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CERTAIN UNIFIED INTEGRALS INVOLVING I- FUNCTION AND M-SERIES WITH GENERALIZED MULTIVARIABLE POLYNOMIALS

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ABSTRACT

In the present paper, we consider new unified integrals associated with the I- function and M-series with generalized polynomial. Some interesting special cases of main results are considered in form of many corollaries. The obtained results of this paper provide an extension of the results given by the literature.

Key Words: *H-function, I- function, M-series, Generalized Mittag-Leffler function, and Generalized polynomials.*

MSC: *33C45, 33C60, 33C70, 33E12.*

1. INTRODUCTION

The I-function, which is more general the fox's H-function [3], introduced by V.P. Saxena [8], by means of the following Mellin-Barnes type contour integral

$$I[z] = I^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n};\ (a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};\ (b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]$$

$$= \frac{1}{2\pi i}\int_L \theta(s) z^s ds \tag{1}$$

Where $\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s).\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s).\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s)\right]}$ (2)

$p_i, q_i\ (i = 1,2,\dots,r), m, n$ are integers satisfying $0 \le n \le p_i,\ \ 0 \le m \le q_i, \propto_i, \beta_j,\ \propto_{ij}, \beta_{ji}$ are real and positive and a_j, b_j, a_{ij}, b_{ji} are complex numbers, L is suitable contour of the Mellin-Barnes type running from $\gamma - i\alpha$ to $\gamma + i\alpha$ (γ is real) in the complex s-plane, Detail regarding existence conditions and various parametric restriction of I-function, we may refer [8].

The generalized polynomials introduced by Srivastava [10] is as follows:

$$S^{m_1,\dots,m_t}_{n_1,\dots,n_t}[z_1,\dots,z_t] = \sum_{\alpha_1=0}^{[n_1/m_1]},\dots,\sum_{\alpha_t=0}^{[n_t/m_t]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!},\dots,\frac{(-n_t)_{m_t\alpha_t}}{\alpha_t!}.A[n_1,\alpha_1;\dots;n_t,\alpha_t]z_1^{\alpha_1},\dots,z_t^{\alpha_t} \tag{3}$$

Where $n_i = 0,1,2,\dots\ \forall\ i = (1,\dots,t), m_1,\dots,m_t$ arbitrary positive integers and the coefficients $A[n_1,\alpha_1;\dots;n_t,\alpha_t]$ are arbitrary constants, real or complex.

The generalized M-series is defined by Sharma and Jain [9]

$${}^{\alpha}_{p}M^{\beta}_{q}(a_1,a_2,\dots,a_p;b_1,b_2,\dots,b_q;z) = {}^{\alpha}_{p}M^{\beta}_{q}\left[(a_j)_1^p;(b_j)_1^q;z\right]$$

$$= \sum_{n=0}^{\infty}\frac{(a_1)_n,\dots,(a_p)_n}{(b_1)_n,\dots,(b_q)_n}.\frac{z^n}{\Gamma(\alpha n+\beta)}\ ,\ z,\alpha,\beta\epsilon\mathbb{C}, R(\alpha) > 0 \tag{4}$$

Where $(a_j)_n, (b_j)_n$ are the know pochammer symbols. The series (1) is defined when none of the parameters $b_j's, j = 1,2,\dots,q$ is a negative integer or zero; if any numerator parameter a_j is a negative integer or zero, then the series terminates to a polynomial in z.

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The generalized M-series (4) can be represented as a Fox H-function [3]

$$ {}_{p}^{\alpha}M_{q}^{\beta}\left[(a_j)_1^p;(b_j)_1^q;z\right]=\tau\, H_{p+1,q+2}^{1,p+1}\left[-z\left|\begin{matrix}(1-a_j,1)_{1,p};(0,1)\\(0,1);(1-b_j,1)_{1,q};(1-\beta,\alpha)\end{matrix}\right.\right] \quad \ldots \quad (5) $$

Where $\tau=\frac{\Pi_{j=1}^{q}\Gamma(b_j)}{\Pi_{j=1}^{p}\Gamma(a_j)}$

The three-parameter Mittag-Leffler function, also called generalized Mittag-Leffler function was introduced by Prabhakar [7] by means of the following series representation

$$ E_{\alpha,\beta}^{\gamma}(z)=\sum_{m=0}^{\infty}\frac{(\gamma)_m}{\Gamma(\alpha m+\beta)}\frac{z^m}{m!} \quad (6) $$

An interesting integral formula given by F. Oberhettinger [6],

$$ \int_0^{\infty}x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda}dx=2\lambda a^{-\lambda}\left(\frac{a}{2}\right)^{\mu}\frac{\Gamma(2\mu)\Gamma(\lambda-\mu)}{\Gamma(1+\lambda+\mu)} \quad (7) $$

2. UNIFIED INTEGRAL INVOLVING I-FUNCTION WITH THE GENERALIZED POLYNOMIALS

We derive the following theorem

Theorem 1: $\int_0^{\infty}x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda} I_{p_i,q_i;r}^{m,n}\left[\frac{y^k}{w^k\left(x+a+\sqrt{x^2+2ax}\right)^k}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]$

$$ S_{n_1,\ldots,n_t}^{m_1,\ldots,m_t}\left[z_1\left(x+a+\sqrt{x^2+2ax}\right)^{-\rho_1},\ldots,z_t\left(x+a+\sqrt{x^2+2ax}\right)^{-\rho_t}\right]dx $$

$$ =2^{1-\mu}a^{\mu-\lambda-\sum_{i=1}^{t}\rho_i\alpha_i}\Gamma(2\mu)\sum_{\alpha_1=0}^{[n_1/m_1]},\ldots,\sum_{\alpha_t=0}^{[n_t/m_t]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!},\ldots,\frac{(-n_t)_{m_t\alpha_t}}{\alpha_t!}.A[n_1,\alpha_1;\ldots;n_t,\alpha_t]z_1^{\alpha_1},\ldots,z_t^{\alpha_t}. $$

$$ I_{p_i+2,q_i+2;r}^{m,n+2}\left[\left(\frac{y}{wa}\right)^k\left|\begin{matrix}(-\lambda-\sum_{i=1}^{t}\rho_i\alpha_i,k);(1-\lambda+\mu-\sum_{i=1}^{t}\rho_i\alpha_i,k);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(1-\lambda-\sum_{i=1}^{t}\rho_i\alpha_i,k);(-\lambda-\mu-\sum_{i=1}^{t}\rho_i\alpha_i,k)\end{matrix}\right.\right] \quad (8) $$

p_i,q_i $(i=1,2,\ldots,r),m,n$ are integers satisfying $0\le n\le p_i, 0\le m\le q_i,\propto_i,\beta_j$, $\propto_{ij},\beta_{ji}$ are real and positive and a_j,b_j,a_{ij},b_{ji} are complex numbers, L is suitable contour of the Mellin-Barnes type running from $\gamma-i\alpha$ to $\gamma+i\alpha$ (γ is real) in the complex s-plane.

(1) $n_i=0,1,2,\ldots\ \forall\ i=(1,\ldots,t),m_1,\ldots,m_t$ arbitrary positive integers and the coefficient $A[n_1,\alpha_1;\ldots;n_t,\alpha_t]$ are arbitrary constants, real or complex.

(2) $k>0,\ Re(\mu,\lambda,\rho)>0$.

(3) $Re(\mu)-Re(\lambda)-k\ min_{1\le j\le m}Re\left[\frac{b_j}{\beta_j}\right]<0$.

Proof: Using the I-function in terms of Mellin-Barnes contour integral given by (1) and the definition of a generalized Polynomials given by (3), then interchanging the order of summation and integration, we obtain L.H.S. of (8) as

$$ =\frac{1}{2\pi i}\sum_{\alpha_1=0}^{\left[\frac{n_1}{m_1}\right]},\ldots,\sum_{\alpha_t=0}^{\left[\frac{n_t}{m_t}\right]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!},\ldots,\frac{(-n_t)_{m_t\alpha_t}}{\alpha_t!}.A[n_1,\alpha_1;\ldots;n_t,\alpha_t].z_1^{\alpha_1},\ldots,z_t^{\alpha_t}. $$

$$ \int_L\left(\frac{y}{w}\right)^{ks}\theta(s)\left[\int_0^{\infty}x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-(\lambda+ks+\sum_{i=1}^{t}\rho_i\alpha_i)}dx\right]ds $$

And evaluating the inner integral by using a integral formula (7) given by F.Oberhettinger [6] and we get

$$ =2^{1-\mu}a^{\mu-\lambda-\sum_{i=1}^{t}\rho_i\alpha_i}\Gamma(2\mu)\sum_{\alpha_1=0}^{\left[\frac{n_1}{m_1}\right]},\ldots,\sum_{\alpha_t=0}^{\left[\frac{n_t}{m_t}\right]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!},\ldots,\frac{(-n_t)_{m_t\alpha_t}}{\alpha_t!}.A[n_1,\alpha_1;\ldots;n_t,\alpha_t].z_1^{\alpha_1},\ldots,z_t^{\alpha_t}. $$

$$ \frac{1}{2\pi i}\int_L\left(\frac{y}{wa}\right)^{ks}\theta(s)\ \frac{\Gamma(\lambda+ks+\sum_{i=1}^{t}\rho_i\alpha_i+1)\Gamma(\lambda+ks+\sum_{i=1}^{t}\rho_i\alpha_i-\mu)}{\Gamma(\lambda+ks+\sum_{i=1}^{t}\rho_i\alpha_i)\Gamma(1+\lambda+ks+\sum_{i=1}^{t}\rho_i\alpha_i+\mu)}ds $$

Now using definition of I-function (1) and (2) we get the desired result.

SPECIAL CASES

Corollary 1.1: Let the condition of theorem 1 be satisfied and set $r=1$, the equation (8) reduces to

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda} H_{p,q}^{m,n}\left[\frac{y^k}{w^k\left(x+a+\sqrt{x^2+2ax}\right)^k}\middle|\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right]$$
$$S_{n_1,\dots,n_t}^{m_1,\dots,m_t}\left[z_1\left(x+a+\sqrt{x^2+2ax}\right)^{-\rho_1},\dots,z_t\left(x+a+\sqrt{x^2+2ax}\right)^{-\rho_t}\right]dx$$

$$=2^{1-\mu}a^{\mu-\lambda-\sum_{i=1}^t\rho_i\alpha_i}\Gamma(2\mu)\sum_{\alpha_1=0}^{[n_1/m_1]},\dots,\sum_{\alpha_t=0}^{[n_t/m_t]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!},\dots,\frac{(-n_t)_{m_t\alpha_t}}{\alpha_t!}.A[n_1,\alpha_1;\dots;n_t,\alpha_t]z_1^{\alpha_1},\dots,z_t^{\alpha_t}.$$
$$H_{p+2,q+2}^{m,n+2}\left[\left(\frac{y}{wa}\right)^k\middle|\begin{matrix}(-\lambda-\sum_{i=1}^t\rho_i\alpha_i,k);(1-\lambda+\mu-\sum_{i=1}^t\rho_i\alpha_i,k);(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q};(1-\lambda-\sum_{i=1}^t\rho_i\alpha_i,k);(-\lambda-\mu-\sum_{i=1}^t\rho_i\alpha_i,k)\end{matrix}\right]\tag{9}$$

Where $H_{p,q}^{m,n}[z]$ is well known H-function [3].

Corollary 1.2: If we take $m_1,\dots,m_t\to m$ and $n_1,\dots,n_t\to n$ i.e. $(1,\dots,t)\to 1$ in the integral (8) and let the condition of theorem 1 be satisfied, we arrive at the following result which is obtained by Mishra & Pandey [5].

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda} I_{p_i,q_i;r}^{m,n}\left[\frac{y^k}{w^k\left(x+a+\sqrt{x^2+2ax}\right)^k}\middle|\begin{matrix}(a_j,\alpha_j)_{1,n},\dots,(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m},\dots,(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right]$$
$$S_n^m\left[z\left(x+a+\sqrt{x^2+2ax}\right)^{-\rho}\right]dx$$

$$=2^{1-\mu}a^{\mu-\lambda-\rho\alpha}\Gamma(2\mu)\sum_{\alpha=0}^{[n/m]}\frac{(-n)_{m\alpha}}{\alpha!}.A[n,\alpha]z^\alpha$$
$$I_{p_i+2,q_i+2;r}^{m,n+2}\left[\left(\frac{y}{wa}\right)^k\middle|\begin{matrix}(-\lambda-\rho\alpha,k);(1-\lambda+\mu-\rho\alpha,k);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(1-\lambda-\rho\alpha,k);(-\lambda-\mu-\rho\alpha,k)\end{matrix}\right]\tag{10}$$

Corollary 1.3: If we take $r=1$ and $m_1,\dots,m_t\to m$ and $n_1,\dots,n_t\to n$ i.e. $(1,\dots,t)\to 1$ in the integral (8) and let the condition of theorem 1 be satisfied, we arrive at the following result which is obtained by Garg and Mittal [4].

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda} H_{p,q}^{m,n}\left[\frac{y^k}{w^k\left(x+a+\sqrt{x^2+2ax}\right)^k}\middle|\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right]S_n^m\left[z\left(x+a+\sqrt{x^2+2ax}\right)^{-\rho}\right]dx$$
$$=2^{1-\mu}a^{\mu-\lambda}\Gamma(2\mu)\sum_{\alpha=0}^{[n/m]}\frac{(-n)_{m\alpha}}{\alpha!}.A[n,\alpha]z^\alpha a^{-\rho\alpha}H_{p+2,q+2}^{m,n+2}\left[\left(\frac{y}{wa}\right)^k\middle|\begin{matrix}(-\lambda-\rho\alpha,k);(1-\lambda+\mu-\rho\alpha,k);(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q};(1-\lambda-\rho\alpha,k);(-\lambda-\mu-\rho\alpha,k)\end{matrix}\right]\tag{11}$$

Corollary 1.4: If we take $r=1$ and $S_{n_1,\dots,n_t}^{m_1,\dots,m_t}\to 1$ in the integral (8) and let the condition of theorem 1 be satisfied, we arrive at the following result which is obtained by Chouhan and Khan [2].

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda} H_{p,q}^{m,n}\left[\frac{y^k}{w^k\left(x+a+\sqrt{x^2+2ax}\right)^k}\middle|\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right]dx$$
$$=2^{1-\mu}a^{\mu-\lambda}\Gamma(2\mu)H_{p+2,q+2}^{m,n+2}\left[\left(\frac{y}{wa}\right)^k\middle|\begin{matrix}(-\lambda,k);(1-\lambda+\mu,k);(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q};(1-\lambda,k);(-\lambda-\mu,k)\end{matrix}\right]\tag{12}$$

Corollary 1.5: If we take $r=1,\alpha_i=\beta_j=1$ and $S_{n_1,\dots,n_t}^{m_1,\dots,m_t}\to 1$ in the integral (8) and let the condition of theorem 1 be satisfied, we arrive at the following result

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda} G_{p,q}^{m,n}\left[\frac{y^k}{w^k\left(x+a+\sqrt{x^2+2ax}\right)^k}\middle|\begin{matrix}(a_i)_{1,p}\\(b_j)_{1,q}\end{matrix}\right]dx$$
$$=2^{1-\mu}a^{\mu-\lambda}\Gamma(2\mu)G_{p+2,q+2}^{m,n+2}\left[\left(\frac{y}{wa}\right)^k\middle|\begin{matrix}(-\lambda,k);(1-\lambda+\mu,k);(a_i)_{1,p}\\(b_j)_{1,q};(1-\lambda,k);(-\lambda-\mu,k)\end{matrix}\right]\tag{13}$$

Where $G_{p,q}^{m,n}[z]$ is well known meijer G-function (fox [3]).

Corollary 1.6: If we take $r=1,S_{n_1,\dots,n_t}^{m_1,\dots,m_t}\to 1,m=1,n=0,p=0,q=2,k=2,w=2,b_1=0,\beta_1=1,b_2=-\nu$, $\beta_2=1,\lambda=\lambda+\nu$ in the integral (8) and let the condition of theorem 1 be satisfied, we arrive at the following result which is obtained by Choi and Agarwal [1].

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda} J_\nu\left[\frac{y}{\left(x+a+\sqrt{x^2+2ax}\right)}\right]dx$$
$$=2^{1-\nu-\mu}a^{\mu-\nu-\lambda}\Gamma(2\mu)\,{}_2\Psi_3\left[\begin{matrix}(1+\lambda+\nu,2);(\lambda+\nu-\mu,2);\\(\lambda+\nu,2);(1+\lambda+\nu+\mu,2);(1+\nu,1);\end{matrix}\ \frac{-y^2}{4a^2}\right]\tag{14}$$

Where $J_v(z)$ is the ordinary Bessel function of first kind [1].

Corollary 1.7: If we take $r = 1, S_{n_1,\dots,n_t}^{m_1,\dots,m_t} \to 1, m = 1, n = 1, p = 1, q = 2, k = 1, w = 1, b_1 = 0, \beta_1 = 1, b_2 = 1 - c$, $\beta_2 = 1, \lambda = \nu, a_1 = 1 - a, \alpha_1 = 1, \mu = \lambda, y = -Y$, in the integral (5) and let the condition of theorem 1 be satisfied, we arrive at the following result which is obtained by Choi and Agarwal [1].

$$\int_0^\infty x^{\lambda-1}\left(x + a + \sqrt{x^2 + 2ax}\right)^{-\nu} {}_1F_1\left[a; c; \frac{Y}{\left(x + a + \sqrt{x^2 + 2ax}\right)}\right] dx$$
$$= 2^{1-\lambda}\left[\frac{\nu a^{\lambda-\nu}\Gamma(2\lambda)\Gamma(\nu-\lambda)}{\Gamma(\nu+\lambda+1)}\right] {}_3F_3\left[\begin{matrix} a, \nu - \lambda, \nu + 1; \ \frac{Y}{a} \\ c, \nu + \lambda + 1, \nu; \end{matrix}\right] \tag{15}$$

3. UNIFIED INTEGRAL INVOLVING M-SERIES WITH THE GENERALIZED POLYNOMAL

We derive the following theorem

Theorem 2: $\int_0^\infty x^{\mu-1}\left(x + a + \sqrt{x^2 + 2ax}\right)^{-\lambda} {}_p^{\alpha}M_q^{\beta}\left[\frac{y^k}{w^k\left(x+a+\sqrt{x^2+2ax}\right)^k}\right]$

$$S_{n_1,\dots,n_t}^{m_1,\dots,m_t}\left[z_1\left(x + a + \sqrt{x^2 + 2ax}\right)^{-\rho_1}, \dots, z_t\left(x + a + \sqrt{x^2 + 2ax}\right)^{-\rho_t}\right] dx$$
$$= 2^{1-\mu} a^{\mu-\lambda-\sum_{i=1}^t \rho_i\alpha_i}\Gamma(2\mu) \sum_{\alpha_1=0}^{[n_1/m_1]}, \dots, \sum_{\alpha_t=0}^{[n_t/m_t]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}, \dots, \frac{(-n_t)_{m_t\alpha_t}}{\alpha_t!}. A[n_1, \alpha_1; \dots; n_t, \alpha_t]. z_1^{\alpha_1}, \dots, z_t^{\alpha_t}. \frac{\prod_{j=1}^q \Gamma(b_j)}{\prod_{j=1}^p \Gamma(a_j)}$$
$$H_{p+3,q+4}^{1,p+3}\left[-\left(\frac{y}{wa}\right)^k \middle| \begin{matrix} (-\lambda - \sum_{i=1}^t \rho_i\alpha_i, k); (1 - \lambda + \mu - \sum_{i=1}^t \rho_i\alpha_i, k); (0,1); (1 - a_j, 1)_{1,p} \\ (0,1); (1 - b_j, 1)_{1,q} (1 - \lambda - \sum_{i=1}^t \rho_i\alpha_i, k); (-\lambda - \mu - \sum_{i=1}^t \rho_i\alpha_i, k); (1 - \beta, \alpha) \end{matrix}\right] \tag{16}$$

$(a_j)_n, (b_j)_n$ are know pochammer symbols and parameters $b_j's, j = 1,2, \dots, q$ is a negative integer or zero; if any numerator parameter a_j is a negative integer or zero, then the series terminates to a polynomial in z.

(1) $n_i = 0,1,2, \dots \forall\ i = (1, \dots, t), m_1, \dots, m_t$ arbitrary positive integers and the coefficient $A[n_1, \alpha_1; \dots; n_t, \alpha_t]$ are arbitrary constants, real or complex.
(2) $k > 0,\ Re(\mu, \lambda, \rho) > 0.$
(3) $Re(\mu) - Re(\lambda) - k\ min_{1 \le j \le m} Re\left[\frac{b_j}{\beta_j}\right] < 0.$

Proof: Using the generalized M-series as the function defined by means of the power series in given by (4) and the definition of a generalized Polynomials given by (3), then interchanging the order of summation and integration, we obtain L.H.S. of (16) as

$$= \sum_{\alpha_1=0}^{\left[\frac{n_1}{m_1}\right]}, \dots, \sum_{\alpha_t=0}^{\left[\frac{n_t}{m_t}\right]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}, \dots, \frac{(-n_t)_{m_t\alpha_t}}{\alpha_t!}. A[n_1, \alpha_1; \dots; n_t, \alpha_t]. z_1^{\alpha_1}, \dots, z_t^{\alpha_t}.$$
$$\sum_{n=0}^\infty \frac{(a_1)_n, \dots, (a_p)_n}{(b_1)_n, \dots, (b_q)_n}. \frac{\left(\frac{y}{w}\right)^{kn}}{\Gamma(\alpha n + \beta)} \int_0^\infty x^{\mu-1}\left(x + a + \sqrt{x^2 + 2ax}\right)^{-(\lambda+kn+\sum_{i=1}^t \rho_i\alpha_i)} dx$$

And evaluating the inner integral by using a integral formula (7) given by F.Oberhettinger [6] and we get

$$= 2^{1-\mu} a^{\mu-\lambda-\sum_{i=1}^t \rho_i\alpha_i}\Gamma(2\mu) \sum_{\alpha_1=0}^{\left[\frac{n_1}{m_1}\right]}, \dots, \sum_{\alpha_t=0}^{\left[\frac{n_t}{m_t}\right]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}, \dots, \frac{(-n_t)_{m_t\alpha_t}}{\alpha_t!}. A[n_1, \alpha_1; \dots; n_t, \alpha_t]. z_1^{\alpha_1}, \dots, z_t^{\alpha_t}$$
$$\sum_{n=0}^\infty \frac{(a_1)_n, \dots, (a_p)_n}{(b_1)_n, \dots, (b_q)_n}. \frac{\left(\frac{y}{aw}\right)^{kn}}{\Gamma(\alpha n + \beta)} \frac{\Gamma(\lambda + kn + \sum_{i=1}^t \rho_i\alpha_i + 1)\Gamma(\lambda + kn + \sum_{i=1}^t \rho_i\alpha_i - \mu)}{\Gamma(\lambda + kn + \sum_{i=1}^t \rho_i\alpha_i)\Gamma(1 + \lambda + kn + \sum_{i=1}^t \rho_i\alpha_i + \mu)}$$

Now using definition of H-function (5) we get the desired result.

SPECIAL CASES

Corollary 2.1: Let the condition of theorem 2 be satisfied and set $p = 1, q = 1$,
$b_j = 1$ the generalized M-series convert in generalized Mittag-leffler function (6) introduced by Prabhakar [7] the equation (16) reduces to

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda} E_{\alpha,\beta}^{\gamma}\left[\frac{y^k}{w^k\left(x+a+\sqrt{x^2+2ax}\right)^k}\right] S_{n_1,\dots,n_t}^{m_1,\dots,m_t}\left[z_1\left(x+a+\sqrt{x^2+2ax}\right)^{-\rho_1},\dots,z_t\left(x+a+\sqrt{x^2+2ax}\right)^{-\rho_t}\right]dx$$

$$= 2^{1-\mu}a^{\mu-\lambda-\sum_{i=1}^{t}\rho_i\alpha_i}\Gamma(2\mu)\sum_{\alpha_1=0}^{[n_1/m_1]},\dots,\sum_{\alpha_t=0}^{[n_t/m_t]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!},\dots,\frac{(-n_t)_{m_t\alpha_t}}{\alpha_t!}.A[n_1,\alpha_1;\dots;n_t,\alpha_t].z_1^{\alpha_1},\dots,z_t^{\alpha_t}.\frac{1}{\Gamma\gamma}.$$
$$H_{4,5}^{1,4}\left[-\left(\frac{y}{wa}\right)^k\left|\begin{matrix}(-\lambda-\sum_{i=1}^{t}\rho_i\alpha_i,k);(1-\lambda+\mu-\sum_{i=1}^{t}\rho_i\alpha_i,k);(0,1);(1-\gamma,1)\\(0,1);(0,1);\ (1-\lambda-\sum_{i=1}^{t}\rho_i\alpha_i,k);(-\lambda-\mu-\sum_{i=1}^{t}\rho_i\alpha_i,k);(1-\beta,\alpha)\end{matrix}\right.\right] \quad (17)$$

Where $H_{p,q}^{m,n}[z]$ is well known H-function [3].

Corollary 2.2: If we take $m_1,\dots,m_t\to m$ and $n_1,\dots,n_t\to n$ i.e. $(1,\dots,t)\to 1$ in the integral (16) and let the condition of theorem 2 be satisfied, we obtained

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda}\ {}_p^{\alpha}M_q^{\beta}\left[\frac{y^k}{w^k\left(x+a+\sqrt{x^2+2ax}\right)^k}\right].S_n^m\left[z\left(x+a+\sqrt{x^2+2ax}\right)^{-\rho}\right]dx$$
$$= 2^{1-\mu}a^{\mu-\lambda-\rho\alpha}\Gamma(2\mu)\sum_{\alpha=0}^{[n/m]}\frac{(-n)_{m\alpha}}{\alpha!}.A[n,\alpha]z^{\alpha}\ \frac{\prod_{j=1}^{q}\Gamma(b_j)}{\prod_{j=1}^{p}\Gamma(a_j)}.$$
$$H_{p+3,q+4}^{1,p+3}\left[-\left(\frac{y}{wa}\right)^k\left|\begin{matrix}(-\lambda-\rho\alpha,k);(1-\lambda+\mu-\rho\alpha,k);(0,1);\left(1-a_j,1\right)_{1,p}\\(0,1);\left(1-b_j,1\right)_{1,q}(1-\lambda-\rho\alpha,k);(-\lambda-\mu-\rho\alpha,k);(1-\beta,\alpha)\end{matrix}\right.\right] \quad (18)$$

Corollary 2.3: If we take $m_1,\dots,m_t\to m$ and $n_1,\dots,n_t\to n$ i.e. $(1,\dots,t)\to 1$ and set $p=1,q=1,\ a_j=\gamma,b_j=1$ the generalized M-series convert in generalized Mittag-leffler function (6) introduced by Prabhakar [7] in the integral (16) and let the condition of theorem 2 be satisfied, we obtained

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda} E_{\alpha,\beta}^{\gamma}\left[\frac{y^k}{w^k\left(x+a+\sqrt{x^2+2ax}\right)^k}\right]S_n^m\left[z\left(x+a+\sqrt{x^2+2ax}\right)^{-\rho}\right]dx$$
$$= 2^{1-\mu}a^{\mu-\lambda-\rho\alpha}\Gamma(2\mu)\sum_{\alpha=0}^{\left[\frac{n}{m}\right]}\frac{(-n)_{m\alpha}}{\alpha!}.A[n,\alpha]z^{\alpha}\ .\frac{1}{\Gamma\gamma}.$$
$$H_{4,5}^{1,4}\left[-\left(\frac{y}{wa}\right)^k\left|\begin{matrix}(-\lambda-\rho\alpha,k);(1-\lambda+\mu-\rho\alpha,k);(0,1);(1-\gamma,1)\\(0,1);(0,1);\ (1-\lambda-\rho\alpha,k);(-\lambda-\mu-\rho\alpha,k);(1-\beta,\alpha)\end{matrix}\right.\right] \quad (19)$$

Corollary 2.4: If we take $S_{n_1,\dots,n_t}^{m_1,\dots,m_t}\to 1$ in the integral (6) and let the condition of theorem 1 be satisfied, we arrive at the following result which is obtained by Chouhan and Khan [2].

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda}\ {}_p^{\alpha}M_q^{\beta}\left[\frac{y^k}{w^k\left(x+a+\sqrt{x^2+2ax}\right)^k}\right]d\ = 2^{1-\mu}a^{\mu-\lambda}\Gamma(2\mu).\frac{\prod_{j=1}^{q}\Gamma(b_j)}{\prod_{j=1}^{p}\Gamma(a_j)}$$
$$H_{p+3,q+4}^{1,p+3}\left[-\left(\frac{y}{wa}\right)^k\left|\begin{matrix}(-\lambda,k);(1-\lambda+\mu,k);(0,1);\left(1-a_j,1\right)_{1,p}\\(0,1);\left(1-b_j,1\right)_{1,q}(1-\lambda,k);(-\lambda-\mu,k);(1-\beta,\alpha)\end{matrix}\right.\right] \quad (20)$$

Corollary 2.5: If we take $S_{n_1,\dots,n_t}^{m_1,\dots,m_t}\to 1$ and set $p=1,q=1,\ a_j=\gamma,b_j=1$ the generalized M-series convert in generalized Mittag-leffler function (6) introduced by Prabhakar [7] in the integral (16) and let the condition of theorem 2 be satisfied, we arrive at the following result

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda} E_{\alpha,\beta}^{\gamma}\left[\frac{y^k}{w^k\left(x+a+\sqrt{x^2+2ax}\right)^k}\right]dx$$
$$= 2^{1-\mu}a^{\mu-\lambda-\rho\alpha}\Gamma(2\mu)\ .\frac{1}{\Gamma\gamma}.H_{4,5}^{1,4}\left[-\left(\frac{y}{wa}\right)^k\left|\begin{matrix}(-\lambda,k);(1-\lambda+\mu,k);(0,1);(1-\gamma,1)\\(0,1);(0,1);\ (1-\lambda,k);(-\lambda-\mu,k);(1-\beta,\alpha)\end{matrix}\right.\right] \quad (21)$$

4. CONCLUSION

In this paper we have presented generalized integral formulas involving Sexana I- function and M-series with generalized polynomials. The result so established may be found useful in several interesting situation appearing in the literature on mathematical analysis. Further many known and unknown results have been established in terms of special cases. The results presented in this paper are easily converted in terms of the Fox *H*-function, generalized Mittag-leffler function, G-function, Bessel function and hyper geometric function after some suitable parametric replacement. We are also trying to find certain possible applications of those results presented here to some other research areas.

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