International Journal of Mathematical Archive-8(3), 2017, 21-29
JMA Available online through www.ijma.info ISSN 2229 – 5046

$(\alpha p)^*$ - CLOSED SETS IN TOPOLOGICAL SPACES

L. ELVINA MARY
Assistant Professor, Nirmala College for Women, Coimbatore, India.

R. SARANYA*
M. Phil Scholar, Nirmala College for Women, Coimbatore, India.

(Received On: 12-02-17; Revised & Accepted On: 13-03-17)

ABSTRACT

In this paper, we introduce a new class of sets called $(\alpha p)^*$ -closed sets and a new class of generalized function called $(\alpha p)^*$ -continuous maps and $(\alpha p)^*$ -irresolute maps in topological spaces. Also we discuss some basic properties and applications of $(\alpha p)^*$ -closed sets, which define a new class of space $T_{(\alpha p)^*}$ spaces.

Key words: *$(\alpha p)^*$ -closed sets, $(\alpha p)^*$ -continuous, $(\alpha p)^*$- irresolute and $T_{(\alpha p)^*}$ spaces.*

1. INTRODUCTION

In 1970 Levine [12] first introduced the concept of generalized closed (briefly g-closed) sets in topological spaces. S.P.Arya and T.Nour [2] defined gs-closed sets in 1990. Dontchev [9] introduced gsp-closed sets. Maki *et al.* [15] defined α g-closed sets in 1994. Levine [13], Mashhour *et al.* [14] introduced semi-open sets, pre-open sets respectively. Maki *et al.* [16] introduced gα-closed sets. The purpose of this paper is to introduce the concept of $(\alpha p)^*$ - closed sets, $(\alpha p)^*$ -continuous maps, $(\alpha p)^*$-irresolute and $T_{(\alpha p)^*}$ spaces and investigate some of their properties.

2. PRELIMINARIES

Throughout this paper (X, τ) represents a non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space (X, τ), cl(A) and int(A) denote the closure and interior of the subset A

Definition: 2.1 A subset A of a topological space (X,τ) is called,

i. a pre- open set[14] if A $\subseteq$ int (cl(A)) and pre- closed set if cl (int(A)) $\subseteq$ A.
ii. a semi-open set[13] if A $\subseteq$ cl (int(A)) and a semi -closed set if int (cl(A)) $\subseteq$ A.
iii. an α-open set[17] if A $\subseteq$ int (cl (int(A))) and if an α-closed if cl (int (cl(A))) $\subseteq$ A.
iv. a semi-preopen set[1] if A $\subseteq$ cl (int (cl(A))) and a semi-preclosed set if int (cl (int(A))) $\subseteq$ A.

Definition: 2.2 A subset A of a topological space (X, τ) is called,

i. a generalized closed set [12] (briefly g-closed) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
ii. a generalized semi-preclosed set [9] (briefly gsp-closed) if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
iii. a α-generalized closed set [15] (briefly αg-closed) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
iv. a generalized α-closed set [16] (briefly gα-closed) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in (X, τ).
v. a semi-generalized closed set [4] (briefly sg-closed) if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ).
vi. a semi-pregeneralized closed set [21] (briefly spg-closed) if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ).
vii. a generalized semiclosed set [2] (briefly gs-closed) if $s$cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X , τ).
viii. a α-generalized semiclosed set [19] (briefly αgs-closed) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi open in (X, τ).

Corresponding Author: R. Saranya
M. Phil Scholar, Nirmala College for Women, Coimbatore, India.*

ix. a ψ -closed set [22] if $s$cl(A) ⊆ U whenever A ⊆ U and U is sg-open in (X,τ).
x. a $(sp)^*$-closed set [10] cl(A) ⊆ U whenever A ⊆ U and U is semi-pre open in(X, τ).
xi. a $(gsp)^*$-closed set [18] cl(A) ⊆ U whenever A ⊆ U and U is gsp-open in (X, τ).

Definition: 2.3 A function f: (X, τ) → (Y, σ) is called
i. an α-contiuous [17] if $f^{-1}(V)$ is an α-closed set of (X, τ) for every closed set V of (Y, σ).
ii. a g-continuous [3] if $f^{-1}(V)$ is a g-closed set of (X, τ) for every closed set V of (Y, σ).
iii. a sg-continuous [4] if $f^{-1}(V)$ is a sg-closed set of (X, τ) for every closed set V of (Y, σ).
iv. a gs-continuous [6] if $f^{-1}(V)$ is a gs-closed set of (X, τ) for every closed set V of (Y, σ).
v. an αg-contiuous [11] if $f^{-1}(V)$ is an αg-closed set of (X, τ) for every closed set V of (Y, σ).
vi. a gα-contiuous [16] if $f^{-1}(V)$ is an $g\alpha$-closed set of (X, τ) for every closed set V of (Y, σ).
vii. a gsp-continuous [9] if $f^{-1}(V)$ is a gsp-closed set of (X, τ) for every closed set V of (Y, σ).
viii. a spg-continuous [21] if $f^{-1}(V)$ is a spg-closed set of (X, τ) for every closed set V of (Y, σ).
ix. an αgs-contiuous [20] if $f^{-1}(V)$ is an αgs-closed set of (X, τ) for every closed set V of (Y, σ).
x. a ψ -continuous [22] if $f^{-1}(V)$ is a ψ -closed set of (X, τ) for every closed set V of (Y, σ).
xi. a $(sp)^*$-continuous [10] if $f^{-1}(V)$ is a $(sp)^*$-closed set of (X, τ) for every closed set V of (Y, σ).
xii. $(gsp)^*$-continuous [18] if $f^{-1}(V)$ is a $(gsp)^*$-closed set of (X, τ) for every closed set V of (Y, σ).

Definition: 2.4 A topological space (X, τ) is said to be,
i. a $T_{1/2}$ space [12] if every g-closed set in it is closed.
ii. a T_b space [7] if every gs-closed set in it is closed.
iii. a T_d space [7] if every gs-closed set in it is g-closed.
iv. a ${}_\alpha T_d$ space [8] if every αg-closed set in it is g-closed.
v. a ${}_\alpha T_b$ space [8] if every αg-closed set in it is closed.
vi. a $T_{g\alpha}$ space [5] if every gα-closed set in it is αg-closed.
vii. T_{gsp} space [23] if every gsp-closed set in it is gs-closed.

3. BASIC PROPERTIES OF $(\alpha p)^*$-CLOSED SETS

We introduce the following definition

Definition 3.1: A subset A of a topological space (X, τ) is called a $(\alpha p)^*$-closed set if αcl(A) ⊆ U whenever A ⊆ U and U is semi-preopen in X.

Theorem 3.2: Every closed set is $(\alpha p)^*$- closed set.

Proof follows from the definition.

The following example supports that an $(\alpha p)^*$-closed set need not be true in general.

Example 3.3: Let X = $\{a, b, c\}$ **,** τ = { ϕ ,X,$\{a\}$,$\{a, b\}$}. Let A = $\{b\}$ is $(\alpha p)^*$ -closed but not closed in (X,τ).

Theorem 3.4: Every $(\alpha p)^*$-closed set is gsp-closed set.

Proof: Let A be $(\alpha p)^*$ -closed. Let A ⊆ U and U be open. Then U is semi-preopen. Since A is $(\alpha p)^*$- closed, then spcl(A) ⊆ αcl(A) ⊆ U. Hence A is gsp-closed set.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.5: Let X = $\{a, b, c\}$**,** τ = { ϕ , X, $\{a\}$, $\{b\}$, $\{a, b\}$ }**.** Let A= $\{a\}$ is gsp-closed but not $(\alpha p)^*$ -closed in (X,τ).

Theorem 3.6: Every $(\alpha p)^*$ -closed set is gs-closed set.

Proof follows from the definition.

The reverse implication does not hold as it can be seen from the following example.

Example 3.7: Let X = $\{a, b, c\}$**,** τ = { ϕ ,X,$\{a\}$,$\{b\}$, $\{a, b\}$ }. Let A = $\{a\}$ is gs- closed but not $(\alpha p)^*$-closed in (X,τ).

Theorem 3.8 Every $(\alpha p)^*$-closed set is a spg-closed set.

Proof: Let A be $(\alpha p)^*$ -closed set. Let A $\subseteq$ U and U be open. Then U is semi-preopen. Since A is $(\alpha p)^*$- closed, then spcl(A) $\subseteq \alpha$cl(A) $\subseteq$ U. Hence A is spg-closed set.

The reverse implication does not hold as it can be seen from the following example.

Example 3.9: Let X = $\{a, b, c\}$, τ = { ϕ , X, $\{a\}$, $\{b\}$, $\{a, b\}$ }. Let A= $\{a\}$ is spg-closed but not $(\alpha p)^*$-closed in (X,τ).

Theorem 3.10: Every $(\alpha p)^*$-closed set is sg-closed set.

Proof: Let A be $(\alpha p)^*$ -closed set. Let A $\subseteq$ U and U be open. Then U is semi-preopen. Since A is $(\alpha p)^*$- closed, then $s$cl(A) $\subseteq \alpha$cl(A) $\subseteq$ U. Hence A is sg-closed set.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.11: Let X = $\{a, b, c\}$, τ = { ϕ , X, $\{b, c\}$ }. Let A= $\{a, b\}$ is a sg-closed but not $(\alpha p)^*$-closed in (X,τ).

Theorem 3.12: Every $(\alpha p)^*$-closed set is αg-closed set.

Proof: Let A be $(\alpha p)^*$ -closed set. Let A $\subseteq$ U and U be open. Then U is semi-preopen. Since A is $(\alpha p)^*$- closed, then αcl(A) $\subseteq$ U. Hence A is αg-closed set.

The reverse implication does not hold as it can be seen from the following example.

Example 3.13: Let X = $\{a, b, c\}$, τ = { ϕ , X, $\{b, c\}$ }. Let A= $\{a, b\}$ is αg-closed but not $(\alpha p)^*$-closed in (X,τ).

Theorem 3.14: Every $(\alpha p)^*$ -closed set is $g\alpha$-closed set.

Proof follows from the definition.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.15: Let X = $\{a, b, c\}$, τ = { ϕ , X, $\{b, c\}$ }. LetA= $\{a, b\}$ is $g\alpha$-closed but not $(\alpha p)^*$-closed in (X,τ).

Theorem 3.16: Every $(\alpha p)^*$-closed set is a ψ -closed set.

Proof: Let A be $(\alpha p)^*$ -closed set. Let A $\subseteq$ U and U be open. Then U is semi-preopen. Since A is $(\alpha p)^*$- closed, then $s$cl(A) $\subseteq \alpha$cl(A) $\subseteq$ U. Hence A is ψ $-$ closed set.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.17: Let X = $\{a, b, c\}$, τ = { ϕ , X, $\{b, c\}$ }. Let A= $\{a, b\}$ is ψ -closed but not $(\alpha p)^*$-closed in (X,τ).

Theorem 3.18: Every $(sp)^*$ -closed set is a $(\alpha p)^*$-closed set.

Proof: Let A be $(sp)^*$ -closed set. Let A $\subseteq$ U and U be semi-preopen. Since A is $(sp)^*$ -closed, then $\alpha cl(A) \subseteq Cl(A) \subseteq$ U. Hence A is $(\alpha p)^*$- closed.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.19: Let X = $\{a, b, c\}$, τ = { ϕ ,X,$\{a\}$,$\{a, b\}$ }.Let A = $\{b\}$ is $(\alpha p)^*$ -closed but not $(sp)^*$-closed in (X,τ).

Theorem 3.20: Every $(\alpha p)^*$ -closed set is αgs-closed set.

Proof follows from the definition.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 3.21: Let X = $\{a, b, c\}$, τ = { ϕ , X, $\{b, c\}$ }. Let A= $\{a, b\}$ is αgs -closed but not $(\alpha p)^*$-closed in (X,τ).

Theorem 3.22: Every $(gsp)^*$ -closed set is a $(\alpha p)^*$-closed set.

Proof follows from the definition.

Example 3.23: Let X = $\{a, b, c\}$, τ = { ϕ , X, $\{a\}$ }. Let A= $\{b\}$ is $(\alpha p)^*$ -closed but not $(gsp)^*$-closed in (X,τ).

Theorem 3.24: A is $(\alpha p)^*$-closed set of (X ,τ) if and only if $\alpha cl(A)\backslash A$ does not contain any nonempty semi-pre closed set

Proof: Necessity: Let F be a semi-pre closed set of (X, τ) such that F$\subseteq$ $\alpha cl(A)\backslash A$. Then A$\subseteq$ X\F.A is $(\alpha p)^*$-closed and X\F is semi-preopen,$\alpha cl(A)\subseteq$X\F. Since F$\subseteq$ $X\backslash\alpha cl(A)So, F\subseteq$ $((X/\alpha cl(A))\cap\alpha cl(A)/A)$ = ϕ. Therefore F = ϕ

Sufficiency: Let A be a subset of (X ,τ) such that $\alpha cl(A)\backslash A$ does not contain any non-empty semi-pre closed set.Let U be a semi-pre open set of (X ,τ) such that A$\subseteq$U. If $\alpha cl(A)\not\subset U$, then $\alpha cl(A)\cap U^c\neq\phi$. and $\alpha cl(A)\cap U^c$ is semi-pre closed.$\therefore$ $\phi\neq\alpha cl(A)\cap U^c\subseteq\alpha cl(A)$\A.$\therefore$ $\alpha cl(A)\backslash A$ contains a non-empty semi-pre closed set, which is a contradiction.$\therefore$ $\alpha cl(A)\subseteq$U $\therefore$ A is $(\alpha p)^*$-closed set.

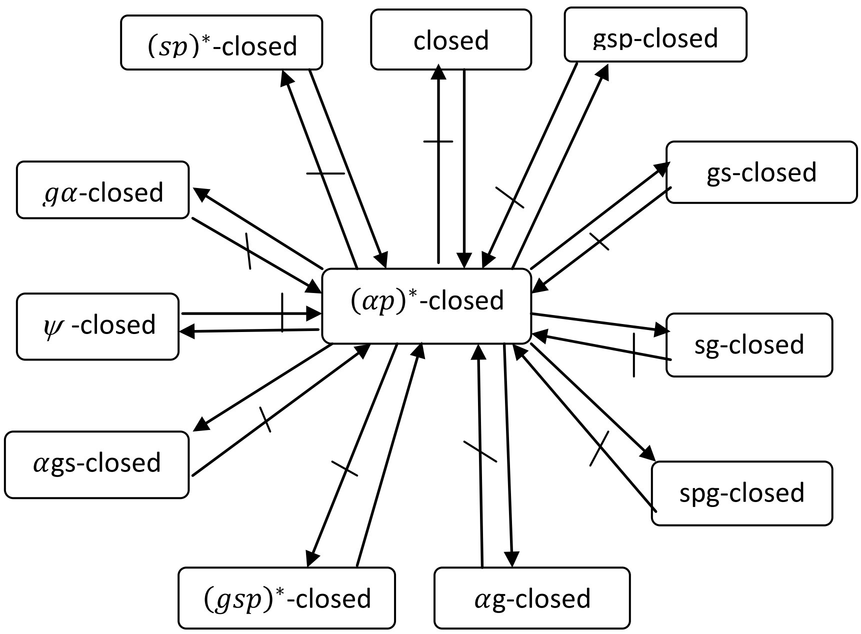
Theorem 3.25: If A is $(\alpha p)^*$-closed set in X and A$\subseteq$B$\subseteq$ $\alpha cl(A)$ then B is also$(\alpha p)^*$-closed set in X.

Proof: It is given that A is $(\alpha p)^*$-closed set in X.To prove B is also $(\alpha p)^*$-closed set of X.Let U be semi-preopen set of X such that B $\subseteq$ U. Since A $\subseteq$ B, we have A $\subseteq$ U. Since A is $(\alpha p)^*$ -closed and $\alpha cl(A)\subseteq$ U. Now $\alpha cl(B)\subseteq\alpha cl(\alpha cl(A)) = \alpha cl(A)\subseteq$U.So that $\alpha cl(B)\subseteq$U. Hence B is $(\alpha p)^*$-closed set in X.

Theorem 3.26: If A is both semi-preopen and $(\alpha p)^*$-closed, then A is α-closed.

Proof: Let A be both semi-preopen and $(\alpha p)^*$-closed. Let A$\subseteq$A, Where A is semi-preopen. Then $\alpha cl(A)\subseteq$ A as A is $(\alpha p)^*$-closed in (X ,τ).But A$\subseteq$ $\alpha cl(A)$ is always true. $\therefore$ A= $\alpha cl(A)$Hence A is α-closed set in (X ,τ)

The above result can be represented as the following diagram



Where A⟶ B represents A implies B and A ⟼ B represents A does not imply B

4. $(\alpha p)^*$-CONTINUOUS AND $(\alpha p)^*$-IRRESOLUTE MAPS

We introduce the following definition

Definition 4.1: A function f: (X, τ) $\rightarrow$ (Y, σ) is called $(\alpha p)^*$-continuous if $f^{-1}(V)$ is a $(\alpha p)^*$-closed set of (X, τ) for every closed set V of (Y, σ).

Theorem 4.2: Every continuous map is $(\alpha p)^*$- continuous.

The following example supports that the converse of the above theorem is not true.

Example 4.3: Let X = $Y = \{a, b, c\}$, τ = { ϕ , X, $\{a\}, \{a, b\}$ } and σ = { ϕ , Y, $\{a, c\}$}. Let f: (X, τ) $\rightarrow$ (Y, σ) be defined by an identity mapping . $f^{-1}\{b\} = \{b\}$ is $(\alpha p)^*$-closed but not closed in (X, τ).

Theorem 4.4: Every $(\alpha p)^*$-continuous map is gsp-continuous.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be $(\alpha p)^*$-continuous map. Let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $(\alpha p)^*$-closed in (X, τ). Since every $(\alpha p)^*$ -closed set is gsp-closed, $f^{-1}(V)$ is gsp-closed in (X, τ). Therefore f is gsp-continuous in (X, τ).

The following example support that the converse of the above theorem need not be true in general.

Example 4.5: Let X = Y = $\{a, b, c\}$, τ = { ϕ , X, $\{a\}$, {b,c}} and σ = { ϕ , Y, $\{b\}$ } Let f : (X , τ) $\rightarrow$ (Y, σ) be defined by an identity mapping. $f^{-1}\{ a , c \} = \{a, c\}$ is gsp-closed but not $(\alpha p)^*$-closed in (X, τ).

Theorem 4.6: Every $(\alpha p)^*$-continuous map is gs-continuous.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be $(\alpha p)^*$-continuous map. Let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $(\alpha p)^*$-closed in (X,τ) Since every $(\alpha p)^*$-closed set is gs-closed,$f^{-1}(V)$ is gs-closed in (X,τ). Therefore f is gs-continuous in (X, τ).

The following example support that the converse of the above theorem need not be true in general.

Example 4.7: Let X = Y = $\{a, b, c\}$, τ = { ϕ , X, $\{a\}, \{b, c\}$} and σ ={ ϕ , Y, $\{b\}$ }. Let f: (X, τ) $\rightarrow$ (Y, σ) be defined by an identity mapping $f^{-1}\{a, c\} = \{a, c\}$ is gs-closed but not $(\alpha p)^*$- closed in (X, τ).

Theorem 4.8: Every $(\alpha p)^*$-continuous map is spg-continuous.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be $(\alpha p)^*$-continuous map. Let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $(\alpha p)^*$-closed in (X, τ) . Since every $(\alpha p)^*$ -closed set is spg-closed, $f^{-1}(V)$ is spg-closed in (X, τ). Therefore f is spg-continuous in (X, τ).

The following example support that the converse of the above theorem need not be true in general.

Example 4.9: Let X = Y = $\{a, b, c\}$, τ = { ϕ , X, $\{a\}, \{b, c\}$ } and σ ={ ϕ , Y, $\{b\}$}. Let f (X , τ) $\rightarrow$ (Y , σ) be defined by an identity mapping $f^{-1}\{a, c\} = \{a, c\}$ is spg-closed but not $(\alpha p)^*$-closed in (X, τ).

Theorem 4.10: Every $(\alpha p)^*$-continuous map is sg-continuous.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be $(\alpha p)^*$-continuous map. Let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $(\alpha p)^*$-closed in (X, τ). Since every $(\alpha p)^*$ -closed set is sg-closed, $f^{-1}(V)$ is sg-closed in (X, τ). Therefore f is sg-continuous in (X, τ).

The following example support that the converse of the above theorem need not be true in general.

Example 4.11: Let X = Y = $\{a, b, c\}$, τ = { ϕ , X, $\{a\}, \{b, c\}$} and σ ={ ϕ , Y, $\{b\}$}. Let f: (X , τ) $\rightarrow$ (Y , σ) be defined by an identity mapping . $f^{-1}\{a, c\} = \{a, c\}$ is sg-closed but not $(\alpha p)^*$-closed in (X, τ).

Theorem 4.12: Every $(\alpha p)^*$-continuous map is αg-continuous.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be $(\alpha p)^*$-continuous map. Let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $(\alpha p)^*$-closed in (X, τ). Since every $(\alpha p)^*$ -closed set is αg -closed, $f^{-1}(V)$ is αg -closed in (X, τ). Therefore f is αg -continuous in (X, τ).

The following example support that the converse of the above theorem need not be true in general.

Example 4.13: Let X = Y = $\{a,b,c\}$, τ = { ϕ, X, $\{a\},\{b,c\}$ } and σ ={ ϕ, Y, $\{b\}$}. Let f: (X, τ) $\rightarrow$ (Y , σ) be defined by an identity mapping .$f^{-1}\{a,c\} = \{a,c\}$ $is\ \alpha g\ -$ closed set but not a $(\alpha p)^*$-closed in (X, τ).

Theorem 4.14: Every $(\alpha p)^*$-continuous map is $g\alpha$ -continuous.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be $(\alpha p)^*$-continuous map. Let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $(\alpha p)^*$-closed in (X, τ). Since every $(\alpha p)^*$ -closed set is $g\alpha$ -closed, $f^{-1}(V)$ is $g\alpha$ -closed in (X, τ). Therefore f is $g\alpha$ -continuous in (X, τ).

The following example support that the converse of the above theorem need not be true in general.

Example 4.15: Let X = Y = $\{a,b,c\}$, τ = { ϕ, X, $\{a\},\{b,c\}$ } and σ ={ ϕ, Y, $\{b\}$}. Let f: (X, τ) $\rightarrow$ (Y , σ) be defined by an identity mapping. $f^{-1}\{a,c\} = \{a,c\}$ $is\ g\alpha\ -$ closed set but not $(\alpha p)^*$- closed in (X, τ).

Theorem 4.16: Every $(\alpha p)^*$-continuous map is ψ -continuous.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be $(\alpha p)^*$-continuous map. Let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $(\alpha p)^*$-closed in (X, τ). Since every $(\alpha p)^*$ -closed set is ψ -closed, $f^{-1}(V)$ is ψ -closed in (X, τ). Therefore f is ψ -continuous in (X, τ).

The following example support that the converse of the above theorem need not be true in general.

Example 4.17: Let X = Y = $\{a,b,c\}$, τ = { ϕ, X, $\{b,c\}$}, σ = { ϕ, Y, $\{c\}$}. Let f: (X, τ) $\rightarrow$ (Y, σ) be defined by an identity mapping .$f^{-1}\{a,b\} = \{a,b\}$ $is\ \psi\ -$ closed set but not $(\alpha p)^*$-closed in (X, τ).

Theorem 4.18: Every $(\alpha p)^*$-continuous map is αgs -continuous.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be $(\alpha p)^*$-continuous map. Let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $(\alpha p)^*$-closed in (X, τ). Since every $(\alpha p)^*$ -closed set is αgs -closed, $f^{-1}(V)$ is αgs -closed in (X, τ). Therefore f is αgs -continuous in (X, τ).

The following example support that the converse of the above theorem need not be true in general

Example 4.19: Let X = Y = $\{a,b,c\}$, τ = { ϕ, X, $\{a\},\{b,c\}$} and σ ={ ϕ, Y, $\{b\}$}. Let f: (X, τ) $\rightarrow$ (Y , σ) be defined by an identity $f^{-1}\{a,c\} = \{a,c\}$ is $\alpha gs - closed$set but not $(\alpha p)^*$- closed in (X, τ).

Theorem 4.20: Every $(sp)^*$ -continuous map is $(\alpha p)^*$ -continuous.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be $(sp)^*$-continuous map. Let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $(sp)^*$-closed in (X, τ). Since every $(sp)^*$ -closed set is $(\alpha p)^*$ -closed, $f^{-1}(V)$ is $(\alpha p)^*$ -closed in (X, τ). Therefore f is $(\alpha p)^*$ -continuous in (X, τ).

Example 4.21: Let X = $Y = \{a,b,c\}$,τ ={ ϕ, X, $\{a\},\{a,b\}$ } and σ ={ ϕ, Y, $\{a,c\}$ }. Let f : (X , τ) $\rightarrow$ (Y , σ) be defined by an identity mapping .$f^{-1}\{b\} = \{b\}$ $is\ (\alpha p)^*\ -$ closed set but not $(sp)^*$- closed in (X, τ).

Theorem 4.22: Every $(gsp)^*$ -continuous is $(\alpha p)^*$ -continuous.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be $(gsp)^*$-continuous map. Let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $(gsp)^*$-closed in (X, τ). Since every $(gsp)^*$ -closed set is (αp) -closed, $f^{-1}(V)$ is (αp) -closed in (X, τ). Therefore f is (αp) -continuous in (X, τ).

Example 4.23: Let X = $Y = \{a,b,c\}$,τ ={ ϕ , X, $\{a\}$ } and σ ={ ϕ , Y, $\{a,c\}$}. Let f: (X , τ) $\rightarrow$ (Y , σ) be defined by an identity mapping .$f^{-1}\{b\} = \{b\}$ is $(\alpha p)^*$ $-$ closed set but not $(gsp)^*$-closed in (X, τ).

Definition 4.24: A function f: (X, τ) $\rightarrow$ (Y,σ) is called $(\alpha p)^*$-irresolute if $f^{-1}(V)$ is a $(\alpha p)^*$-closed set of (X, τ) for every $(\alpha p)^*$-closed set V of (Y, σ)

Theorem 4.25: Every $(\alpha p)^*$ -irresolute is $(\alpha p)^*$ -continuous

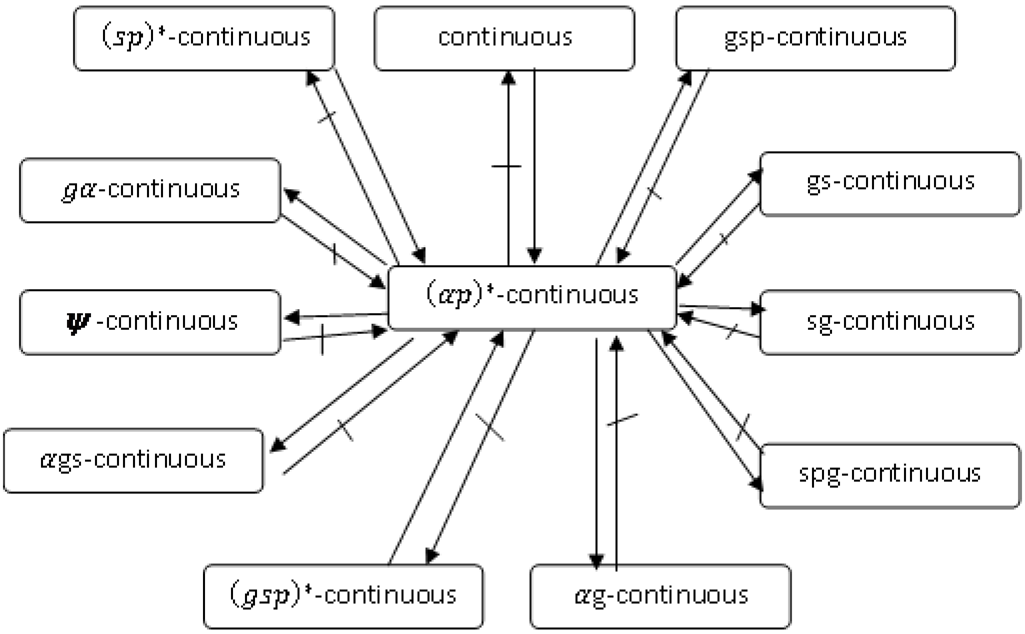
Proof: Let f: (X , τ) $\rightarrow$ (Y , σ) be an $(\alpha p)^*$ - irresolute. Let V be a closed set in (Y, σ) Every closed set is $(\alpha p)^*$-closed. Therefore V is $(\alpha p)^*$-closed.Then $f^{-1}(V)$ is $(\alpha p)^*$ $-$closed since f is $(\alpha p)^*$ $-$irresolute and hence f is $(\alpha p)^*$ -continuous. The converse of the above theorem is not true in general as it can be seen from the following example

Example 4.26: Let X = $Y = \{a,b,c\}$, τ = { ϕ , X, $\{a\}$, $\{b,c\}$} and σ = { ϕ , Y, $\{c\}$} Define f: (X , τ) $\rightarrow$ (Y, σ) by f(a) = c, f(b) = a and f(c) = b. $f^{-1}\{a,b\} = \{b,c\}$ is $(\alpha p)^*$ closed set in (X , τ) .Therefore f is $(\alpha p)^*$-continuous. $\{b,c\}$ is $(\alpha p)^*$ closed set in (Y, σ) . $f^{-1}\{a\} = \{b\}$, $f^{-1}\{b\} = \{c\}$ is not $(\alpha p)^*$ closed set in (X, τ). Therefore f is not $(\alpha p)^*$- irresolute.

Theorem 4.27: Let f: (X, τ) $\rightarrow$ (Y,σ) and g : (Y,σ) $\rightarrow$ (Z,η) be any two functions then,
(i).g∘f: (X, τ) $\rightarrow$ (Z,η) is $(\alpha p)^*$-continuous if f is $(\alpha p)^*$-irresolute and g is $(\alpha p)^*$-continuous.
(ii).g∘f : (X, τ) $\rightarrow$ (Z,η) is $(\alpha p)^*$- irresolute if f and g are $(\alpha p)^*$- irresolute.
(iii).g∘f: (X, τ) $\rightarrow$ (Z,η) is $(\alpha p)^*$-continuous if f is $(\alpha p)^*$-continuous and g is continuous.

Proof: (i).Let F be closed set in (Z,η).Then g^{-1}(F) is $(\alpha p)^*$-closed in (Y,σ) implies $f^{-1}(g^{-1}(F))$ is $(\alpha p)^*$-closed in (X, τ) . Therefore $(f\circ g)^{-1}(F)$ is $(\alpha p)^*$-closed in (X, τ). Hence (f∘g) is $(\alpha p)^*$-continuous. (ii).Let F be $(\alpha p)^*$-closed in (Z,η). Then g^{-1}(F) is $(\alpha p)^*$-closed in (Y,σ).Therefore $f^{-1}(g^{-1}(F))$ is $(\alpha p)^*$-closed in (X, τ). Therefore $(f\circ g)^{-1}(F)$ is $(\alpha p)^*$-closed in (X, τ).Hence (f∘g) is $(\alpha p)^*$-irresolute. (iii).Let F be closed in (Z,η). Then g^{-1}(F) is $(\alpha p)^*$-closed in (Y,σ) . Therefore $f^{-1}(g^{-1}(F))$ is $(\alpha p)^*$-closed in (X, τ). Therefore $(f\circ g)^{-1}(F)$ is $(\alpha p)^*$-closed in (X,τ).Hence (f∘g) is $(\alpha p)^*$-continuous.

Thus we have the following diagram.



Where A⟶B represents A implies B and A⟶̸B represents A does not imply B

5. APPLICATIONS OF $(\alpha p)^*$-CLOSED SETS

As applications of $(\alpha p)^*$- closed set, a new space $T_{(\alpha p)^*}$ is introduced.

Definition 5.1: A space (X,τ) is called a $T_{(\alpha p)^*}$ space if every $(\alpha p)^*$- closed set is closed.

Theorem 5.2: Every T_b space is a $T_{(\alpha p)^*}$ space.

Proof follows from the definition of T_b space and $T_{(\alpha p)^*}$ space.

The reverse implication does not hold as it can be seen from the following example.

Example 5.3: Let X = $\{a,b,c\}$,τ ={ ϕ ,X,$\{a\}$,$\{b\}$, $\{a,b\}$ }, then (X ,τ) is a $T_{(\alpha p)^*}$ space. A={a} is gs-closed set but not a closed set. Therefore (X ,τ) is a T_b space.

Theorem 5.4: Every ${}_{\alpha}T_b$ space is a $T_{(\alpha p)^*}$ space.

Proof follows from the definition of ${}_{\alpha}T_b$ space and $T_{(\alpha p)^*}$ space.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 5.5: Let X = $\{a,b,c\}$,τ ={ ϕ ,X,$\{b,c\}$}. $(\alpha p)^*$ -closed sets are ϕ ,X,$\{a\}$ and αg-closed sets are ϕ ,X,$\{a\}$, {a,b}, {a,c}. Since every $(\alpha p)^*$ -closed set is αg-closed, the space (X,τ) is a $T_{(\alpha p)^*}$ space. A = {a, b} is αg-closed but not closed. Therefore the space is not a ${}_{\alpha}T_b$ space.

Theorem 5.6: Every $T_{(\alpha p)^*}$ space is a $T_{(gsp)^*}$ space.

Proof follows from the definition of $T_{(\alpha p)^*}$ space and $T_{(gsp)^*}$ space.

Example 5.7: Let X = $\{a,b,c\}$,τ ={ ϕ ,X,$\{a\}$}. $(gsp)^*$ -closed sets are ϕ , X, $\{b,c\}$ and $(\alpha p)^*$ -closed sets are ϕ , X,$\{b\}$, {c}, {b,c}.Since every $(gsp)^*$-closed set is $(\alpha p)^*$ -closed, the space (X,τ) is a $T_{(gsp)^*}$ space. A={b} is $(\alpha p)^*$ -closed but not closed .Therefore the space is not a $T_{(\alpha p)^*}$ space.

Theorem 5.8: A space (X , τ) which is both ${}_{\alpha}T_b$ and $T_{g\alpha}$ is a $T_{(\alpha p)^*}$ space.

Theorem 5.9: A space (X , τ) which is both T_b and T_{gsp} is a $T_{(\alpha p)^*}$ space.

Theorem 5.10: A space (X , τ) which is both $T_{1/2}$ and ${}_{\alpha}T_d$ is a $T_{(\alpha p)^*}$ space.

Theorem 5.11: A space (X , τ) which is both $T_{1/2}$ and T_d is a $T_{(\alpha p)^*}$ space.

Theorem 5.12: Let (X , τ) be a $T_{(\alpha p)^*}$ space and f : (X, τ) $\rightarrow$ (Y, σ) be *an* $(\alpha p)^*$ - irresolute.then f is continuous.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be an$(\alpha p)^*$ - irresolute. Let V be a closed set of (Y, σ). Every closed set is $(\alpha p)^*$-closed set. Then $f^{-1}(V)$ is an $(\alpha p)^*$ $-$closed since f is $(\alpha p)^*$-irresolute. Every $(\alpha p)^*$-closed set is closed in X. Since (X, τ) is a $T_{(\alpha p)^*}$ space. Therefore $f^{-1}(V)$ is closed and hence f is continuous.

Theorem 5.13: Let (X , τ) be a $T_{(\alpha p)^*}$ space and f: (X, τ) $\rightarrow$ (Y, σ) be continuous. Then f is $(\alpha p)^*$ - irresolute.

Proof: Let A be an $(\alpha p)^*$- closed set in Y. Then A is closed, Since (Y , σ) is a $T_{(\alpha p)^*}$ space. Then $f^{-1}(A)$ is closed, since f is continuous. Every closed set is $(\alpha p)^*$- closed set . Therefore f is an $(\alpha p)^*$ - irresolute.

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Source of support: Nil, Conflict of interest: None Declared.