$$

The induced edge labeling are,

$$f^*[(u_iu_{i+1})] = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 0 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

Here, $ev_f(1) = ev_f(0) + 1$ for $n \equiv 3 \bmod 4$ and
$ev_f(0) = ev_f(1) + 1$ for $n \equiv 0,1,2 \bmod 4$.

Therefore, the path P_n satisfies the condition $|ev_f(0)-ev_f(1)| \le 1$.

Hence, the path P_n is Total Homo-Cordial Graph.

Example 3.3: Consider the following graph P_7,

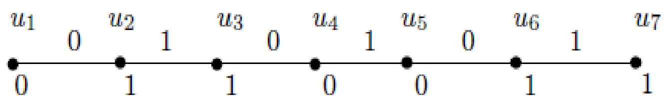

Figure 3.1

Here, $ev_f(1) = 7$ and $ev_f(0) = 6$.

Therefore, the path P_7 satisfies the condition $|ev_f(0)-ev_f(1)| \le 1$.

Hence, the graph P_7 is Total Homo-Cordial Graph.

Theorem 3.4: Cycle C_n ($n \equiv 0 \bmod 4$) is Total Homo-Cordial Graph.

Proof: Let $V(C_n) = \{u_i: 1 \le i \le n\}$ and $E(C_n)=\{(u_iu_{i+1}): 1 \le i \le n-1\}\cup\{(u_1u_n)\}$.

Define $f : V(C_n)\rightarrow\{0,1\}$.

The vertex labeling are,

$$f(u_i) = \begin{cases} 1 & i \equiv 0,1 \bmod 4 \\ 0 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1 \le i \le n$$

The induced edge labeling are,

$$f^*[(u_iu_{i+1})] = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 0 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

$$f^*[(u_1u_n)] = 1$$

Here, $ev_f(1) = ev_f(0)$ for $n \equiv 0 \bmod 4$.

Therefore, the cycle C_n ($n \equiv 0 \bmod 4$) satisfies the condition $|ev_f(0)-ev_f(1)| \le 1$.

Hence, the cycle C_n ($n \equiv 0 \bmod 4$) is Total Homo-Cordial Graph.

Example 3.5: Consider the following graph C_4,

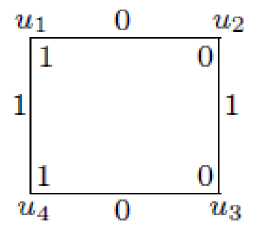

Figure 3.2

Here, $ev_f(1) = 4$ and $ev_f(0) = 4$.

Therefore, the cycle C_4 satisfies the condition $|ev_f(0) - ev_f(1)| \leq 1$.

Hence, the cycle C_4 is Total Homo-Cordial Graph.

Theorem 3.6: Cycle C_n is not Total Homo-Cordial Graph.

Proof: Let $V(C_n) = \{u_i : 1 \leq i \leq n\}$ and $E(C_n) = \{(u_i u_{i+1}): 1 \leq i \leq n-1\} \cup \{(u_1 u_n)\}$.

Define $f: V(C_n) \rightarrow \{0,1\}$.

The vertex labeling are,

$$f(u_i) = \begin{cases} 1 & i \equiv 0,1 \bmod 4 \\ 0 & i \equiv 2,3 \bmod 4 \end{cases} \qquad 1 \leq i \leq n$$

The induced edge labeling are,

$$f^*[(u_i u_{i+1})] = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 0 & i \equiv 1 \bmod 2 \end{cases} \qquad 1 \leq i \leq n-1$$

$$f^*[(u_1 u_n)] = \begin{cases} 1 & n \equiv 1 \bmod 4 \\ 0 & n \equiv 2,3 \bmod 4 \end{cases}$$

Here, $ev_f(0) = ev_f(1) + 2$ for $n \equiv 2,3 \bmod 4$ and
$ev_f(1) = ev_f(0) + 2$ for $n \equiv 1 \bmod 4$.

Therefore, the cycle C_n does not satisfies the condition $|ev_f(0) - ev_f(1)| \leq 1$.

Hence, the cycle C_n is not Total Homo-Cordial Graph.

Example 3.7: Consider the following graph C_5,

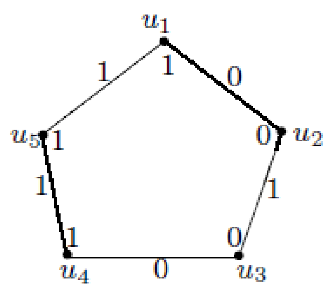

Figure 3.3

Here, $ev_f(1) = 6$ and $ev_f(0) = 4$.

Therefore, the cycle C_5 does not satisfy the condition $|ev_f(0) - ev_f(1)| \leq 1$.

Hence, the graph C_5 is not Total Homo-Cordial Graph.

Theorem 3.8: Wheel W_n is Total Homo-Cordial Graph.

Proof: Let $V(W_n)=\{u, u_i: 1 \le i \le n\}$ and $E(W_n) = \{(uu_i) : 1 \le i \le n\} \cup \{(u_iu_{i+1}): 1 \le i \le n-1\} \cup \{(u_1u_n)\}$.

Define $f: V(W_n) \to \{0,1\}$.

Case-1: When $n \equiv 1 \bmod 4$.

The vertex labeling are,

$f(u) = 0$

$$f(u)=\begin{cases}1 & i \equiv 0,1 \bmod 4 \\ 0 & i \equiv 2,3 \bmod 4\end{cases} \quad 1 \le i \le n$$

The induced edge labeling are,

$$f^*[(uu_i)] = \begin{cases}1 & i \equiv 2,3 \bmod 4 \\ 0 & i \equiv 0,1 \bmod 4\end{cases} \quad 1 \le i \le n$$

$$f^*[(u_iu_{i+1})] = \begin{cases}1 & i \equiv 0 \bmod 2 \\ 0 & i \equiv 1 \bmod 2\end{cases} \quad 1 \le i \le n-1$$

$f^*[(u_1u_n)] = 1$

Here, $ev_f(0) = ev_f(1)$ for all n.

Case-2: When n is even.
The vertex labeling are,

$f(u) = 0$

$$f(u_i) = \begin{cases}1 & i \equiv 1,2 \bmod 4 \\ 0 & i \equiv 0,3 \bmod 4\end{cases} \quad 1 \le i \le n$$

The induced edge labeling are,

$$f(uu_i) = \begin{cases}1 & i \equiv 0,3 \bmod 4 \\ 0 & i \equiv 1,2 \bmod 4\end{cases} \quad 1 \le i \le n$$

$$f^*[(u_iu_{i+1})] = \begin{cases}1 & i \equiv 1 \bmod 2 \\ 0 & i \equiv 0 \bmod 2\end{cases} \quad 1 \le i \le n-1$$

$$f^*[(u_1u_n)] = \begin{cases}1 & n \equiv 2 \bmod 4 \\ 0 & n \equiv 0 \bmod 4\end{cases}$$

Here, $ev_f(1) = ev_f(0)+1$ for $n \equiv 2 \bmod 4$ and
$ev_f(0) = ev_f(1)+1$ for $n \equiv 0 \bmod 4$.

Therefore, the wheel W_n is satisfies the conditions $|ev_f(0)-ev_f(1)| \le 1$.

Hence, the wheel W_n is Total Homo-Cordial Graph.

Example 3.6: Consider the following graph W_5,

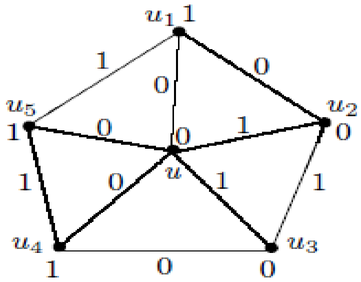

Figure 3.4

Here, $ev_f(1) = 8$ and $ev_f(0) = 8$.

Therefore, the wheel W_5 satisfies the condition $|ev_f(0)-ev_f(1)| \leq 1$.

Hence, the graph W_5 is Total Homo-Cordial Graph.

Theorem 3.9: Comp $P_n \odot K_1$ is Total Homo-Cordial Graph.

Proof: Let $V(P_n \odot K_1) = \{u_i, v_i: 1 \leq i \leq n\}$ and $E(P_n \odot K_1) = \{(u_i u_{i+1}): 1 \leq i \leq n-1\} \cup \{(u_i v_i): 1 \leq i \leq n\}$.

Define $f: V(P_n \odot K_1) \to \{0,1\}$.

The vertex labeling are,

$f(u_i) = 0 \quad 1 \leq i \leq n$

$f(v_i) = 1 \quad 1 \leq i \leq n$

The induced edge labeling are,

$f^*[(u_i u_{i+1}] = 1 \quad 1 \leq i \leq n-1$

$f^*[(u_i v_i] = 0 \quad 1 \leq i \leq n$

Here, $ev_f(0) = ev_f(1) + 1$ for all n.

Therefore, the comp $P_n \odot K_1$ satisfies the conditions $|ev_f(0)-ev_f(1)| \leq 1$.

Hence, comp $P_n \odot K_1$ is Total Homo-Cordial Graph.

Example 3.10: Consider the following graph $P_5 \odot K_1$,

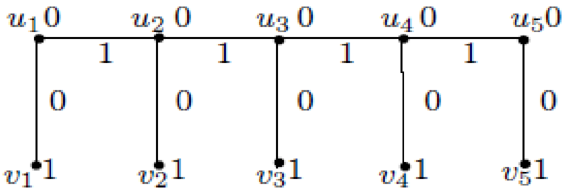

Figure 3.5

Here, $ev_f(1) = 9$ and $ev_f(0) = 10$.

Therefore, the comp $P_5 \odot K_1$ satisfies the condition $|ev_f(0)-ev_f(1)| \leq 1$.

Hence, the graph $P_5 \odot K_1$ is Total Homo-Cordial Graph.

Theorem 3.11: Fan P_n+K_1 is Total Homo-Cordial Graph.

Proof: Let $V(P_n+K_1) = \{u, u_i: 1 \leq i \leq n\}$ and $E(P_n+K_1) = \{(uu_i): 1 \leq i \leq n\} \cup \{(u_i u_{i+1}): 1 \leq i \leq n-1\}$.

Define $f: V(P_n+K_1) \to \{0,1\}$.

Case-1: When n is odd

The vertex labeling are,

$f(u) = 0$

$$f(u_i) = \begin{cases} 1 & i \equiv 1,2 \bmod 4 \\ 0 & i \equiv 0,3 \bmod 4 \end{cases} \qquad 1 \leq i \leq n$$

The induced edge labeling are,

$$f^*[(uu_i)] = \begin{cases} 1 & i \equiv 0,3 \bmod 4 \\ 0 & i \equiv 1,2 \bmod 4 \end{cases} \qquad 1 \leq i \leq n$$

$$f^*[(u_i u_{i+1})] = \begin{cases} 1 & i \equiv 1 \bmod 2 \\ 0 & i \equiv 0 \bmod 2 \end{cases} \qquad 1 \leq i \leq n-1$$

Here, $ev_f(0) = ev_f(1) + 1$ for all n.

Case-2: When n is even.

The vertex labeling are,

$$f(u) = 1$$

$$f(u_i) = \begin{cases} 1 & i \equiv 2,3 \bmod 4 \\ 0 & i \equiv 0,1 \bmod 4 \end{cases} \qquad 1 \le i \le n$$

The induced edge labeling are,

$$f^*[(uu_i)] = \begin{cases} 1 & i \equiv 2,3 \bmod 4 \\ 0 & i \equiv 0,1 \bmod 4 \end{cases} \qquad 1 \le i \le n$$

$$f^*[(u_iu_{i+1})] = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 0 & i \equiv 1 \bmod 2 \end{cases} \qquad 1 \le i \le n-1$$

Here, $ev_f(0) = ev_f(1)$ for all n.

Therefore, the fan P_n+K_1 satisfies the condition $|ev_f(0)-ev_f(1)| \le 1$.

Hence, the fan P_n+K_1 is Total Homo-Cordial Graph.

Example 3.12: Consider the following graph P_5+K_1,

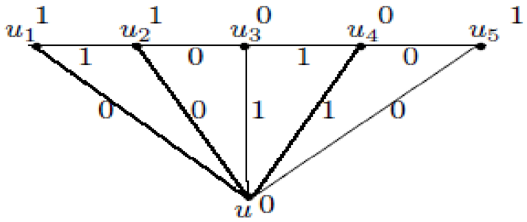

Figure 3.6

Here, $ev_f(1) = 7$ and $ev_f(0) = 8$.

Therefore, the fan P_5+K_1 satisfies the conditions $|ev_f(0)-ev_f(1)| \le 1$.

Hence, the graph P_5+K_1 is Total Homo-Cordial Graph.

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