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CONTRA sgα- CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce and investigate the notion of Contra sgα- Continuous Functions. We obtain fundamental properties and characterization of contra sgα-continuous functions and discuss the relation- ships between contra-sgα-continuity and other related functions.

Subject Classification: *54C05, 54C08, 54C10.*

Keywords: *sgα-closed sets, sgα-continuous functions, contra sgα-continuous functions.*

1. INTRODUCTION

N. Levine [15] introduced generalized closed sets (briefly g-closed set) in 1970. N. Levine [14] introduced the concepts of semi-open sets in 1963. Bhattacharya and Lahiri [6] introduced and investigated semi-generalized closed (briefly sg- closed) sets in 1987. Arya and Nour [3] defined generalized semi-closed (briefly gs-closed) sets for obtaining some characterization of s-normal spaces in 1990. O.Njastad in 1965 defined α-open sets [22].

In 1996, Dontchev [10] introduced a new class of functions called contra- continuous functions. A new weaker form of this class of functions called contra semi-continuous function is introduced and investigated by Dontchev and Noiri [11].

In this paper, the notion of sgα-closed sets [8] in topological spaces is applied to introduce and study a new class of functions called contra sgα- continuous functions, as a new generalization of contra continuity, and to obtain some of their characterizations and properties. Also the relationships with some other functions are discussed.

2. PRELIMINARIES

Through this paper (X, τ), (Y, σ) and (Z, η) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and the interior of A are denoted by cl(A) and int(A) respectively. (X, τ) will be replaced by X if there is no chance of confusion. Let us recall the following definitions as pre requests.

A subset A of a topological space X is said to be open if $A \in \tau$. A subset.

A of a topological space X is said to be closed if the set X−A is open.

The interior of a subset A of a topological space X is defined as the union of all open sets contained in A. It is denoted by int(A). The closure of a subset A of a topological space X is defined as the intersection of all closed sets containing A. It is denoted by cl(A).

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Definitions 2.1: A subset A of a space (X, τ) is said to be

1. semi open [14] if A ⊆ cl (int(A)) and semi closed if int(cl (A)) ⊆ A.
2. α-open [22] if A ⊆ int (cl (int (A))) and α-closed if cl (int (cl (A))) ⊆ A.
3. β-open or semi pre-open [1] if A ⊆ cl (int(cl (A))) and β-closed or semi pre-closed if int (cl (int (A))) ⊆ A. pre-open [20] if A ⊆ int (cl (A)) and pre-closed if cl (int(A)) ⊆ A.
   The complement of a semi-open (resp.pre-open, α-open, β-open) set is called semi-closed (resp.pre-closed, α-closed, β-closed). The intersection of all semi-closed (resp.pre-closed, α-closed, β-closed) sets containing A is called the semi-closure (resp.pre-closure, α-closure, β-closure) of A and is denoted by scl(A)(resp. pcl(A), α-cl(A), β-cl(A)). The union of all semi-open (resp.pre-open, α-open, β-open) sets contained in A is called the semi-interior (resp.pre-interior, α-interior, β-interior) of A and is de- noted by sint(A)(resp.pint(A), α-int(A), β-int(A)). The family of all semi- open (resp.pre-open, α-open, β-open) sets is denoted by SO(X) (resp. P O(X), α − O(X), β − O(X)). The family of all semi-closed (resp.pre-closed, α-closed, β-closed)sets is denoted by SCl(X) (resp. P Cl(X), α-C l(X), β-Cl(X)).

Definitions 2.2: A subset A of a space (X, τ) is called

1. g-closed [15] if cl (A) ⊆ U, whenever A ⊆ U and U is open in (X, τ).The complement of a g-closed set is called g-open set.
2. gs-closed set [7] if scl (A) ⊆ U, whenever A ⊆ U and U is open in (X, τ).
3. sg-closed set [6] if scl (A) ⊆ U, whenever A ⊆ U and U is semi-open in (X, τ).
4. αg-closed [16] if α (cl (A)) ⊆ U, whenever A ⊆ U and U is open in (X, τ).
5. gα-closed [17] i f α (cl (A)) ⊆ U, whenever A ⊆ U and U is α-open in (X, τ).
6. gp-closed [18] if pcl (A) ⊆ U, whenever A ⊆ U and U is open in (X, τ).

Definition 2.3: Let X and Y be topological spaces. A function f: X → Y is said to be

1. continuous [13] if for each open set V of Y the set f^{-1}(V) is an open subset of X.
2. α-continuous [22] if f^{-1}(V) is a α-closed set of (X, τ) for every closed set V of (Y, σ).
3. β-continuous [1] if f^{-1}(V) is a β-closed set of (X, τ) for every closed set V of (Y, σ).
4. pre-continuous [20] if f^{-1}(V) is a pre-closed set of (X, τ) for every closed set V of (Y, σ).
5. semi-continuous [14] if f^{-1}(V) is a semi-closed set of (X, τ) for every closed set V of (Y, σ).

Definition 2.4: A function f: (X, τ) → (Y, σ) is said to be

1. g-continuous [15] if f^{-1}(V) is a g-closed set of (X, τ) for every closed set V of (Y, σ).
2. gs-continuous [7] if f^{-1}(V) is a gs-closed set of (X, τ) for every closed set V of (Y, σ).
3. sg-continuous [6] if f^{-1}(V) is a sg-closed set of (X, τ) for every closed set V of (Y, σ).
4. αg-continuous [16] if f^{-1}(V) is a αg-closed set of (X, τ) for every closed set V of (Y, σ).
5. gα-continuous [17] if f^{-1}(V) is a gα-closed set of (X, τ) for every closed set V of (Y, σ).
6. gp-continuous [18] if f^{-1}(V) is a gp-closed set of (X, τ) for every closed set V of (Y, σ).

Definitions 2.5 [21]: A function f: (X, τ) → (Y, σ) is said to be almost continuous if for every open set V of Y, f^{-1}(V) is regular open in X.

Definitions 2.6 [8]: A subset A of space (X, τ) is called sgα-closed if scl (A) ⊆ U, whenever A ⊆ U and U is α-open in X.

The family of all sgα-closed subsets of the space X is denoted by SGαC (X).

Definitions 2.7 [8]: The intersection of all sgα-closed sets containing a set A is called sgα-closure of A and is denoted by sgα-cl(A).

A set A is sgα-closed set if and only if sgα Cl (A) = A.

Definitions 2.8 [8]: A subset A in X is called sgα-open in X if A^c is sgα-closed in X.

The family of a sgα-open sets is denoted by SGαO(X).

Definitions 2.9 [8]: The union of all sgα-open sets containing a set A is called sgα-interior of A and is denoted by sgα-Int(A).

A set A is sgα-open set if and only if sgα Int (A) = A.

Lemma 2.10 [12]: The following properties hold for subsets A and B of a space X.
1. $x \in$ ker (A) if and only if $A \cap F = \varphi$ for any closed set F of X containing x.
2. $A \subset$ ker (A) and A = ker(A) if A is open in X.
3. if $A \subset B$, then ker (A) $\subset$ ker (B)

3. CONTRA − sgα -CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

In this section, the notion of a new class of functions called contra sgα- continuous functions is introduced and we obtain some of their characterizations and properties. Also, the relationships with some other related functions are discussed.

Definition 3.1: A function $f:(X, \tau)\to(Y, \sigma)$ is called sgα-continuous if $f^{-1}(V)$ is sgα-closed in (X, τ) for every closed set V of (Y, σ).

Definition 3.2: A function $f: X \to Y$ is said to be Contra sgα-Continuous if $f^{-1}(V)$ is sgα-closed in X for each open set V of Y.

Remark 3.3: From the following examples, it is clear that both contra sgα- continuous and sgα-continuous are independent notions of each other.

Example 3.4: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{a, b\}\}$ be topologies on X and Y respectively. Define a function $f: X \to Y$ by f(a) = a, f(b) = b and f(c) = c. Then f is sgα-continuous function but not contra sgα-continuous, because for the open set {a, b} in Y, $f^{-1}(\{a, b\}) = \{a, b\}$ is not sgα-closed in X.

Example 3.5: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{a, c\}\}$ be topologies on X and Y respectively. Define a function $f: X \to Y$ by f(a) = c, f(b) = b and f(c) = a. Then f is contra sgα-continuous function but not sgα-continuous, because for the open set {a} in Y, $f^{-1}(\{a\}) = \{c\}$ is not sgα-open in X.

Theorem 3.6: If $f: X \to Y$ is contra continuous, then it is contra sgα- continuous.

Proof: Let V be an open set in Y. Since f is contra continuous, $f^{-1}(V)$ is closed in X. Since every closed set is sgα-closed, $f^{-1}(V)$ is sgα-closed in X. Therefore f is contra sgα-continuous.

Remark 3.7: Converse of the above theorem need be true in general as seen from the following examples.

Example 3.8: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{a, c\}\}$ be topologies on X and Y respectively. Define a function $f: X \to Y$ by f(a) = b, f(b) = a and f(c) = c. Then f is contra sgα-continuous function but not contra-continuous, because for the open set {a} in Y, and $f^{-1}(\{a\}) = \{b\}$ is not closed in X.

Theorem 3.9: If $f: X \to Y$ is contra semi-continuous, then it is contra sgα-continuous.

Proof: Let V be an open set in Y. Since f is contra semi-continuous, $f^{-1}(V)$ is semi-closed in X. Since every semi-closed set is sgα-closed, $f^{-1}(V)$ is sgα-closed in X. Therefore f is contra sgα-continuous.

Remark 3.10: Converse of the above theorem need be true in general as seen from the following examples.

Example 3.11: Let and $\sigma = \{Y, \varphi, \{a\}, \{b\}, \{a, b\}\}$ be topologies on X and Y respectively. Define a function $f: X \to Y$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is contra sgα-continuous function but not contra semi-continuous, because for the open set $\{b\}$ in Y and $f^{-1}(\{b\}) = \{a\}$ is not semi-closed in X.

Theorem 3.12: The following are equivalent for a function $f: X \to Y$

1. f is contra sgα-continuous.
2. for every closed set F of Y, $f^{-1}(F)$ is sgα-open set of X.
3. for each $x \in X$ and each closed set F of Y containing $f(x)$, there exist sgα-open set U containing x such that $f(U) \subset F$.
4. for each $x \in X$ and each other open set F of Y containing $f(x)$, there exists sgα-closed set K not containing x such that $f^{-1}(V) \subset K$.
5. $f(sg\alpha - Cl(A)) \subset \ker(f(A))$ for every subset A of X.
6. $sg\alpha - Cl(f^{-1}(B)) \subset f^{-1}(\ker(B))$ for every subset B of Y.

Proof:

(1) ⇒(2): Let F be a closed set in Y. Then $Y-F$ is an open set in Y. By (1), $f^{-1}(Y- F) = X - f^{-1}(F)$ is sgα-closed set in X. This implies $f^{-1}(F)$ is sgα-open set in X. Therefore (2) holds.

(2) ⇒(1): Let G be an open set of Y. Then $Y-G$ is a closed set in Y. By (2), $f^{-1}(Y- G) = X- f^{-1}(G)$ is sgα-open set in X, which implies $f^{-1}(G)$ is sgα-closed set in X. Therefore (1) holds.

(2) ⇒(3): Let F be a closed set in Y containing $f(x)$. Then $x \in f^{-1}(F)$. By (2), $f^{-1}(F)$ is sgα-open set in X containing x. Let $U = f^{-1}(F)$. Then $f(U) = f(f^{-1}(F)) \subset F$. Therefore (3) holds.

(3) ⇒(2): Let F be a closed set in Y containing $f(x)$. Then $x \in f^{-1}(F)$. From (3), there exists sgα-open set U_X in X containing x such that $f(U_X) \subset F$. That is $U_X \subset f^{-1}(F)$. Thus $f^{-1}(F) = \cup\{U_X : x \in f^{-1}(F)\}$, which is union of sgα-open sets. Since union of sgα-open sets is a sgα-open sets, $f^{-1}(F)$ is sgα-open set of X.

(3) ⇒(4): Let V be an open set in Y not containing $f(x)$. Then $Y- V$ is closed set in Y containing $f(x)$. From (3), there exists a sgα-open set U in X containing x such that $f(U) \subset Y - V$. This implies $U \subset f^{-1}(Y - V) = X - f^{-1}(V)$. Hence, $f^{-1}(V) \subset X - U$. Set $K = X - U$, then K is sgα-closed set not containing x in X such that $f^{-1}(V) \subset K$.

(4) ⇒(3): Let F be a closed set in Y containing $f(x)$. Then $Y-F$ is an open set in Y not containing $f(x)$. From (4), there exists sgα-closed set K in X not containing x such that $f^{-1}(Y - F) \subset K$. This implies $X - f^{-1}(F) \subset K$. Hence, $X - K \subset f^{-1}(F)$, that is $f(X - K) \subset F$. Set $U = X - K$, then U is sgα-open set containing x in X such that $f(U) \subset F$.

(2) ⇒(5): Let A be any subset of X. Suppose $y \notin \ker(f(A))$. Then by lemma 4.1, there exists a closed set F in Y containing y such that $f(A) \cap F = \varphi$. Thus, $A \subset f^{-1}(F) = \varphi$. Therefore $A \subset X - f^{-1}(F)$. By (2), $f^{-1}(F)$ is sgα-open set in X and hence $X - f^{-1}(F)$ is sgα-closed set in X. Therefore, $sg\alpha - Cl(X - f^{-1}(F)) = X - f^{-1}(F)$. Now $A \subset X - f^{-1}(F)$, which implies $sg\alpha - Cl(A) \subset sg\alpha - Cl(X - f^{-1}(F)) = X - f^{-1}(F)$. Therefore $sg\alpha - Cl(A) \cap f^{-1}(F) = \varphi$, which implies $f(sg\alpha - Cl(A)) \cap F = \varphi$ and hence $y \notin sg\alpha - Cl(A)$. Therefore $f(sg\alpha - Cl(A)) \subset \ker(f(A))$ for every subset A of X.

(5) ⇒(6): Let $B \subset Y$. Then $f^{-1}(B) \subset X$. By (4) and lemma 2.10, $f(sg\alpha - Cl(f^{-1}(B))) \subset \ker(f(f^{-1}(B))) \subset \ker(B)$. Thus $sg\alpha - cl(f^{-1}(B)) \subset f^{-1}(\ker(B))$ for every subset B of Y.

(6) ⇒(1): Let V be any open subset of Y. Then by (6) and lemma 2.10, sgα−C l (f^{-1}(V))⊂f^{-1}(ker (V)) = f^{-1}(V) and sgα − C l (f^{-1}(V))=f^{-1}(V). Therefore f^{-1}(V) is sgα-closed set in X. This shows that f is contra sgα-continuous.

Theorem 3.13: If a function f: X →Y is contra sgα-continuous and Y is regular, then f is sgα-continuous.

Proof: Let x∈X and V be an open set in Y containing f(x). Since Y is regular, there exists an open set W in Y containing f(x) such that Cl (W)⊂V. Since f is contra sgα-continuous, by theorem 3.12 (3), there exists sgα-open set U in X containing x such that f (U)⊂Cl (W). Then f (U)⊂Cl (W)⊂V. Therefore f is sgα-continuous.

Theorem 3.14: If a function f:X → Y is contra sgα-continuous and X is Tsgα-space, then f is contra continuous.

Proof: Let U be an open set in Y. Since f is contra sgα-continuous, f^{-1}(U) is sgα-closed in X. Since X is Tsgα-space f^{-1}(U) is a closed set in X. There- fore f is contra continuous.

Theorem 3.15: If a function f: X →Y is contra sgα-continuous and X is sgα T1/2-space, then f is contra semi continuous.

Proof: Let U be an open set in Y. Since f is contra sgα-continuous, f^{-1}(U) is sgα-closed in X. Since X is sgα T1/2-space, f^{-1}(U) is a semi closed set in X. Therefore f is contra semi continuous.

Definition 3.16: A space X is called locally sgα-indiscrete if every sgα- open set is closed in X.

Theorem 3.17: If a function f: X→Y is contra sgα-continuous and X is locally sgα-indiscrete space, then f is continuous.

Proof: Let U be an open set in Y. Since f is contra sgα-continuous and X is locally sgα-indiscrete space, f^{-1}(U) is an open set in X. Therefore f is continuous.

Definition 3.18: If a function f: X → Y is called almost sgα-continuous if for each x ∈X and each open set V of Y containing f(x), there exists U∈SGαO(X, x) such that f (U)⊂Int (cl (V)).

Definition 3.19: If a function f: X →Y is called quasi sgα-open if image of every sgα-open set of X is open set in Y .

Theorem 3.20: If a function f: X →Y is contra sgα-continuous, quasi sgα-open, then f is almost sgα-continuous function.

Proof: Let x be any arbitrary point of X and V be an open set in Y containing f(x). Then Cl (V) is a closed set in Y containing f(x). Since f is contra sgα- continuous, then by theorem3.12 (3), there exists U∈SGαO(X, x) such that f (U) ⊂cl (V). Since f is quasi sgα-open, f (U) is open in Y. Therefore f (U) = Int (Cl (U)). Thus, f (U)⊂Int (f (V)). This shows that f is almost sgα-continuous function.

Definition 3.21: If a function f: X →Y is called weakly sgα-continuous if for each x ∈X and each open set V of Y containing f(x), there exists U∈SGαO(X, x) such that f (U)⊂scl (V).

Theorem 3.22: If a function f: X →Y is contra sgα-continuous, then f is weakly sgα-continuous function.

Proof: Let V be an open set in Y. Since Cl (V) is closed in Y, by theorem3.12 (2), f^{-1}(Cl (V)) is sgα-open set in X. Set U=f^{-1}(C l (V)) then f (U)⊂f (f^{-1}(C l (V)))⊂C l (V). This shows that f is almost weakly sgα-continuous function.

Definition 3.23: Let A be a subset of X. Then sgα-C l (A)-sgα-Int (A) is called sgα-frontier of A and is denoted by sgα-F r (A)

Theorem 3.24: The set of all points of x of X at which $f: X \to Y$ is not contra sgα-continuous is identical with the union of sgα-frontier of the inverse images of closed sets of Y containing $f(x)$.

Proof: Assume that f is not contra sgα-continuous at $x \in X$. Then by theorem 3.12(3), there exists $F \in C(Y, f(x))$ such that $f(U) \cap (Y-F) = \varphi$. For every $U \in SG\alpha O(X, x)$. This implies $U \cap f^{-1}(Y-F) = \varphi$, for every $U \in SG\alpha O(X, x)$. Therefore, $x \in sg\alpha - Cl(f^{-1}(Y-F)) = sg\alpha - Cl(X - f^{-1}(F))$. Also $x \in f^{-1}(F) \subset sg\alpha - Cl(f^{-1}(F))$. Thus, $x \in sg\alpha - Cl(f^{-1}(F)) \cap sg\alpha - Cl(X - f^{-1}(F))$. This implies $x \in sg\alpha - Cl(f^{-1}(F)) - sg\alpha - Int(f^{-1}(F))$. Therefore, $x \in sg\alpha - Fr(f^{-1}(F))$.

Conversely, Suppose $x \in sg\alpha - Fr(f^{-1}(F))$ for some $F \in C(Y, f(x))$ and f is contra sgα-continuous at $x \in X$, then there exists $U \in SG\alpha O(X, x)$ such that $f(U) \subset F$. Therefore, $x \in U \subset f^{-1}(F)$ and hence $x \in sg\alpha - Int(f^{-1}(F)) \subset X - sg\alpha - Fr(f^{-1}(F))$. This contradicts the fact that $x \in sg\alpha - Fr(f^{-1}(F))$. Therefore f is not contra sgα-continuous.

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