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SIMILARITY AND COMPARISON OF SUMUDU TRANSFORM WITH ELZAKI TRANSFORM IN SOLVING ORDINARY DIFFERENTIAL EQUATIONS WITH VARIABLE COEFFICIENTS

H. K. UNDEGAONKAR*
Assistant Professor, Bahirji Smarak Mahavidyalaya Basmathnagar, India.

R. N. INGLE
Principal, Bahirji Smarak, Mahavidyalaya Basmathnagar, India.

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ABSTRACT

Sumudu transforms and Elzaki transforms both are applicable in solving second order ordinary and partial differential equations but are not used widely. In this paper our aim is to show similarity and comparison of these two integral transforms in solving ordinary differential equations with variable coefficients.

Keywords: *Sumudu transform, Elzaki transform, ordinary differential equations.*

INTRODUCTION

In Mathematics transformation is such a device which we use for the convertion of an integrable function f (t) in to another new function of new variable like transforms like f (p) , f (s).There are various integral transforms like

Laplace, Melin, Fourier, Elzaki etc. applicable for different purposes. Laplace transform technique was developed by the French Mathematician Pierre Simon de Laplace in 1779. Oliver Heaviside applied this technique for solving differential equations in Electrical engineering in 1900 [1].

The most important application of Laplace transform is in solving ordinary differential equation is that without finding general solution and particular solution initial value problems can be solved.

In 1990 Gamage K. watugala has introduced new transform namely Sumudu transform which is similar to Laplace transform .He introduced this transform to solve differential equations and control engineering problems [2].

In [3] some fundamental properties of Sumudu transform were established.

Anew integral transform namely Elzaki transform was introduced by Tarig Elzaki in 2010 [5]. Elzaki transform is a modified form of Sumudu and Laplace transform

.
[1] Some definitions and theorems of Sumudu transform which are given in [4]

Def. 1.1 The Sumudu transform of a function f(x) is defined by

$$G\ (w) = \int_0^{\infty} \frac{1}{w} e^{\frac{-x}{w}} f\ (x)\ dx$$

Theorem 1.1: If sumudu transform of f (x) is G (w) then

(a) $S[x\ f'(x)] = w^2 \frac{d}{dw}\left[\frac{G(W)-f(0)}{w}\right] + w\left[\frac{G(W)-f(0)}{w}\right]$

(b) $S[x\ f''((x)] = w^2 \frac{d}{dw}\left[\frac{G(W)-f(0)-f'(0)}{w^2}\right] + w\left[\frac{G(W)-f(0)-f'(0)}{w^2}\right]$

(c) $S\ [x^2 f''(x)] = w^4 \frac{d2}{dw2}\left[\frac{G(W)-f(0)-f'(0)}{w^2}\right] + 4w^3 \frac{d}{dw}\left[\frac{G(W)-f(0)-f'(0)}{w^2}\right] + 2w^2\left[\left[\frac{G(W)-f(0)-f'(0)}{w^2}\right]\right]$

Corresponding Author: H. K. Undegaonkar*
Assistant Professor, Bahirji Smarak Mahavidyalaya Basmathnagar

[2]. Some definitions and theorems of Elzaki transform given in [6].

Def.2.1 The Elzaki transform of f (x) denoted by E {f (x)} is defined by

$$E\{(x)\} = F(w) = w^2 \int_0^{\infty} f(wx)e^{-x} dx$$

Theorem 2.1: If F (w) is the Elzaki transform of the function f (x) then

(a) $E[x f(x)] = w^2 \frac{d}{dw} F(w) - wF(w)$

(b) $E[x^2 f(x)] = w^4 \frac{d2}{dw2} F(w)$

Theorem 2.2: If F (w) is the Elzaki transform of the function f (x) then

(a) $E[x f'(x)] = w^2 \frac{d}{dw}\left[\frac{F(w)}{w} - wf(0)\right] - w\left[\frac{F(w)}{w} - wf(0)\right]$

(b) $E[x f''(x)] = w^2 \frac{d}{dw}\left[\frac{F(w)}{w^2} - f(0) - wf'(0)\right] - w\left[\frac{F(w)}{w^2} - f(0) - wf'(0)\right]$

(c) $E[x^2 f''(x)] = w^4 \frac{d2}{dw2}\left[\frac{F(w)}{w^2} - f(0) - wf'(0)\right]$

[3]. Consider first order ordinary differential equation with variable coefficients given by

$$\alpha[xy'(x)] + \beta\, y(x) = \gamma\, x \quad \text{with initial condition } y(0) = a \tag{3.1}$$

Taking Sumudu transform of equation (3.1) we have

$$\alpha S[xy'(x)] + \beta S[y(x)] = \gamma S[x]$$

$$\alpha w^2 \frac{d}{dw}\left[\frac{G(W)-y(0)}{w}\right] + \alpha\, w\left[\frac{G(W)-y(0)}{w}\right] + \beta\, G(w) = \gamma w$$

Applying initial condition we have

$$\alpha w^2 \frac{d}{dw}\left[\frac{G(W)-a}{w}\right] + \alpha\, w\left[\frac{G(W)-a}{w}\right] + \beta\, G(w) = \gamma w$$

$$\alpha w^2 \frac{d}{dw}\left[\frac{G(W)}{w}\right] - a\, \alpha w^2 \frac{d}{dw}\left[\frac{1}{w}\right] + \alpha\, G(w) - a\, \alpha + \beta\, G(w) = \gamma w$$

$$\alpha\, wG'(w) - \alpha\, G(w) + a\, \alpha + \alpha\, G(w) - a\, \alpha + \beta\, G(w) = \gamma w$$

$$G'(w) + \frac{\beta}{\alpha\, w}\, G(w) = \frac{\gamma}{\alpha}$$

Which is again O.D.E of first order with variable coefficients hence cannot solved by sumudu transform

Now applying Elzaki transform to the same equation (3.1) we have

$$\alpha\; E[xy'(x)] + \beta\; E[y(x)] = \gamma\; E[x]$$

$$\alpha w^2 \frac{d}{dw}\left[\frac{F(w)}{w} - wy(0)\right] - w\left[\frac{F(w)}{w} - wy(0)\right] + \beta\, F(w) = \gamma w^3$$

After applying initial condition and simplifying the above equation we obtain

$$\alpha w\, F'(w) + (\beta - 2\alpha)\, F(w) = \gamma w^3$$

$$F'(w) + \frac{\beta - 2\alpha}{\alpha w}\, F(w) = \frac{\gamma w^2}{\alpha} \tag{3.2}$$

Which is again O.D.E of first order with variable coefficients hence cannot solved by Elzaki transform But if we change the initial condition as $y(0) = 0$ and taking $\alpha=2$, $\beta=4$ then equation (3.2) becomes

$$F'(w) = \gamma w^2$$

which implies that $F(w) = \frac{\gamma w^3}{3}$ w (3.3)

Taking Inverse Elzaki transform of (3.3) with $\gamma = 3$ we have $y(x) = x$ which is solution of equation (3.1)

[4]. Consider the linear second order ordinary differential equation with variable coefficients given by

$$\alpha x^2 y''(x) + \beta\, xy'(x) + \gamma y(x) = \delta\, x^2, \quad y(0) = a, \quad y'(0) = b \tag{4.1}$$

Taking sumudu transform to equation (3.1) we have

αS $[x^2y''(x)] + \beta$S $[xy'(x)]$ +γ S $[y(x)]$ = δ S $[x^2]$

$\alpha w^4 \frac{d2}{dw2}\left[\frac{G(W)-y(0)-y'(0)}{w^2}\right]$ +$4\alpha w^3 \frac{d}{dw}\left[\frac{G(W)-y(0)-y'(0)}{w^2}\right]$+$2\alpha w^2 \left[\frac{G(W)-y(0)-y'(0)}{w^2}\right]$ +$\beta\ w^2 \frac{d}{dw}\left[\frac{G(W)-y(0)}{w}\right]$ +$\beta\ w\left[\frac{G(W)-y(0)}{w}\right]$+$\gamma\ G(w)$ =2 δw^2

Applying initial conditions we obtain

$\alpha w^4 \frac{d2}{dw2}\left[\frac{G(W)-a-b}{w^2}\right]$ +$4\alpha w^3 \frac{d}{dw}\left[\frac{G(W)a-b)}{w^2}\right]$+$2\alpha w^2\left[\frac{G(W)-a-b}{w^2}\right]$ +$\beta\ w^2 \frac{d}{dw}\left[\frac{G(W)-a}{w}\right]$ +$\beta\ w\left[\frac{G(W)-a}{w}\right]$+$\gamma\ G(w)$ =2 δw^2

Therefore we have

$\alpha w^4 \frac{d2}{dw2}\left[\frac{G(w)}{w^2}\right]$+$4\alpha w^3 \frac{d}{dw}\left[\frac{G(w)}{w^2}\right]$+2$\alpha$ $G(w)$ -2 αa-2 αb+ $\beta\ w^2 \frac{d}{dw}\left[\frac{G(w)}{w^2}\right]$ + $\beta\ G(w) - \beta a + \gamma\ G(w)$ =2 δw^2

After simplification we obtain

$\alpha w^2 G''(w) + \beta\ w\ G'(w) + \gamma\ G(w)$= 2 δw^2+2 αa +2 αb (4.2)

This is again the linear second order ordinary differential equation with variable coefficients. Hence we are not able to solve equation (3.1) by Sumudu transform.

If we change the conditions by $y(0)$ =0, $y'(0)$=0 and taking α =1, β=4, γ=2 then the above equation (3.2) becomes

$w^2G''(w)$ + 4 $w\ G'(w)$ + 2 $G(w)$= 2 δw^2

which is again the linear second order ordinary differential equation with variable coefficients.Hence we are not able to solve equation (4.1) by Sumudu transform.

Now in this part we will try to solve equation (3.1) by applying Elzaki transform Taking Elzaki transform to equation (3.1) we have

αE $[x^2y''(x)]$ + βE $[xy'(x)]$ + γE $[y(x)]$ = δE $[x^2]$

$\alpha w^4 \frac{d2}{dw2}\left[\frac{F(w)}{w^2} - y(0) - wy'(0)\right]$ + $\beta w^2 \frac{d}{dw}\left[\frac{F(w)}{w} - wf(0)\right]$ - β w$\left[\frac{F(w)}{w} - y(0)\right]$ +$\gamma\ F(w)$ =2δw^4

Applying initial conditions we obtain

$\alpha w^4 \frac{d2}{dw2}\left[\frac{F(w)}{w^2} - a - wb\right]$ + $\beta w^2 \frac{d}{dw}\left[\frac{F(w)}{w} - wa\right]$ - β w$\left[\frac{F(w)}{w} - wa\right]$ +$\gamma\ F(w)$ =2δw^4

$\alpha w^4 \frac{d2}{dw2}\left[\frac{F(w)}{w^2}\right]$ –b $\alpha w^4 \frac{d2}{dw2}[w]$ + $\beta w^2 \frac{d}{dw}\left[\frac{F(w)}{w}\right]$ – $\beta w^2 a \frac{d}{dw}[w]$ – $\beta\ F(w)$+ $\beta w^2 a$ + $\gamma\ F(w) = 2\delta w^4$

$\alpha w^4 \frac{d}{dw}\left[\frac{w^2F'(w)-2wF(w)}{w^4}\right]$ + $\beta w^2\left[\frac{wF'(w)-F(w)}{w^2}\right]$ - $\beta w^2 a$ - $\beta\ F(w)$+ $\beta w^2 a + \gamma\ F(w)$ = 2δw^4

$\alpha w^4 \frac{d}{dw}\left[\frac{F'(w)}{w^2}\right]$ -2 $\alpha w^4 \frac{d}{dw}\left[\frac{F(w)}{w^3}\right]$ + $\beta\ wF'(w)$ - $\beta\ F(w) - \beta w^2 a$ -- $\beta\ F(w)$+ $\beta w^2 a + \gamma\ F(w)$ = 2δw^4

$\alpha w^4\left[\frac{w^2F''(w)-2wF'(w)}{w^4}\right]$-2 $\alpha w^4\left[\frac{w^3F'(W)-3W^2F(W)}{W^6}\right]$ + $\beta\ wF'(w) - \beta\ F(w) - \beta w^2 a - \beta\ F(w)$+ $\beta w^2 a$ + $\gamma\ F(w)$ =2δw^4

Simplifying the above equation we get

$\alpha w^2F''(w)$ -2 $\alpha wF'(w)$ -2 $\alpha wF'(w)$ +6 $\alpha\ F(w) + \beta\ wF'(w)$ - $\beta\ F(w) - \beta w^2 a$ -- $\beta\ F(w)$+ $\beta w^2 a + \gamma\ F(w)$ = 2δw^4

$\alpha w^2F''(w)$ + (β -4α) $wF'(w)$ + (6 α-2 β+ γ) $F(w)$ =2δw^4 (4.3)

Thus after applying Elzaki transform to equation (4.1) we get second order differential equation with variable coefficients hence we are not able to solve it .

Now if we take some other initial condition $y\ (0)$ = 0, $y'(0)$ = 0 and taking α =1, β=4, γ=2, $\delta = 12$ then the above equation (3.3) becomes

$$w^2 F''(w) = 24w^4 \quad (4.4)$$

Integrating equation (4.4) we obtain

$$F'(w) = \frac{24w^3}{3} + c$$

Again integrating the above equation we

$$F(w) = 2w^4 + c\, w + d$$

Taking inverse Elzaki transform of the above equation and using initial conditions we get c = d = 0 and we have

$$y(x) = x^2$$

Thus we have solved this equation by applying Elzaki transform successfully.

Similarly if we take same initial conditions and taking α =2, β=8, γ=4, $\delta = 24$ then also equation (4.1) can be solved by Elzaki transform which we could not solved by Sumudu transform.

CONCLUSION

In this paper after applying Sumudu transform and Elzaki transform to first and second order ordinary differential equation with variable coefficients (3.1) and (4.1) respectively there is similarity that equation (3.1) & (4.1) can not solved by these two integral transforms. But after changing initial condition and constant coefficients the same equations can be solved by Elzaki transform only. Finally if we compare these two transforms we conclude that Elzaki transform is more effective than Sumudu transform in solving first & second order ordinary differential equation with variable coefficients.

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