International Journal of Mathematical Archive-8(3), 2017, 13-20
Available online through www.ijma.info ISSN 2229 – 5046

VARIATION OF FLOW RATE IN CASE OF MHD FLOW OVER A MOVING INFINITE VERTICAL POROUS PLATE IN THE PRESENCE OF THERMAL RADIATION AND UNIFORM HEAT FLUX

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(Received On: 08-02-17; Revised & Accepted On: 10-03-17)

ABSTRACT

The case of variation of flow rate with respect to various flow entities in situation of MHD flow over a moving infinite vertical porous plate in the presence of thermal radiation has been examined in detail in this paper. It is observed that, as the pore size decreases the flow rate is found to be decreasing. In each of these illustrations, it is seen that the applied transverse magnetic field influences the flow rate significantly. Further, as the magnetic intensity increases the flow rate is found to be inversely proportional. Also, increase in the intensity of the magnetic field contributes to the decrease in flow rate. For a constant pore size of the bounding surface and the frequency of excitation, increase in the magnetic intensity amounts to the decrease in the flow rate. Also, a back flow is observed in certain cases and is illustrated graphically. Further, decrease in Grashoff number (Gr) contributes to the increase in the flow rate.

Keywords*: MHD flow, Heat and Mass transfer, Heat flux, impulsively started vertical plate, Radiation.*

NOMENCLATURE

C_P	:	Specific heat at constant pressure
g	:	Acceleration due to gravity
G_r	:	Thermal Grashoff number
κ	:	Thermal conductivity of the fluid
P_r	:	Prandtl number
p	:	Pressure
q_r	:	Radiative heat flux in the y-direction
k	:	Thermal diffusivity
K	:	Porosity
M	:	Magnetic field
N	:	Radiation parameter
t	:	Time
T	:	Temperature of the fluid near the plate
T_W	:	Temperature of the plate
T_∞	:	Temperature of the fluid far away from the plate
u	:	Velocity of the fluid in the x-direction
u_0	:	Velocity of the fluid plate
U	:	Dimensionless velocity
y	:	Coordinate axis normal to the plate
y'	:	Dimensionless coordinate axis normal to the plate
k^*	:	Mean absorption coefficient
α	:	Thermal diffusivity
β	:	Volumetric coefficient of thermal expansion

μ	:	Coefficient of viscosity
ν	:	Kinematic viscosity
ρ	:	Density
σ	:	Stefan-Boltzmann constant
τ	:	Dimensionless skin-friction
θ	:	Dimensionless temperature

INTRODUCTION

Generally in industrial situations like nuclear power plants, gas turbines and propulsion devises for air craft, missiles and space vehicles are some situations where in radiative convective heat transfer occurs more often. The applications wide in general and are found mostly in cooling chambers, fossil fuel combustion, energy processes, astrophysical flows, solar power technology and space vehicle re-entry. Mostly, radiative heat transfer is found to have an important role in manufacturing sectors for the design of highly precision equipment.

Stokes[1] examined the problem of viscous incompressible fluid past an impulsively started infinite horizontal moving plate in its own plane. Thereafter Brinkman[2] analyzed the viscous force imparted by flowing fluid. Subsequently, Stewartson[3] examined the problem of viscous flow past and impulsively started semi infinite horizontal plate for an analytical solution. The case of two dimensional steady state flow of an incompressible fluid with rigid parallel porous walls was investigated by Berman[4]. Thereafter, Mori[5] examined the flow between two vertical plates which are electrically non - conducting while the wall temperature varies linearly in the direction of the flow. Later, the flow in renal tubules was analyzed by Macy[6]. Subsequently, Hall[7] studied similar problems using finite differences method while Chang[8] *et al* analyzed the effects of radioactive heat transfer on free convection regimes in specialized applications that occur in geophysics and geothermal reservoirs. Later, Mahajan[9] *et al* examined the influence of viscous heat dissipating effect in natural convective flows. It was established that the heat transfer rates are reduced by an increase in the dissipation parameter. Subsequently, Soundalgekar and Thaker[10] examined thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Later, Das[11] *et al* examined higher order numerical approximation for mass transfer effects on the study flow past an accelerated vertical porous plate. Using Rossland's approximation Hossain[12] *et al* studied the radiation effects on a mixed convective flow along a vertical plate with a uniform surface temperature. Raptis and Perdikis[13] studied the effect of thermal radiation and convective flow past a moving infinite vertical plate. The effects of thermal radiation on the flow past a semi infinite vertical isothermal plate with uniform heat flux was studied exhaustively by Chandrakala and Antony Raj[14]. Recently, Jhansi Rani and Ramana Murthy[15] examined the MHD flow over a moving infinite vertical porous plate in the presence of thermal radiation and studied in detail the factors influencing the velocity and skin friction.

In all above papers the nature of velocity with reference to all critical parameters that appears in the field equations were just analysed. The most important factor i.e flow rate has been totally ignored and was not addressed by any of the investigators. Also, not much of prime importance was given to the bounding surface and factors influencing the flow rate. The flow rate is found to have an important application in several situations where efficient transfer of heat and mass transfer take place. Hence, such an important concept cannot be ignored while studying the situation of thermal radiation. Therefore, in this paper the influence of various critical parameters which appear in the field equations and there effects on the flow rate is examined. Such an analysis provides the knowledge of the transfer of fluid from one reaction chamber to another chamber and the amount of fluid that adheres to the walls of the container.

MATHEMATICAL FORMULATION

The geometry of the bounding surface and fluid motion is as given below. As usual the x- axis is taken along the plate in the vertical direction and y – axis is taken normal to the plate. The fluid is assumed to be incompressible, laminar, viscous radiating fluid past an impulsively started infinite vertical plate

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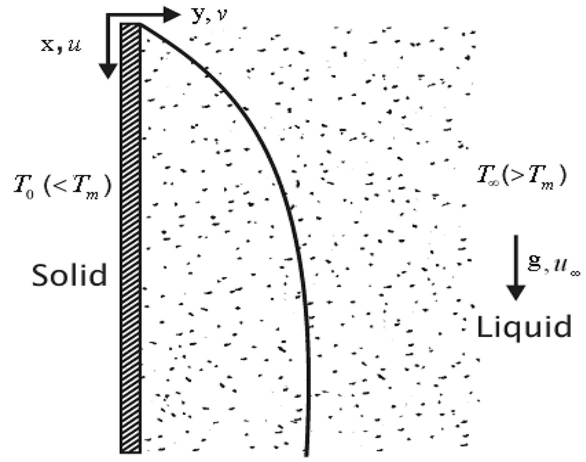

Figure – 1: Schematic representation of the problem

When $t \le 0$ the plate and fluid are maintained at same temperature and in stationary condition. When $t > 0$ the plate is given an impulsive motion in the vertical direction and in opposite to the gravitational field with constant velocity u_0. During such a time the plate is heated through which the fluid temperature also rises at uniform rate. The fluid is considered to be absorbing, emitting radiation. Using Boussinesq's approximation we have

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

In view of Rossland approximation q_r is given by:

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \tag{3}$$

While, the initial and boundary conditions are:

$$\left.\begin{aligned} &When\ t' \le 0 : u = 0, T = T_\infty \text{ for all } y' \\ &and\ if\ t' > 0 : u = u_0, \frac{\partial T}{\partial y} = -\frac{q}{k} \text{ at } y' = 0 \\ &Also\ u = 0, T \to T_\infty \text{ as } y' \to \infty \end{aligned}\right\} \tag{4}$$

Assuming that the temperature differences within the flow are negligible than T^4 can be expressed as a linear function of the temperature. By neglecting higher order terms T^4 in Taylor's series about T_∞ can be expressed as

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{5}$$

By using equations (4) and (5), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{6}$$

So as to make the Governing equations, initial and boundary conditions we introduce the following non – dimensional terms as

$$U = \frac{u}{u_0}, t = \frac{t' u_0^2}{\nu}, y = \frac{y' u_0}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr = \frac{g\beta\nu(T_w - T\infty)}{u_0^3}, \Pr = \frac{\mu C_p}{k}, N = \frac{\kappa^* k}{4\sigma T_\infty^3} \tag{7}$$

By considering the magnetic intensity as M and the permeability of the boundary as K Eqs. (1) to (6), can be re defined as:

$$\frac{\partial U}{\partial t} = Gr\theta + \frac{\partial^2 U}{\partial y^2} + (M + \frac{1}{K})u \tag{8}$$

$$3N\,\mathrm{Pr}\frac{\partial \theta}{\partial t} = (3N + \mathrm{Pr})\frac{\partial^2 \theta}{\partial y^2} \tag{9}$$

The initial and boundary conditions in non-dimensionless form are

$$\left.\begin{aligned} &u = 0, \theta = 0, \text{ For all } y, t \le 0 \\ &t > 0 : u = 1, \frac{\partial \theta}{\partial y} = -1 \text{ at } y = 0 \\ &u = 0, \theta \to 0, \text{ As } y \to \infty \end{aligned}\right\} \tag{10}$$

METHODOLOGY FOR SOLUTION

Assuming the solutions for Eqn (8) and Eqn (9) as:

$$u(x,t) = u_0(y)e^{i\omega t} \tag{11}$$

$$\theta(y,t) = \theta_0(y)e^{i\omega t} \tag{12}$$

The modified initial and boundary conditions will be as follows:

$$\left.\begin{aligned} &u_0 = 0, \theta_0 = 0, \text{ for all } y, t \le 0 \\ &t > 0 : u_0 = e^{-i\omega t}, \frac{d\theta_0}{dy} = e^{-i\omega t} \text{ at } y = 0 \\ &u_0 = 0, \theta_0 \to 0 \text{ as } y \to \infty \end{aligned}\right\} \tag{13}$$

Using Eqns (11), (12) and (13) in Eqns (8) and (9)

$$u(y,t) = \frac{Gr}{R1}(\exp(-m_2 y) - \exp(-m_1 y)) + \exp(-m_2 y) \tag{14}$$

Flow rate $Q = \int_0^h u(y,t)\, dy$ (15)

From equation (15)

$$Q = \frac{Gr}{R1}\left[\frac{\exp(-m_1 h) - 1}{m_1} + \frac{1 - \exp(-m_2 h)}{m_2}\right] + \left[\frac{1 - \exp(-m_2 h)}{m_2}\right]$$

Where

$$m_1 = \sqrt{\frac{3N\,\mathrm{Pr}\, i\omega}{3N + \mathrm{Pr}}}, m_2 = \sqrt{(i\omega - M - \frac{1}{K})},$$

$$R_1 = m_1(m_1^2 - (i\omega - M - \frac{1}{K}))$$

RESULTS AND DISCUSSIONS

1. The influence of magnetic field with respect to Grashoff number (Gr) for different Pore sizes of the fluid bed are illustrated in Figure 2, Figure 3. In general it is observed as the pore size decrease the flow rate is found to be decreasing. As the magnetic intensity increases the flow rate is found to be decreasing. This can be attributed to the fact that the magnetic field suppresses the velocity field hence the flow rate is found to be decreasing. Further, it is observed that for a constant value of magnetic and porosity parameters, increase in Grashoff number (Gr) contributes to the decrease in the flow rate.

2. Figure 4 and Figure 5 shows the influence of applied magnetic field with respect to the porosity and the Prandtl number (Pr). In general it is observed that, as the porosity decreases the flow rate is also found to be decreasing. But such a decrease is observed to be very minimal. In each of these illustrations, the applied transverse magnetic field influences the flow rate quite significantly. Increase in the intensity of the magnetic field decreases the flow rate. For a constant porosity and applied magnetic intensity change in the Prandtl number (Pr) does not influence the flow rate. In all these situations increase in Prandtl number (Pr) contributes to the constant flow rate.
3. When the pore size of the fluid bed is held constant, the effect of the magnetic field with respect to the frequency of excitation has been shown in Figure 6 and Figure 7. In general, it is seen that as the pore size decreases the flow rate is found to be decreasing. It is seen that the magnetic intensity influences the flow rate quite significantly. Also, it is seen that the back flow is observed in the cases exhibited. In a situation where the porosity decreases significantly the flow rate is found to be positive. However, increase in the magnetic intensity does not qualitatively alter in nature of profiles for the flow rate.
4. The effect of Grashoff number (Gr) with respect to the Prandtl number (Pr) on the flow rate has been illustrated in Figure 8 and Figure 9. It is observed that as the Grashoff number (Gr) decreases the flow rate is observed to be increasing. However, a situation of slight back flow is noticed when the Grashoff number (Gr) is 1.0. The magnetic field suppresses the flow rate significantly. Increase in the Prandtl number (Pr) marginally increases the flow rate.
5. The effect of the Prandtl number (Pr) and the magnetic intensity with respect to the Grashoff number (Gr) has been illustrated in Figure 10. It is noticed that as the magnetic intensity increases the flow rate is inversely proporsional. Further, increase in the Grashoff number (Gr) contributes to the decrease in the flow rate and such a change in the flow rate is observed to be almost negligible.

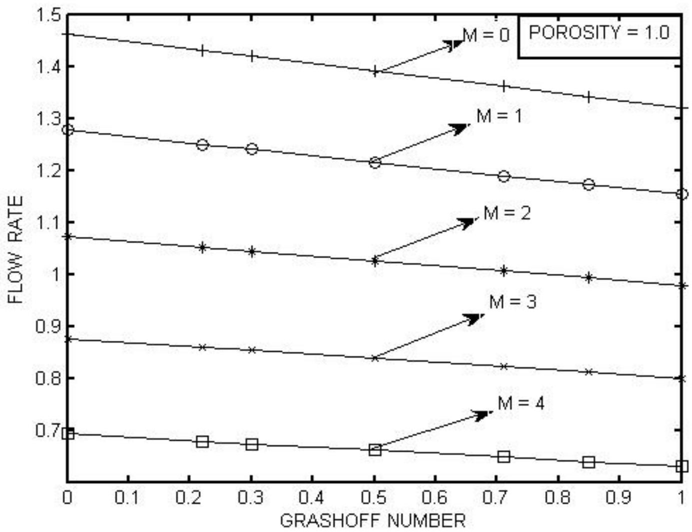

Figure 2: Influence of Magnetic Field (M) on flow rate

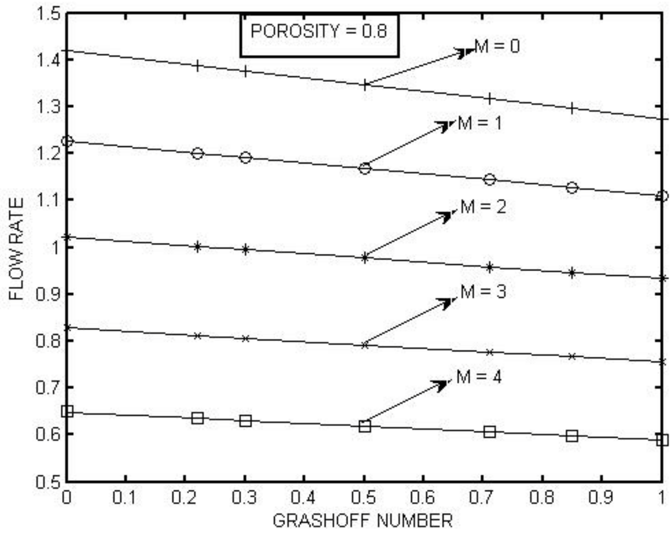

Figure 3: Effect of magnetic field (M) on flow rate

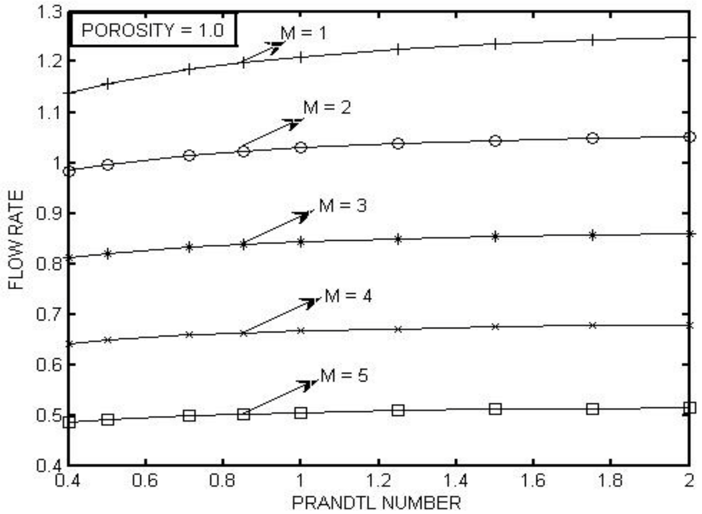

Figure 4: Influence of magnetic field (M) on flow rate

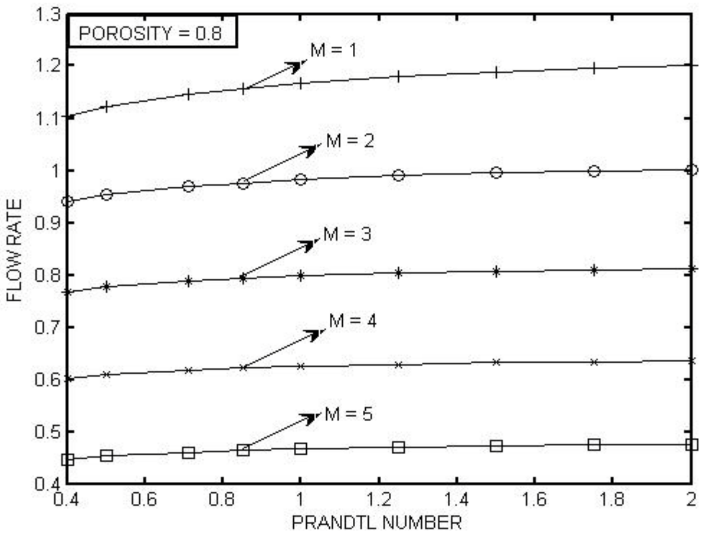

Figure 5: Effect of both magnetic field (M) and prandtl number (Pr) on flow rate

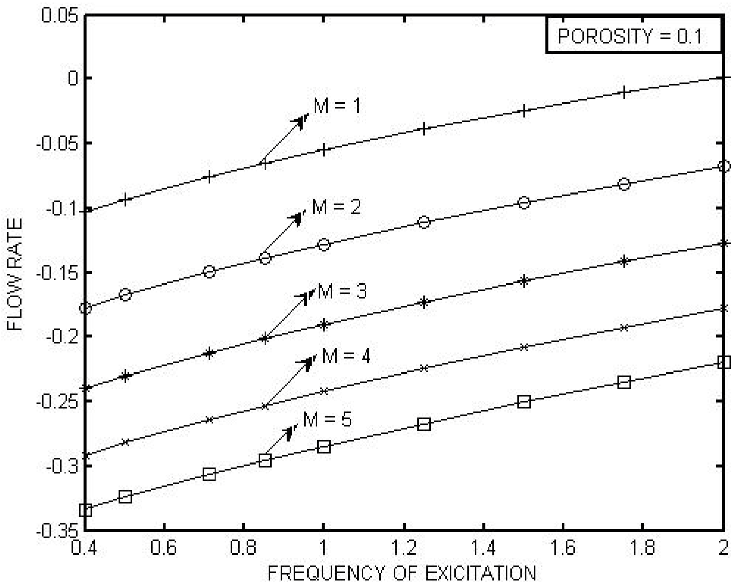

Figure – 6: Effect of frequency excitation on flow rate

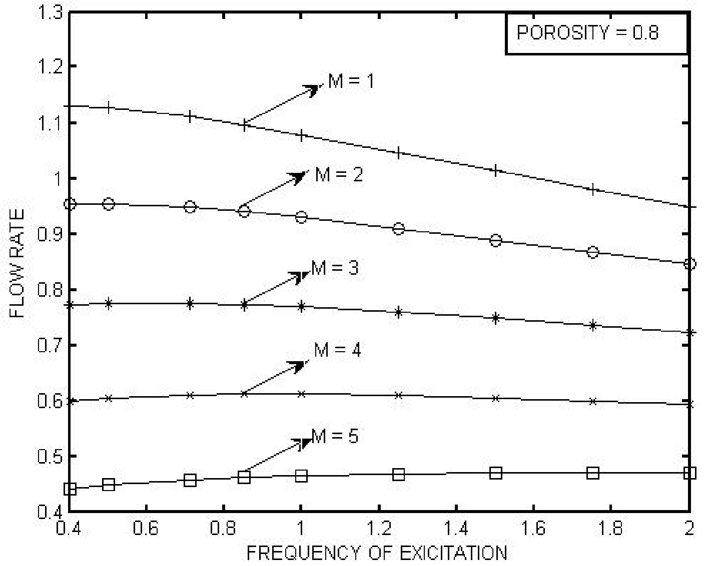

Figure – 7: Combined effect of frequency of excitation and magnetic field on flow rate

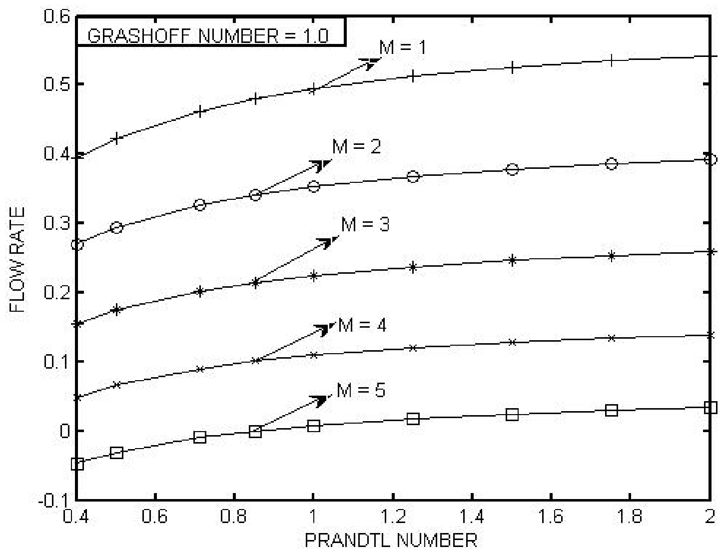

Figure – 8: Influence of Grashoff number (Gr) on flow rate

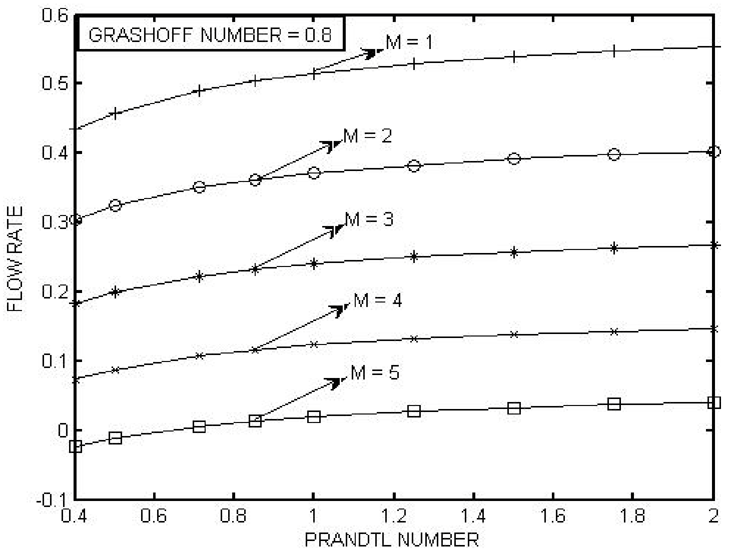

Figure – 9: Consolidated effect of Prandtl number and Grashoff number on flow rate

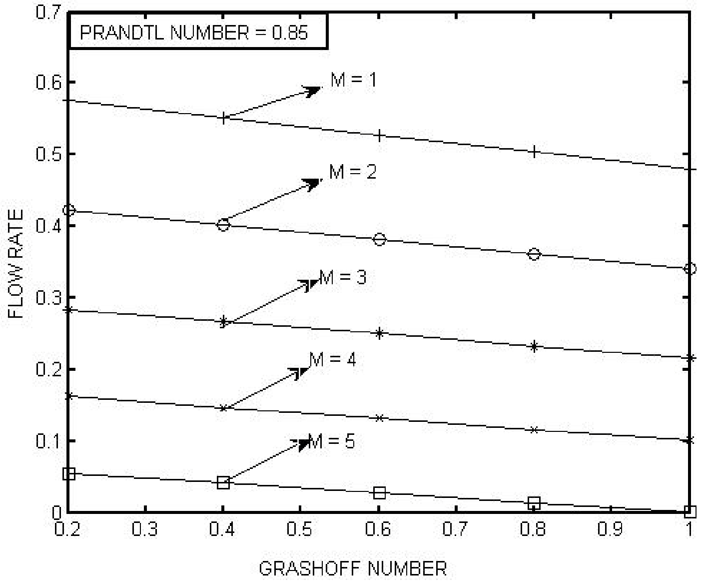

Figure – 10: Effect of Prandtl number (Pr) on flow rate

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Source of support: Nil, Conflict of interest: None Declared.