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ON (sg)* CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce a new class of sets namely (sg)- closed sets and a new class of generalized functions namely (sg)* - continuous maps, (sg)*- irresolute maps. Further the separation axioms namely* T_S*-space and $^*T_S$*-space are introduced and its basic properties are discussed.*

Keywords: *(sg)*- closed set, (sg)* - continuous maps, (sg)*- irresolute map,* T_S*-space and $^*T_S$*-space.*

1. INTRODUCTION

Levine [7] introduced the class of generalized closed sets in 1970. Andrijevic [1] defined semi-pre- open sets in 1986. Balachandran, Sundaram and Maki [2] introduced on generalized continuous maps in topological spaces in 1991. Devi [3], Maki and Balachandran introduced Semi-generalized closed maps and generalized closed maps in 1993. Veerakumar [16] defined g* closed sets which lies between closed sets and g-closed sets in 1996. Levine [8], and Njasted [11] introduced semi-open sets, pre-open sets, α-closed sets. Pauline Mary Helen, Ponnuthai Selvarani and Veronica Vijayan [13], introduced g**-closed sets in topological spaces in 2012. M.Pauline Mary Helen, Monika.P [14] introduced sg** -closed sets in topological spaces in 2013.

In this paper we introduce and study the concept of (sg)*- closed set, (sg)* - continuous maps, (sg)*- irresolute maps, T_S*-space and $^*T_S$*-space.

2. PRELIMINARIES

Throughout this paper (X,τ), (Y,σ), (Z,) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of space (X,τ), cl(A) and int(A) denote the closure and interior of A respectively.

Definition 2.1: A subset A of a topological space (X,τ) is called
1) *a semi – open set* [8] if A⊆cl(int(A)) and semi – closed set if int(cl(A))⊆A.
2) *a semi – pre open* [1] set if A⊆cl(int(cl(A))) and semi – pre closed set if int(cl(int(A)))⊆A.
3) *an α – open set* [10] if A⊆ int(cl(int(cl(A))) and an α – closedset if cl(int(cl(A))) ⊆ A.

Definition 2.2: A subset A of a topological space (X,τ) is called
1) *generalized closed set* (briefly g-closed) [7] if cl(A)⊆U whenever A⊆U and U is open in (X,τ).
2) *generalized* closed set* (briefly g*- closed) [16] if cl(A)⊆U whenever A⊆U and U is g - open in (X,τ).
3) *generalized** closed set* (briefly g**-closed) [13] if cl(A)⊆U whenever A⊆U and U is g* - open in (X,τ).
4) *generalized semi-pre closed set* (briefly gsp-closed) [8] if spcl(A)⊆U whenever A⊆U and U is open in (X,τ).
5) *generalized# semi- closed set* (briefly $g^{\#}s$ - closed) [12] if scl(A)⊆U whenever A⊆U and U is αg - open in (X,τ).
6) *generalized * semi closed set* (briefly g*s-closed) [16] if scl(A)⊆U whenever A⊆U and U is gs - open in (X,τ).
7) *semi α generalized* closed set* (briefly sαg ∗-closed)[9] if αcl(A)⊆U whenever A⊆U and U is g*- open in (X,τ).

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8) αg^{**} - *closed*[15] if αcl(A)⊆U whenever A⊆U and U is g** - open
9) *sg*** - *closed*[14] if scl(A)⊆U whenever A⊆U and U is g** - open

Definition 2.3: A function f: (X,τ) → (Y,σ) is called
1) *Continuous* [18] if the inverse image of every closed set in (Y,σ) is closed in (X,τ).
2) *gsp – continuous* [5] if the inverse image of every closed set in (Y,σ) is gsp - closed in (X,τ).
3) *sαg** - *continuous* [9] if the inverse image of every closed set in (Y,σ) is sαg* - closed in (X,τ).
4) *g#s – continuous* [12] if the inverse image of every closed set in (Y,σ) is g#s - closed in (X,τ).
5) *g*s – continuous* [16] if the inverse image of every closed set in (Y,σ) is g*s - closed in (X,τ).
6) αg^{**} - *continuous* [15] if the inverse image of every closed set in (Y,σ) is αg**- closed in (X,τ).
7) *sg*** - *continuous* [14] if the inverse image of every closed set in (Y,σ) is sg**- closed in (X,τ).

Definition 2.4: A map f: (X,τ) → (Y,σ) is called a g* - irresolute **[13]** if the inverse image of every g* - closed set in (Y,σ) is g*- closed in (X,τ).

Definition 2.5: A topological space (X,τ) is said to be
1) *a* $T_{1/2}$* - *space* [13] if every g*-closed set in (X,τ) is closed in (X,τ).
2) *a* T_α ** - *space* [15] if every αg**-closed set in (X,τ) is closed in (X,τ).
3) *a* $*T_\alpha*$ - *space* [15] if every αg**-closed set in (X,τ) is g* - closed in (X,τ).

3. BASIC PROPERTIES OF (sg)*- CLOSED SETS

We now introduce the following definition

Definition 3.1: A subset A of a topological space (X,τ) is said to be *(sg)*- closed set* if scl(A)⊆A, whenever A⊆U and U is *g*- open* in X.

Theorem 3.2: Every closed set is *(sg)* closed set.*

Proof follows from the definition.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.3: Let X={a, b, c} τ={X, ∅,{a},{a, b}}. Let A={b} then A is (sg)* but it is not closed set in (X, τ).

Theorem 3.4: Every *(sg)* - closed set* is *gsp - closed set.*

Let A be a (sg)* - closed set. Let A⊆U and U is open. Then U is g*-open. Since A is (sg)*-closed, then spcl(A)⊆scl(A)⊆U. Hence A is gsp closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.5: Let X={a, b, c} τ={X,∅,{b, c}}. Let A={b}, then A is gsp but it is not *(sg)* - closed set* in (X, τ).

Theorem 3.6: Every *g#s closed set* is *(sg)* - closed set.*

Let A be $g^{\#}$s - closed set. Let A⊆U and U is g* - open. Then U is αg - open. Since A is $g^{\#}$s – closed, scl (A) ⊆U Hence A is (sg)* - closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.7: Let X={a, b, c} τ={X,∅, {a},{a,b}}. Let A={a, c}, then A is (sg)*, but it is not g#s - closed in (X, τ).

Theorem 3.8: Every *g*s closed set* is *(sg)* - closed set.*

Proof follows from the definition.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.9: Let X={a, b, c} τ={X,∅, {a},{a, c}}. Let A={a, b}, then A is (sg)* - closed, but it is not g*s - closed in (X, τ).

Theorem 3.10: Every *g* - closed set* is *(sg)* - closed set*.

Proof follows from the definition.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.1: Let X={a, b, c} τ={X,∅, {a},{a, b}}. Let A={b}, then A is (sg)*- closed, but it is not g* - closed set in (X, τ).

Theorem 3.12: Every *$s\alpha g$* closed set* is *(sg)* - closed set*.

Let A be $s\alpha$g* - closed set. Let A⊆U where U is g* - open. Since A is $s\alpha$g* - closed, αcl(A) ⊆ U whenever A⊆U and U is g* - open. Therefore, scl(A) ⊆αcl(A) ⊆U. Hence A is (sg)* - closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.13: Let X={a, b, c} τ={X,∅, {a},{b},{a,b}}. Let A={a}, then A is (sg)* - closed, but it is not sαg* - closed set in (X, τ).

Theorem 3.14: Every *αg** closed set* is *(sg)* - closed set*.

Let A be αg** - closed set. Let A⊆U where U is g* - open, then U is g** - open. Since A is αg** - closed, scl(A) ⊆αcl(A) ⊆U. Hence A is (sg)* - closed.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.15: Let X={a, b, c} τ={X,∅, {a}}. Let A={a}, then A is (sg)* -closed, but it is not αg**-closed set in (X,τ).

Theorem 3.16: Every *sg** closed set* is *(sg)* - closed set*.

Proof follows from the definition.

The converse of the above proposition need not be true in general as seen in the following example.

Example 3.17: Let X={a, b, c} τ={X,∅, {a}}, Let A={a,b}, then A is (sg)* - closed, but not sg** - closed set in (X, τ).

Theorem 3.18: If A is *(sg)*- closed set* of (X,τ) such that A ⊆ B ⊆ scl(A), then B is *(sg)*- closed set* of (X, τ).

Proof: It is given that A is (sg)* - closed set in X. Let U be g*- open set of X, such that B⊆U. Since A is (sg)*- closed and scl(A)⊆U.But B ⊆ scl(A),now scl(B)⊆ scl(scl(A)) ⊆ scl(A) ⊆U. Therefore scl(B) ⊆U. Hence B is (sg)* - closed in X.

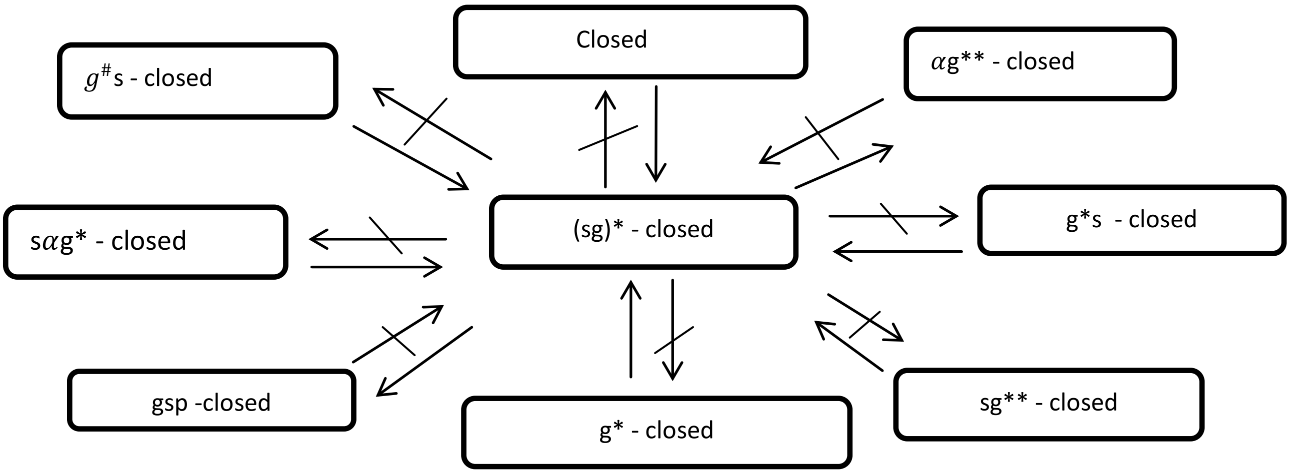
Theorem 3.19: If A is both *g*- open* and *(sg)* - closed*, then A is semi closed.

Proof: Let A be both g*-open and (sg)*-closed. Let A⊆A, where A is g*-open.Then scl(A) ⊆ A as A is (sg)*-closed in (X,τ). But A ⊆ scl(A) is always true. Therefore A = scl(A). Hence A is semi closed set in (X,τ).

Theorem 3.20: A is an *(sg)* - closed* set of (X,τ) then scl(A)\A does not contain any non – empty *g* - closed* set.

Proof: Let F be a g* - closed set of (X,τ) such that F⊆scl(A)\A ,then A⊆X\F. Since A is (sg)* - closed and X\F is g* - open, scl(A) ⊆ X\F. This implies F⊆X\scl(A) so F⊆(X\scl(A))

The above results can be represented in the following diagram.



Where A ⟶ B represents A implies B and A ⟶ B represents A does not imply B.

4. (sg)*- CONTINUOUS and (sg)*-IRRESOLUTE MAPS

We now introduce the following definitions

Definition 4.1: A function f:(X,τ) → (Y,σ) is called *(sg)*- continuous* if f^{-1}(V) is *(sg)*- closed* set of (X,τ) for every closed set V of (Y,σ).

Theorem 4.2: Every continuous map is *(sg)*- continuous*.

Proof: Let f:(X,τ) → (Y,σ) be continuous. Let V be a closed set in (Y,σ).Then f^{-1}(V) is closed in (X, τ).Since every closed set is (sg)* - closed, f^{-1}(V) is (sg)* -closed in (X, τ). Therefore f is (sg)* - continuous.

The following example supports that the converse of the above theorem is not true.

Example 4.3: Let X=Y={a,b,c}, τ = {X,∅,{a},{a,b}},σ={Y,∅,{b}}

Let f: (X,τ)→(Y,σ) be defined by identity mapping f(a)=a, f(b)=b, f(c)=c. f is (sg)* - continuous but not continuous. Since f^{-1}{a, c}= {a, c} is (sg)* - closed set of (X,τ). But not closed set of (X,τ).

Theorem 4.4: Every *(sg)* continuous* map is *gsp continuous*.

Proof: Let f: (X,τ)→(Y,σ) be (sg)* - continuous map. Let V be a closed set in (Y,σ), then f^{-1}(V) is (sg)* - closed in (X, τ).Since every (sg)*-closed set is gsp - closed, f^{-1}(V) is gsp – closed in (X, τ). Therefore f is gsp - continuous in (X, τ).

The following example supports that the converse of the above theorem is not true.

Example 4.5: Let X=Y={a, b, c}, τ = {X,∅,{b, c}},σ={Y,∅,{a, c}}

Let f: (X,τ)→(Y,σ) be defined by identity mapping f(a)=a, f(b)=b,f(c)=c. f is (sg)* -continuous but not gsp continuous. Since f^{-1}{b}={b} is gsp - closed set of (X,τ) but not (sg)*-closed set of (X,τ).

Theorem 4.6: Every *$g^{\#}$s - continuous* map is *(sg)* - continuous*.

Proof: Let f: (X,τ)→(Y,σ) be $g^{\#}$s - continuous map. Let V be a closed set in (Y,σ), then f^{-1}(V) is $g^{\#}$s - closed in (X, τ).Since every $g^{\#}$s - closed set is (sg)* - closed, f^{-1}(V) is (sg)*– closed in (X, τ). Therefore f is (sg)* - continuous.

The following example supports that the converse of the above theorem is not true.

Example 4.7: Let X=Y={a, b, c}, τ = {X,∅,{a},{a,b}},σ={Y,∅,{b}}

Let f: (X,τ)→(Y,σ) be defined by identity mapping f(a)=a, f(b)=b, f(c)=c. f is (sg)* - continuous but not $g^{\#}$s - continuous. Since f^{-1}{a, c}= {a, c} is (sg)*-closed set of (X,τ) but not $g^{\#}$s - continuous.

Theorem 4.8: Every *g*s - continuous* map is *(sg)* - continuous*.

Proof: Let f: (X,τ)→(Y,σ) . Let V be a closed set in (Y,σ), then f^{-1}(V) is g*s - closed in (X, τ). Since, every g*s - closed set is (sg)* - closed, f^{-1}(V) is (sg)* - closed in (X,τ). Therefore f is (sg)* - continuous.

The following example supports that the converse of the above theorem is not true.

Example 4.9 : Let X=Y={a, b, c}, τ = {X,∅,{a},{a,c}},σ={Y,∅,{c}}

Let f: (X,τ)→(Y,σ) be defined by identity mapping f(a)=a, f(b)=b,f(c)=c.f is (sg)* -continuous but not g*s - continuous. Since f^{-1}{a, b}= {a, b} is (sg)*-closed set of (X,τ) but not g*s - continuous.

Theorem 4.10: Every *g* - continuous map* is *(sg)* - continuous*.

Proof: Let f be a map from (X,τ) to (Y,σ). Let V be a closed set in (Y,σ), then f^{-1}(V) is g* - closed in (X, τ). Since every g* - closed set is (sg)* - closed. f^{-1}(V) is (sg)* - closed in (X,τ). Therefore f is (sg)* - continuous.

The following example supports that the converse of the above theorem is not true.

Example 4.11: Let X=Y={a, b, c}, τ = {X, ∅, {a},{a, b}}, σ = {Y, ∅, {a, c}}

Let f: (X,τ)→(Y,σ) be defined by identity mapping f(a)=a, f(b)=b, f(c)=c. f is (sg)* -continuous but not g*- continuous. Since {b} is (sg)*-closed set of (X,τ) but not g* - continuous.

Theorem 4.12: Every *sαg* - continuous* map is *(sg)* - continuous*.

Proof: Let f be a map from (X,τ) to (Y,σ). Let V be a closed set in (Y,σ) then f^{-1}(V) is sαg* - closed in (X, τ). Since every sαg* - closed set is (sg)* - closed, f^{-1}(V) is (sg)* –closed in (X, τ). Therefore f is (sg)* - continuous

The following example supports that the converse of the above theorem is not true.

Example 4.13: Let X=Y={a, b, c}, τ = {X, ∅, {a},{b},{a, b}}, σ={Y, ∅, {b, c}}.

Let f: (X,τ)→(Y,σ) be defined by identity mapping f(a) = a, f(b) = b, f(c) = c. f is (sg)*-continuous but not sαg* - continuous. Since f^{-1}{a}={a} is (sg)*-closed set of (X,τ) but not sαg* - continuous.

Theorem 4.14: Every *αg** - continuous* map is *(sg)* - continuous*.

Proof: Let f be a map from (X,τ) to (Y,σ). Let V be a closed set in (Y,σ), then f^{-1}(V) is αg** - closed in (X, τ).Since every αg** - closed set is (sg)* - closed, f^{-1}(V) is (sg)* –closed in (X, τ). Therefore f is (sg)* - continuous in (X, τ).

The following example supports that the converse of the above theorem is not true.

Example 4.15: Let X=Y={a, b, c}, τ = {X, ∅, {a}}, σ={Y, ∅, {b, c}}

Let f:(X,τ)→(Y,σ) be defined by identity mapping f(a) = a, f(b) =b, f(c) = c. f is (sg)*-continuous but not αg**-continuous. Since f^{-1}{a}={a} is (sg)*-closed set of (X,τ) but not αg** - continuous.

Theorem 4.16: Every *sg** - continuous* map is *(sg)* - continuous*.

Let f be a map from (X,τ) to (Y,σ). Let V be a closed set in (Y,σ), then f^{-1}(V) is sg** - closed in (X, τ). Since every sg** - closed set is (sg)* - closed, which implies f^{-1}(V) is (sg)* –closed in (X, τ). Therefore f is (sg)* - continuous.

The following example supports that the converse of the above theorem is not true.

Example 4.17: Let X=Y={a, b, c}, τ = {X, ∅, {a}}, σ={Y, ∅, {b, c}}.

Let f: (X,τ)→(Y,σ) be defined by identity mapping f(a) = a, f(b) = b, f(c) = c. f is (sg)* -continuous but not sg** - continuous. Since {a} is (sg)*-closed set of (X,τ) but not sg** - continuous.

Definition 4.18: A function f:(X,τ) → (Y,σ) is called *(sg)*- irresolute* if f^{-1}(V) is (sg)*- closed set of (X,τ) for every *(sg)*- closed* set V of (Y,σ) .

Theorem 4.19: Every *g*- irresolute map* is *(sg)* - continuous*

Proof follows from the definition.

Theorem 4.20: Let f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z, η) be any two functions then,

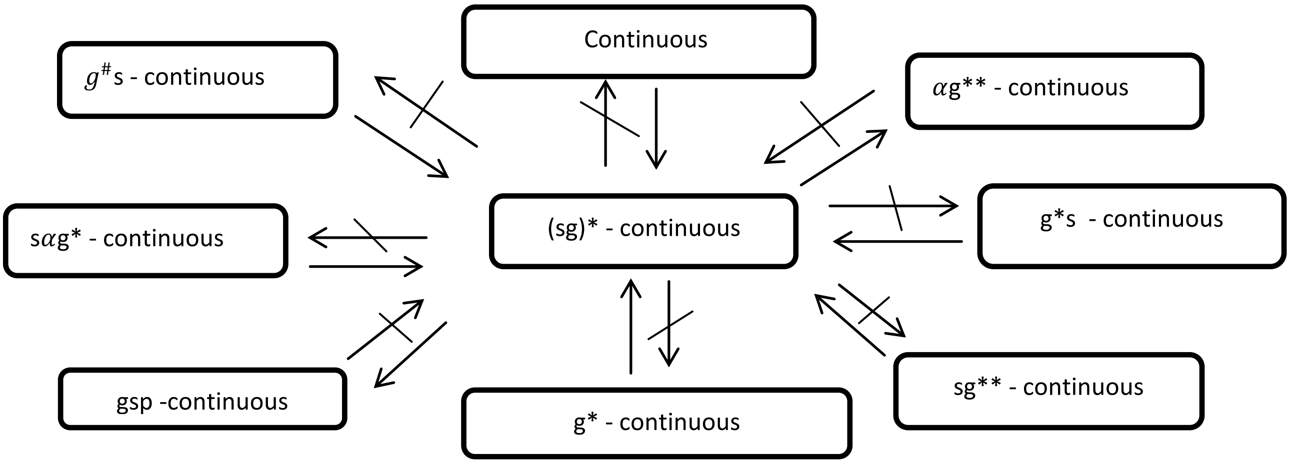
(i) gof: (X,τ) → (Z, η) is *(sg)* - continuous* if f is *(sg)*-irresolute* and g is *(sg)*- continuous*.

(ii) gof: (X,τ) → (Z, η) is *(sg)* - irresolute* if f and g are *(sg)*- irresolute*.

(iii) gof: (X,τ) → (Z, η) is *(sg)* - continuous* if f is *(sg)*- continuous* and g is continuous.

Proof follows from the definition

The above theorems can be represented in the following diagram



Where A ⟶ B represents A implies B and A ⟶̸ B represents A does not imply B.

5. APPLICATIONS OF (sg)* - CLOSED SETS IN TOPOLOGICAL SPACES

Definition 5.1: A space (X, τ) is said to be T_S* - *space* if every (sg)* - closed set in (X,τ) is closed in (X,τ).

Theorem 5.2: Every T_S* - *space* is $T^*_{1/2}$ - *space* but not conversely.

Proof follows from the definition

Example 5.3: Let X={a, b, c} τ={ ∅, X,{a}}**.** g*- closed sets are ∅, X,{b,c}**.** Therefore (X,τ) is $T^*_{1/2}$ - space. (sg)* - closed sets are ∅, X, {a},{b},{c},{a,b},{a,c},{b,c} and closed sets are ∅, X,{b,c}. Let A={b} then A is not closed in (X,τ). Therefore (sg)*-closed set is not closed in (X,τ). Hence (X,τ) is not T_S* - space.

Theorem 5.4: Every T_S* - space is T_α** - space but not conversely.

Proof follows from the definition.

Definition 5.5: A space (X,τ) is said to be *T_S* - space if every (sg)* - closed set in (X,τ) is g*- closed in (X,τ).

Theorem 5.6: Every *T_S* - space is *T_α* - space but not conversely.

Proof: Let (X,τ) be *T_S* - space. Let A be αg** - closed in (X,τ). But by proposition, Every αg** - closed set is (sg)* - closed. Since (X,τ) is *T_S* - space, A is g*-closed in (X,τ). It implies αg**- closed set in (X,τ) is g* - closed in (X,τ). Therefore (X,τ) is *T_α* - space.

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