International Journal of Mathematical Archive-8(3), 2017, 99-103 \$MA Available online through www.ijma.info ISSN 2229-5046

MATHEMATICAL FORMULA "RELATIONSHIP OF LCM \& HCF OF 3 NUMBERS IN AP"
SHARANBASAWESHWAR. M. PATIL*
A student of Class $\mathbf{X}$, JSS Shri Manjunatheshwara Central School, Vidyagiri, Dharwad, Karnataka, India.

DEEPA.SHETTY, PARESH.D.NAYAK AND SADHANA S
Asst. Teachers, Mathematics, JSS Shri Manjunatheshwara Central School, Vidyagiri, Dharwad, Karnataka, India.
(Received On: 25-02-17; Revised \& Accepted On: 21-03-17)


#### Abstract

In the following pages- $a, b, c$ are any three numbers which are in arithmetic progression.


LCM (numbers in A.P) of three numbers say $a, b, c=$

$$
\frac{a \times b \times c}{(H C F)^{2}} \quad \text { or } \quad \frac{a \times b \times c}{2(H C F)^{2}}
$$

i.e LCM is equal to product of three numbers/square of HCF.
or
LCM is equal to product of three numbers/twice the square of HCF.

## INTRODUCTION

In the following pages I put forth two formulae which hold good for finding LCM of any three numbers which are having common difference between them.

It also shows the relation between HCF and LCM of three numbers when their product is given. It is a simple method to find LCM and HCF.

These terms are used in the following pages

- Co-primes: In number theory, two integers a and $b$ are said to be relatively prime, mutually prime, or co-prime if the only positive integer that divides both of them is 1 . That is, the only common positive factor of the two numbers is 1 .


## Example:

1. 2, 3, 5
2. $11,12,13$ etc.

- Arithmetic progression: Arithmetic progression (AP) or arithmetic sequence is a sequence of numbers such that the difference between the consecutive terms is constant.

Example: 5, 7, 9, 11, 13, 15...... is arithmetic progression with common difference of 2.

- HCF: The Highest Common Factor (H.C.F) of two (or more) numbers is the largest number that divides evenly into both numbers.


## Example:

1. HCF of 12 and 18 is 6 .
2. HCF of 100 and 150 is 50 .

- LCM: Least Common Multiple is the smallest positive integer that is divisible by two or more numbers.


## Example:

1. LCM of 12 and 18 is 36 .
2. LCM of 100 and 150 is 300 .

## EXISTING METHODS

These are some methods which exist for finding LCM of three numbers.

- Prime factorization method:

Let us take three numbers- $8,10,12$

| 2 | 8, | 10, | 12 |
| :--- | :--- | :--- | :--- |
| 2 | 4, | 5, | 6 |
| 2 | 2, | 5, | 3 |
| 3 | 1, | 5, | 3 |
| 5 | 1, | 5, | 1 |
|  | 1, | 1, | 1 |

$L C M=2 \times 2 \times 2 \times 3 \times 5=120$

- $\quad \operatorname{LCM}(\operatorname{LCM}(a, b), c)=\operatorname{LCM}(a, b, c)$

Let $a=18, b=24, c=30$
$\operatorname{LCM}(18,24,30)=\operatorname{LCM}(\operatorname{LCM}(18,24), 30)$
LCM of 18 and 24 is 72
$\operatorname{LCM}(18,24,30)=\operatorname{LCM}(72,30)$
LCM of 72 and 30 is 360.
$\operatorname{LCM}(18,24,30)=360$

- $\operatorname{LCM}(p, q, r)=\frac{p q r \times \operatorname{HCF}(p, q, r)}{\operatorname{HCF}(p, q) \times H C F(q, r) \times H C F(p, r)}$

Let $p=10, q=20, r=30$
$\operatorname{LCM}(10,20,30)=\frac{10 \times 20 \times 30 \times H C F(10,20,30)}{H C F(10,20 \times H C F(20,30) \times H C F(10,30)}$
$\operatorname{HCF}(10,20,30)=10$

```
\(\operatorname{HCF}(10,20)=10\)
\(\operatorname{HCF}(20,30)=10\)
\(\operatorname{HCF}(10,30)=10\)
\(\operatorname{LCM}(10,20,30)=\frac{10 \times 20 \times 30 \times 10}{10 \times 10 \times 10}\)
\(\operatorname{LCM}(10,20,30)=60\)
```


## PROPOSED METHODOLOGY

For following three numbers in Arithmetic progression I have found a relation between their LCM and HCF.

- Arithmetic progression is series of numbers where there is common difference between any two numbers.

The following formulae hold good to find LCM of any three numbers which are in sequence.
LCM (numbers in A.P) of three numbers say

$$
a, b, c=\frac{a \times b \times c}{(H C F)^{2}} \quad \text { or } \quad \frac{a \times b \times c}{2(H C F)^{2}}
$$

i.e LCM is equal to product of three numbers/square of HCF.
or
LCM is equal to product of three numbers/twice the square of HCF.
So, How to find when to use which formula?
When we divide three numbers by their HCF, if we get all three numbers co-primes to each other, we use $\frac{a \times b \times c}{(H C F)^{2}}$

$$
\begin{array}{l|ccc} 
\\
\text { i.e } & 46, & 40, & 44 \\
\cline { 2 - 2 } & 9, & 10, & 11
\end{array}
$$

Here, 9, 10, 11 are co-primes.
If we get two out of three numbers having common factor other than 1 , we use $\frac{a \times b \times c}{2(H C F)^{2}}$

i.e $\quad 50 |$| 100, | 150, | 200 |
| :---: | :---: | :---: |
|  | 2, | 3, |

Here, 2 and 4 are'nt co-primes

## RESULT OF THE FINDINGS

LCM (numbers in A.P) of three numbers $=\frac{a \times b \times c}{(H C F)^{2}}$

- 2, 4, 6

2, 4, 6 are in A.P
i.e $\mathrm{d}=2, \mathrm{HCF}=2$
$\mathrm{a}=2, \mathrm{~b}=4, \mathrm{c}=6$
$2 \times 4 \times 6 /(2)^{2}=\underline{12}$

$L C M=2 \times 2 \times 3=12$

LHS $=$ RHS

- 10, 20, 30

10, 20, 30 are in A.P
i.e $\mathrm{d}=10, \mathrm{HCF}=10$
$\mathrm{a}=10, \mathrm{~b}=20, \mathrm{c}=30$
$10 \times 20 \times 30 /(10)^{2}=\underline{60}$

| 2 | 10, | 20, | 30 |
| :--- | ---: | ---: | ---: |
| 5 | 5, | 10 | 15 |
| 2 | 1, | 2, | 3 |
| 3 | 1, | 1, | 3 |
|  | 1, | 1, | 1 |

LHS $=$ RHS

- 210, 230, 250

210, 230, 250 are in A.P
i.e $d=20, H C F=10$
$\mathrm{a}=210, \mathrm{~b}=230, \mathrm{c}=250$
$210 \times 230 \times 250 /(10)^{2}=\underline{120750}$

| 5 | 210, | 230, | 250 |
| :---: | :---: | :---: | :---: |
| 2 | 42, | 46, | 50 |
| 23 | 21, | 23, | 25 |
| 7 | 21, | 1, | 25 |
| 3 | 3, | 1, | 25 |
| 5 | 1, | 1, | 25 |
| 5 | 1, | 1, | 5 |
|  | 1, | 1, | 1 |

LHS = RHS
or LCM (numbers in A.P) of three numbers $=\frac{a \times b \times c}{2(H C F)^{2}}$

- 600, 602, 604
$600,602,604$ are in A.P
i.e $d=2, H C F=2$
$a=600, b=602, c=604$
$600 \times 602 \times 604 / 2 \times(2)^{2}=$ $\underline{27270600}$

| 2 | 600, | 602, | 604 |
| :---: | :---: | :---: | :---: |
| 2 | 300, | 301, | 302 |
| 2 | 150, | 301, | 151 |
| 7 | 75, | 301, | 151 |
| 151 | 75, | 43, | 151 |
| 43 | 75, | 43, | 151 |
| 5 | 75, | 1, | 1 |
| 3 | 15, | 1, | 1 |
| 5 | 5, | 1, | 1 |
|  | 1, | 1, | 1 |

LHS = RHS

- $4,6,8$

4, 6, 8 are in A.P
i.e $d=2, H C F=2$
$\mathrm{a}=4, \mathrm{~b}=6, \mathrm{c}=8$
$4 \times 6 \times 8 / 2 \times(2)^{2}=\underline{24}$

| 2 | 4, | 6, | 8 |
| :--- | :--- | :--- | :--- |
|  | 2, | 3, | 4 |
|  | 1, | 3, | 2 |
|  | 1, | 3, | 1 |
|  | 1, | 1, | 1 |

$L C M=2 \times 2 \times 2 \times 3=2$
$L C M=2 \times 2 \times 5 \times 5 \times 3 \times 2=600$

## REFERENCES

- www.math-only-math.com
- www.math.stackexchange.com
- www.wikipedia.org
- www.qc.edu.hk
- Maths textbook of CBSE class X

Source of support: Nil, Conflict of interest: None Declared.
[Copy right © 2017. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]

