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MATHEMATICAL FORMULA "RELATIONSHIP OF LCM & HCF OF 3 NUMBERS IN AP"

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ABSTRACT

In the following pages- a, b, c are any three numbers which are in arithmetic progression.

LCM (numbers in A.P) of three numbers say $a, b, c =$

$$\frac{a \times b \times c}{(HCF)^2} \quad or \quad \frac{a \times b \times c}{2(HCF)^2}$$

i.e LCM is equal to product of three numbers/square of HCF.

or

LCM is equal to product of three numbers/twice the square of HCF.

INTRODUCTION

In the following pages I put forth two formulae which hold good for finding LCM of any three numbers which are having common difference between them.

It also shows the relation between HCF and LCM of three numbers when their product is given. It is a simple method to find LCM and HCF.

These terms are used in the following pages

- **Co-primes:** In number theory, two integers a and b are said to be relatively prime, mutually prime, or co-prime if the only positive integer that divides both of them is 1. That is, the only common positive factor of the two numbers is 1.

  Example:
  1. 2, 3, 5
  2. 11, 12, 13 etc.

- **Arithmetic progression:** Arithmetic progression (AP) or arithmetic sequence is a sequence of numbers such that the difference between the consecutive terms is constant.

  Example: 5, 7, 9, 11, 13, 15…….. is arithmetic progression with common difference of 2.

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- **HCF:** The Highest Common Factor (H.C.F) of two (or more) numbers is the largest number that divides evenly into both numbers.

  Example:
  1. HCF of 12 and 18 is 6.
  2. HCF of 100 and 150 is 50.

- **LCM:** Least Common Multiple is the smallest positive integer that is divisible by two or more numbers.

  Example:
  1. LCM of 12 and 18 is 36.
  2. LCM of 100 and 150 is 300.

EXISTING METHODS

These are some methods which exist for finding LCM of three numbers.

- **Prime factorization method:**

  Let us take three numbers- 8, 10, 12

2	8, 10, 12
2	4, 5, 6
2	2, 5, 3
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

  $LCM = 2 \times 2 \times 2 \times 3 \times 5 = 120$

- **LCM(LCM(a, b) , c) = LCM(a, b, c)**

  Let $a = 18, b = 24, c = 30$

  $LCM(18, 24, 30) = LCM(LCM(18, 24), 30)$

  $LCM \ of \ 18 \ and \ 24 \ is \ 72$

  $LCM(18, 24, 30) = LCM(72, 30)$

  $LCM \ of \ 72 \ and \ 30 \ is \ 360.$

  $LCM(18, 24, 30) = 360$

- $LCM(p, q, r) = \dfrac{pqr \times HCF(p, q, r)}{HCF(p, q) \times HCF(q, r) \times HCF(p, r)}$

  Let $p = 10, q = 20, r = 30$

  $$LCM(10, 20, 30) = \frac{10 \times 20 \times 30 \times HCF(10, 20, 30)}{HCF(10, 20 \times HCF(20, 30) \times HCF(10, 30)}$$

  $HCF(10, 20, 30) = 10$

$$HCF(10,20)=10$$
$$HCF(20,30)=10$$
$$HCF(10,30)=10$$
$$LCM(10,20,30)=\frac{10\times20\times30\times10}{10\times10\times10}$$
$$LCM(10,20,30)=60$$

PROPOSED METHODOLOGY

For following three numbers in Arithmetic progression I have found a relation between their LCM and HCF.

- Arithmetic progression is series of numbers where there is common difference between any two numbers.

  The following formulae hold good to find LCM of any three numbers which are in sequence.

  LCM (numbers in A.P) of three numbers say

$$a,b,c=\frac{a\times b\times c}{(HCF)^2} \qquad or \qquad \frac{a\times b\times c}{2(HCF)^2}$$

  i.e LCM is equal to product of three numbers/square of HCF.
  or
  LCM is equal to product of three numbers/twice the square of HCF.

  So, How to find when to use which formula?

  When we divide three numbers by their HCF, if we get all three numbers co-primes to each other, we use $\frac{a\times b\times c}{(HCF)^2}$

  i.e

4	36,	40,	44
	9,	10,	11

  Here, 9, 10, 11 are co-primes.

  If we get two out of three numbers having common factor other than 1, we use $\frac{a\times b\times c}{2(HCF)^2}$

  i.e

50	100,	150,	200
	2,	3,	4

  Here, 2 and 4 are'nt co-primes

RESULT OF THE FINDINGS

LCM (numbers in A.P) of three numbers = $\frac{a\times b\times c}{(HCF)^2}$

- 2, 4, 6
  2, 4, 6 are in A.P
  i.e d = 2 , HCF = 2
  a = 2, b = 4, c = 6
  2 X 4 X 6/ $(2)^2$ = 12

2	2,	4,	6
2	1,	2,	3
3	1,	1,	3
	1,	1,	1

$$LCM=2\times2\times3=12$$

  LHS = RHS

- 10, 20, 30
  10, 20, 30 are in A.P
  i.e d = 10 , HCF = 10
  a = 10, b = 20, c = 30
  10 X 20 X 30/ $(10)^2$ = 60

2	10, 20, 30
5	5, 10, 15
2	1, 2, 3
3	1, 1, 3
	1, 1, 1

$LCM = 2\times 2\times 3\times 5 = 60$

LHS = RHS

- 210, 230, 250
  210, 230, 250 are in A.P
  i.e d = 20 , HCF = 10
  a = 210, b = 230, c = 250
  210 X 230 X 250/ $(10)^2$ = 120750

5	210, 230, 250
2	42, 46, 50
23	21, 23, 25
7	21, 1, 25
3	3, 1, 25
5	1, 1, 25
5	1, 1, 5
	1, 1, 1

$LCM = 5\times 2\times 2\times 3\times 7\times 3\times 5\times 5$
$= 120750$

LHS = RHS

or **LCM (numbers in A.P) of three numbers =** $\dfrac{a\times b\times c}{2\left(HCF\right)^2}$

- 600, 602, 604
  600, 602, 604 are in A.P
  i.e d = 2 , HCF = 2
  a = 600, b = 602, c = 604
  600 X 602 X 604/ 2 X $(2)^2$ = 27270600

2	600, 602, 604
2	300, 301, 302
2	150, 301, 151
7	75, 301, 151
151	75, 43, 151
43	75, 43, 151
5	75, 1, 1
3	15, 1, 1
5	5, 1, 1
	1, 1, 1

$LCM = 2\times 2\times 2\times 7\times 151\times 43\times 5\times 3\times 5$
$= 27270600$

LHS = RHS

- 4, 6, 8
  4, 6, 8 are in A.P
  i.e d = 2, HCF = 2
  a = 4, b = 6, c = 8
  4 X 6 X 8/ 2 X $(2)^2$ = 24

2	4, 6, 8
2	2, 3, 4
2	1, 3, 2
3	1, 3, 1
	1, 1, 1

$LCM = 2\times 2\times 2\times 3 = 2$

LHS = RHS

- 100, 150, 200
  100, 150, 200 are in A.P
  d = 50, HCF = 50
  a = 100, b = 150, c = 200
  100 X 150 X 200/ 2 X $(50)^2$ = 600

2	100, 150, 200
2	50, 75, 100
5	25, 75, 50
5	5, 15, 10
3	1, 3, 2
2	1, 1, 2
	1, 1, 1

$LCM = 2\times 2\times 5\times 5\times 3\times 2 = 600$

LHS = RHS

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