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NUMERICAL STUDY OF NON-NEWTONIAN JEFFREY FLUID FLOW IN AN INCLINED VERTICAL PLATE

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ABSTRACT

The non-linear steady state boundary layer flow, heat transfer of an incompressible non-Newtonian Jeffery's fluid from an inclined vertical plate is considered in this study. The transformed conservation equations are solved numerically subject to physically appropriate boundary conditions using a versatile, implicit, finite-difference technique. The influence of non-dimensional parameters, namely Deborah number (De), Prandtl number (Pr), ratio of relaxation to retardation times (λ) and dimensionless tangential coordinate (ξ) on velocity, temperature evolution in the boundary layer region are examined in details. It is observed that the velocity is reduced when Deborah number increases. where as temperature is enhanced. Increasing λ enhances the velocity, but temperature reduced.

Keywords: *non-Newtonian Jeffrey's Fluid; Heat transfer; Keller Box finite difference method; inclined plate.*

I. INTRODUCTION

In the recent years, the investigation of the flow of heat and mass transfer of a non-Newtonian fluid has gained considerable attention because of its extensive engineering applications. As a sub class of a non-Newtonian fluid, Casson fluid model is found to be good in representing the pseudo-plastic and rheological behaviour of the fluid. Several fluids in chemical engineering, multiphase mixtures, pharmaceutical formulations, paints, synthetic lubricants, jams, soups, jellies, sewage sludge etc. are non-Newtonian. During the last few years, there has been an increasing interest of non-Newtonian fluids due to the applications in science and engineering including thermal oil recovery, the plastic manufacture, food and slurry transportation, performance of lubricants, polymer and food processing etc. A variety of non-Newtonian fluid models have been proposed in the literature keeping in view of their several rheological features. In these fluids, the constitution relationships between stress and rate of strain are much complicated in comparison to the Navier-stokes equations. There is one subclass of non-Newtonian fluids known as the Jeffery fluid [1-6] which has investigated considerably in recent years in view of its simplicity. This fluid model is capable of describing the characteristics of relaxation and retardation times.

Recently, Zeeshan and Majeed [7] performed of the flow and heat transfer of Jeffery fluid past a linearly stretching sheet with the presence of a magnetic dipole. Unsteady natural convection flow through a fluid-saturated porous medium of a viscous, incompressible, fluid past an impulsively moving semi-infinite vertical plate with MHD and convective surface boundary condition is carried out by Seth [8]. Most of the studies are related to vertical plate, the articles on the heat transfer in non-Newtonian Jeffrey's flow from an inclined vertical plate are very limited. Boundary Layer Flows of non-Newtonian fluid from an inclined vertical plate in the presence of hydrodynamic and thermal slip was the base of investigation by Subba Rao *et.al.* [9] and he has used the Keller Box finite difference method. One subclass of non-Newtonian fluids known as the *Jeffery fluid* [10] is particularly useful owing to its simplicity. This fluid model is capable of describing the characteristics of relaxation and retardation times which arise in complex polymeric flows. Furthermore the Jeffrey type model utilizes time derivatives rather than convected derivatives, which make it more amenable for numerical simulations. Recently the Jeffery model has received considerable attention. Interesting studies employing this model include peristaltic magnetohydrodynamic non-Newtonian flow [11], variable-viscosity peristaltic flow [12], convective-radiative flow in porous media [13] and stretching sheet flows [14, 15].

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In the present paper, we investigate analytical solution for two dimensional incompressible viscoelastic Jeffrey's non-Newtonian fluid flow from an inclined vertical plate. Numerical solutions for the velocity and the temperature distributions are obtained using a powerful technique namely Keller-Box finite difference method. The graphs are plotted and discussed for the variations of different involved parameters.

II. MATHEMATICAL FLOW MODEL

We considered the steady, two-dimensional, incompressible boundary layer flow, heat transfer of a Jeffrey's fluid from an inclined vertical plate and as illustrated in F**igure 1.** Both the plate and the Jeffrey's fluid are maintained at a constant temperature. Instantaneously they are raised to a temperature $T_w > T_\infty$, the ambient temperature of the fluid which remains unchanged. The x-coordinate (tangential) is measured from the leading edge of the plate and y-coordinate (radial) is measured normal to the plate.

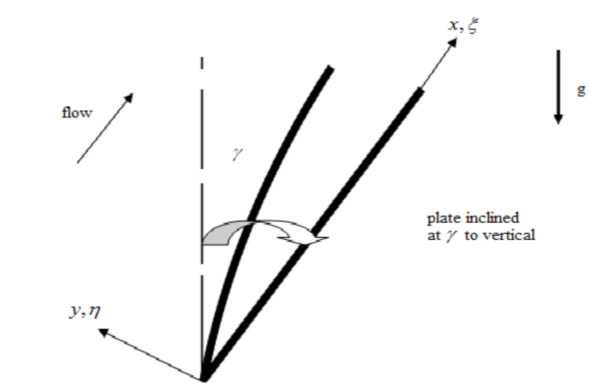

Figure 1: Physical model and coordinate

The corresponding velocities in the x and y directions are u and v respectively. The governing conservation equations can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta\left(T-T_\infty\right)\cos\gamma+\frac{\nu}{1+\lambda}\left[\frac{\partial^2 u}{\partial y^2}+\lambda_1\left(u\frac{\partial^3 u}{\partial x\partial y^2}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}\right)\right] \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

The boundary conditions are prescribed at the surface and the edge of the boundary layer regime, respectively as follows:

$$u=0,\ v=0,\ T=T_w,\ at\ y=0 \tag{4}$$

$$u\to u_\infty,\ T=T_\infty,\ as\ y\to\infty$$

The stream function ψ is defined by $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$, and therefore, the continuity equation is automatically satisfied. In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\eta=y\left(\frac{u_\infty}{\nu x}\right)^{1/2},\xi=\xi(x),\ f(\xi,x)=\frac{\psi(x,y)}{(\nu u_\infty x)^{1/2}},Gr=\frac{g\beta(T_w-T_\infty)x^3}{\nu^2},\ De=\frac{\lambda_1 U_\infty}{x},Pr=\frac{\nu}{\alpha}, \tag{5}$$

$$Re_x=\frac{U_\infty x}{\nu},R_i=\frac{Gr}{Re^2{}_x},B=R_i\operatorname{Cos}\gamma,\theta(\xi,x)=\frac{(T-T_\infty)}{(T-T_w)}$$

Where u and v are the velocity components in the x - and y - directions respectively, $\nu=\frac{\mu}{\rho}$ - the kinematic viscosity of the conducting fluid, β - is the coefficient of thermal expansion, α - the thermal diffusivity, k - the thermal conductivity, γ - inclination of the plate to the vertical, ρ - is the density of the fluid, c_p – the specific heat at constant pressure, λ - the ratio of relaxation to retardation times, λ_1 - the retardation time. $\Pr$ - Prandtl number, Gr - Grashof number, De - Deborah number, T_∞ - the free stream temperature, R_i - Mixed convection parameter, Re_x - Local Reynolds number respectively.

In view of eqn. (5), eqns. (2) - (3) reduce to the following coupled, nonlinear, dimensionless partial differential equations for momentum and energy for the regime:

$$\frac{1}{1+\lambda} f''' + \frac{1}{2} ff'' - \frac{De}{1+\lambda}\left(ff''' + \frac{1}{2} f''^2 - \frac{1}{2} ff^{iv} \right) + B\xi\theta = \xi\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi} - \frac{De}{1+\lambda}\left(f'\frac{\partial f'''}{\partial \xi} - f'''\frac{\partial f'}{\partial \xi} + f''\frac{\partial f''}{\partial \xi} - f^{iv}\frac{\partial f}{\partial \xi} \right)\right) \tag{6}$$

$$\frac{\theta''}{\Pr} + \frac{1}{2} f\theta' = \xi\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi} \right) \tag{7}$$

The dimensionless form of the boundary conditions are:

$$\text{At } \eta = 0, \quad f = 0, \quad f' = 0, \quad \theta = 1$$
$$\text{As } \eta \to \infty, \quad f' = 1, \quad \theta = 0 \tag{8}$$

The engineering design quantities of physical interest include the skin-friction coefficient and Nusselt number, which are given by:

$$\frac{1}{2} C_f \mathrm{Re}_x^{-1/2} = f''(\xi, 0) \tag{9}$$

$$\frac{Nu}{\sqrt{\mathrm{Re}_x}} = -\theta'(\xi, 0) \tag{10}$$

III. NUMERICAL SOLUTION

In this study, the efficient Keller-Box implicit difference method has been employed to solve the general flow model defined by equations (6) – (7) with boundary conditions (8).This method was originally developed for low speed aerodynamic boundary layers and this system is developed by Keller [16]. These include Casson slip boundary layer flows [17] and many other flows by [18-26]. This method remains among the most powerful, versatile and accurate computational finite difference schemes employed in modern viscous fluid dynamics simulations. This method has been used extensively and effectively for over three decades in a large spectrum of nonlinear fluid mechanics problems. Keller's method provides unconditional stability and rapid convergence for strongly non-linear flows. It involves four key stages, summarized below.

1) Reduction of the N^{th} order partial differential equation system to N first order equations
2) Finite difference discretization of reduced equations
3) Quasilinearization of non-linear Keller algebraic equations
4) Block-tridiagonal elimination of linearized Keller algebraic equations

IV. RESULTS AND DISCUSSIONS

Comprehensive solutions have been obtain and are presented in **Figs.2-5.** The numerical problem comprises two independent variables (ξ,η), two dependent fluid dynamic variables $\left(f, \theta \right)$ and 5 thermo-physical and body force control parameters, namely $\Pr$, De, λ, ξ and γ . The following default parameter values *i.e.* $\Pr = 0.71, De = 0.1, \lambda = 0.2, \xi = 1.0, \gamma = 70^0$ are prescribed (unless otherwise stated).

Figures 2(a)-2(b) illustrates the effect of the ratio of relaxation to retardation times i.e. λ on the velocity and temperature distributions through the boundary layer regime. Velocity is significantly decreased with increasing λ, in particular close to the plate surface. The polymer flow is therefore considerably *decelerated* with an increase in relaxation time (or decrease in retardation time). Conversely temperature is depressed slightly with increasing values of λ. The mathematical model reduces to the *Newtonian viscous flow model* as $\lambda \to 0$ and $De \to 0$, since this negates relaxation, retardation and elasticity effects. The thermal boundary layer equation (6) remains unchanged. Effectively with greater relaxation time of the polymer the *thermal boundary layer thickness* is reduced. However with greater relaxation times, the *momentum boundary layer thickness* is only decreased *near* the plate surface whereas further away it is enhanced since the flow is strongly accelerated there.

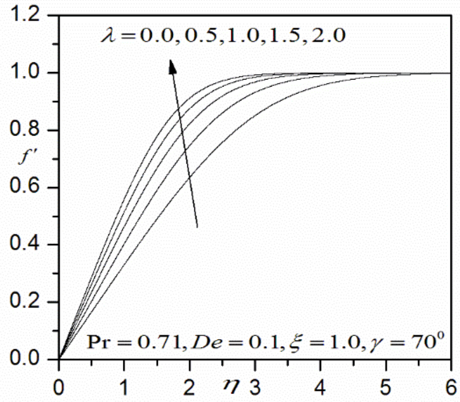

Fig 2(a): Influence of λ on velocity profiles

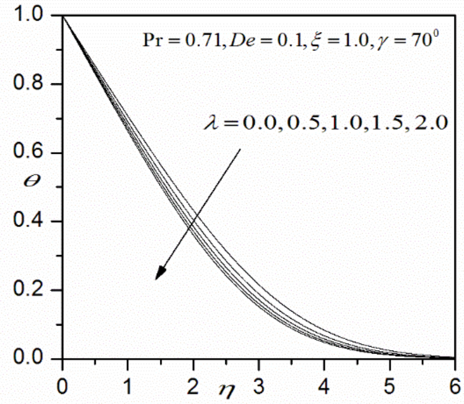

Fig 2(b): Influence of λ on temperature profiles

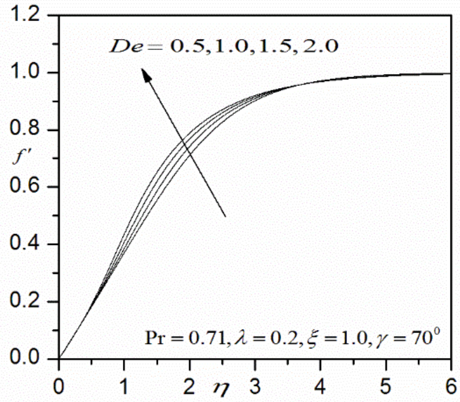

Fig 3(a): Influence of De on velocity profiles

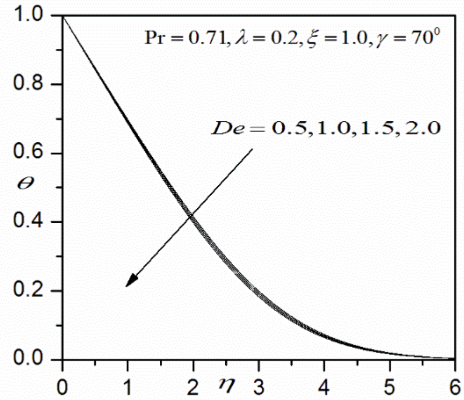

Fig 3(b): Influence of De on temperature profiles

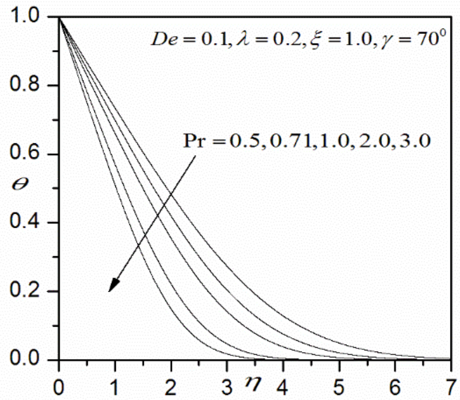

Fig 4: Influence of Pr on temperature profiles

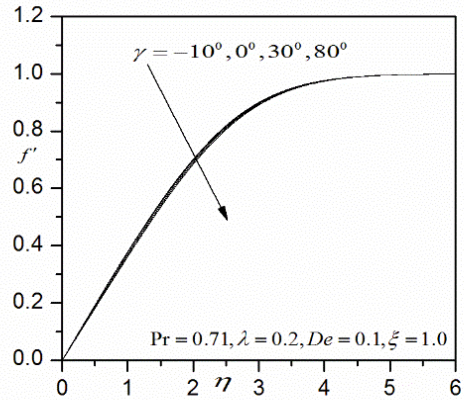

Fig 5(a): Influence of γ on velocity profiles

In Figures 3(a)–3(b), the evolution of velocity and temperature functions with a variation in Deborah number, *De*, is depicted. Dimensionless velocity component (fig. 3a) is considerably reduced with increasing *De* near the cylinder surface and for some distance into the boundary layer. *De* clearly arises in connection with some high order derivatives in the momentum boundary layer equation, (6) i.e. $\dfrac{De}{1+\lambda}[f^{//2} - ff^{iv}]$ and also

$$\xi\left(-\frac{De}{1+\lambda}\left[f^{/}\frac{\partial f^{///}}{\partial\xi} - f^{///}\frac{\partial f^{/}}{\partial\xi} + f^{//}\frac{\partial f^{//}}{\partial\xi} - f^{iv}\frac{\partial f}{\partial\xi}\right]\right).$$

It therefore is intimately associated with the *shearing characteristics* of the polymer flow. For polymers, larger *De* values imply that the polymer becomes *highly oriented* in one direction and stretched. Generally this arises when the polymer takes longer to relax in comparison with the rate at which the flow is deforming it. When such fluids are stretched there is a delay in their return to the unstressed state. For very large Deborah numbers, the fluid movement is too fast for elastic forces to relax and the material then acts like a purely elastic solid. Large Deborah numbers are therefore not relevant to the present simulations. For small Deborah numbers, the time scale of fluid movement is much greater than the relaxation time of elastic forces in the polymer and the polymer then behaves as a *simple viscous fluid*, as elaborated by Bég and Makinde [27]. Vrentas *et al.* [28] have also indicated that the Deborah number can be utilized in characterizing diffusional transport in amorphous polymer-solvent systems. Further from the cylinder surface we observe that there is a slight increase in velocity i.e. the flow is *accelerated* with increasing Deborah number. With greater distance from the solid boundary, the polymer is therefore assisted in flowing even with higher elastic effects. Clearly the responses in the near-wall region and far-field region are very different. In fig. 3b, an increase in Deborah number is seen to considerably enhance temperatures throughout the boundary layer regime. This has also been observed by Hayat *et al.* [29]. Thermal boundary layer thickness is also elevated with increasing Deborah number.

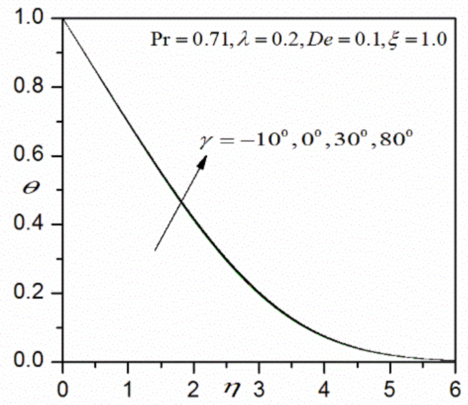

Fig 5(b): Influence of γ on Temperature Profiles

Figure 4 depict the profile for temperature distribution for various values of Prandtl number, Pr. It is observed that an increase in the Prandtl number the temperature reduced.

Figures 5(a)-5(b) presents the influence of the plate inclination on the dimensionless velocity and temperature. When $\gamma < 0$ i.e. negative plate inclination, in Figure 5a, the velocity is reduced. Conversely in Figure 5b, with negative plate inclination $\gamma < 0$ the temperature decreases slightly. Further temperatures are increased marginally with positive inclination of the plate.

V. CONCLUSIONS

A mathematical model has been developed for boundary layer flow of a Jeffrey's non-Newtonian fluid from an inclined vertical plate. The transformed conservation equations have been solved with prescribed boundary conditions using implicit Keller-box finite difference method. The present simulations have shown that:

1. Increasing the Deborah number (De), reduces the velocity, skin friction- $f''(\xi,0)$ and heat transfer rate $\left(-\theta'(\xi,0)\right)$ whereas it enhances temperature.
2. Increasing the parameter ratio of relaxation and retardation times (λ), increases velocity, skin friction coefficient $f''(\xi,0)$, heat transfer rate $\left(-\theta'(\xi,0)\right)$ it reduces whereas temperature for all values of radial coordinate.

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